your name_____

Physics 321 Practice Exam #2 - Wednesday, Nov. 19

FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$\begin{aligned} x &= A_1 e^{-\beta t}\cos\omega' t + A_2 e^{-\beta t}\sin\omega' t \;\; \omega' = \sqrt{\omega_0^2 - \beta^2} \quad \text{(under damped)} \\ x &= Ae^{-\beta t} + Bte^{-\beta t}, \quad \text{(critically damped)} \\ x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).} \end{aligned}$$

Coriolis and centrifugal forces

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}_{\text{real}} - m\vec{\omega}\times\vec{\omega}\times\vec{r} - 2m\vec{\omega}\times\vec{v}.$$

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1. A small particle of mass m is aimed at a heavy target. They are attracted by a potential,

   $$V(r) = -\frac{\alpha}{r^4}.$$

   Find the cross sectional area for impacting the origin if the incoming energy of the particle is E.

2. A particle is in a circular orbit with angular velocity $\dot{\theta}$ due to a potential

   $$V(r) = V_0 \ln(r/a).$$

   If the radius is given a small perturbation, what is the frequency ω with which the particle's radius oscillates about the original value?

3. A particle is fired directly upward from a point on the equator with muzzle velocity $v_0 = 500$ m/s. Neglecting air resistance, Where does the particle land relative to the firing point. Take into account the Coriolis force.

4. Additionally, one problem on the midterm will be a reprise from a previous quiz or midterm.