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Physics 321 Midterm #2 - Wednesday, Nov. 9, 2022

Some integrals:

$$\begin{aligned}
\int \frac{dx}{1+x^2} &= \tan^{-1}(x),\\
\int \frac{dx}{1-x^2} &= \tanh^{-1}(x),\\
\int \frac{dx}{\sqrt{1-x^2}} &= \sin^{-1}(x),\\
\int \frac{dx}{\sqrt{1+x^2}} &= \sinh^{-1}(x),\\
\int dx\ \tan(x+\phi) &= -\ln(\cos(x+\phi))\\
\int dx\ \tanh(x+a) &= \ln(\cosh(x+a)).
\end{aligned}$$

$$\begin{aligned}
F(t) &= \frac{f_0}{2} + \sum_n f_n \cos(n\omega t) + g_n \sin(n\omega t),\\
f_n &= \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} dt\ F(t)\cos(2n\pi t/\tau),\\
g_n &= \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} dt\ F(t)\sin(2n\pi t/\tau).
\end{aligned}$$

For the equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

The solutions are:

$$\begin{aligned}
x &= Ae^{i\omega t},\\
\omega &= i\beta \pm \sqrt{\omega_0^2 - \beta^2},\\
x(t) &= Ae^{-\beta t}e^{\pm i\sqrt{\omega_0^2-\beta^2}t},\ \text{underdamped},\\
&= Ae^{-\beta t \pm\sqrt{\beta^2-\omega_0^2}t},\ \text{overdamped}.
\end{aligned}$$

Write your name on EVERY page!

80 points possible

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your name______________________________________

1. (15 pts) A mass m is connected to a spring with spring constant $k = m\omega_0^2$ and has a very small damping. The mass also feels a periodic force

$$F(t) = F_0 \sin(\omega t),$$
$$\omega \neq \omega_0.$$

Assuming the force has been applied for a very very long time, find the displacement $x(t)$ for $t > 0$.

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Extra Space for No.1

Solution: After long a long time the homogeneous part of the solution fades away. You need only the particular part.

$$
\begin{aligned}
x &= A\cos\omega t + B\sin\omega t, \\
\ddot{x} + \omega_0^2 x &= \frac{F_0}{m}\sin\omega t, \\
-\omega^2(A\cos\omega t + B\sin\omega t) + \omega_0^2(A\cos\omega t + B\sin\omega t) &= (F_0/m)\sin\omega t, \\
B &= 0, \\
A &= \frac{(F_0/m)}{\omega_0^2 - \omega^2}.
\end{aligned}
$$

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2. A mass m is connected to a spring with spring constant $k = m\omega_0^2$ and has an very small damping constant. It is subject to a periodic force,

$$F(t) = \begin{cases} -F_0, & -\tau/2 < t < 0, \\ F_0, & 0 < t < \tau/2 \end{cases},$$
$$F(t+\tau) = F(t).$$

   (a) (15 pts) Expressing the force as a series

$$F(t) = \frac{f_0}{2} + \sum_n f_n \cos(n\omega t) + g_n \sin(n\omega t),$$
$$\omega = 2\pi/\tau,$$

   what are the coefficients f_n and g_n? Note that $\omega \neq \omega_0$.

   (b) (10 pts) Assuming the force has been active for a very very long time, express the displacement $x(t)$ in terms of the coefficients (as a sum) .

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Extra Space for No.2

Solution: a)

$$\begin{aligned}
f_n &= 0 \;\; \text{integrating odd function gives zero} \\
g_n &= \frac{2}{\tau} 2 \int_0^{\tau/2} dt \;\; \sin(n\omega t) F_0, \\
&= \frac{4F_0}{n\omega\tau}[1 - \cos(n\omega\tau/2)] \\
&= \frac{4F_0}{n\omega\tau}[1 - \cos(n\pi)] \\
&= \begin{cases} \frac{4F_0}{\pi n} & n = 1, 3, 5 \cdots \\ 0 & n = 2, 4, 6 \cdots \end{cases}
\end{aligned}$$

b) By inspecting #1,

$$x(t) = \sum_{n=1,3,5\cdots} \frac{8F_0}{2\pi n m(\omega_0^2 - n^2\omega^2)} \sin(n\omega t).$$

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3. Consider a particle of mass m and energy E which experiences an attractive spherically symmetric potential,

$$V(r) = -\frac{V_0}{r^2}.$$

If the particle reaches $r = 0$ it will annihilate.

(a) (5 pts) Sketch the effective (real plus centrifugal) potential for a particle with angular momentum L. Show potentials for a few values of L to demonstrate how V_{eff} depends on L.

(b) (15 pts) Calculate the cross section for annihilation.

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Extra Space for No.3

a)

b) Criteria for annihilation is

$$\frac{L^2}{2m} < V_0.$$

The impact parameter is related to L by

$$L = bp_{\text{far away}} = b\sqrt{2mE}. \quad (1)$$

The cross section for annihilation is

$$\begin{aligned}
\sigma_{\text{ann.}} &= \pi b^2 = \pi L^2 / p_{\text{far away}}^2 \\
&= \pi \frac{2mV_0}{p_{\text{far away}}^2} \\
&= \pi \frac{V_0}{E}.
\end{aligned}$$

your name____________________________________

4. (20 pts, no points awarded if answer has wrong dimension) You are an experimentalist planning a scattering experiment using a gold target and a much lighter projectile. You go online and find the mass density of gold, ρ_{Au}, and the mass of a gold atom, m_{Au}. Somebody sends you a target, but they didn't tell you the dimensions of the thin foil. You measure the width and length, W and L, then measure the mass of the foil, M, but you can't measure the thickness. The accelerator provides a beam, with N_{beam} particles incident on the foil over the duration of the experiment. One of your detectors is positioned at a scattering angle $\theta_s = 45°$. The detector has a cross-sectional area a and is placed a distance R from the center of the target. Over the six days you record N_{counts} in the detector. What is $d\sigma/d\Omega$ for $\theta_s = 45°$?

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Extra Space for No.4

Solution: Probability of scattering into detector element is

$$\text{Prob} = \frac{N_{\text{target atoms}} d\sigma/d\Omega\delta\Omega}{A}.$$

The number of target atoms per area is

$$\frac{N_{\text{target atoms}}}{A} = \frac{M}{mWL}.$$

So, the Probability is

$$\text{Prob} = \frac{M}{mWL}\frac{d\sigma}{d\Omega}\Delta\Omega.$$

The angular coverage of the detector is

$$\Delta\Omega = \frac{a}{R^2},$$

and the probability is $N_{\text{counts}}/N_{\text{beam}}$. This gives

$$\frac{d\sigma}{d\Omega} = \frac{N_{\text{counts}}}{N_{\text{beam}}}\frac{R^2}{a}\frac{mWL}{M}.$$