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Physics 321 Practice Midterm #2 - Monday, Nov.7, 2022

Some integrals:

$$\begin{aligned}
\int \frac{dx}{1+x^2} &= \tan^{-1}(x),\\
\int \frac{dx}{1-x^2} &= \tanh^{-1}(x),\\
\int \frac{dx}{\sqrt{1-x^2}} &= \sin^{-1}(x),\\
\int \frac{dx}{\sqrt{1+x^2}} &= \sinh^{-1}(x),\\
\int dx\ \tan(x+\phi) &= -\ln(\cos(x+\phi))\\
\int dx\ \tanh(x+a) &= \ln(\cosh(x+a)).
\end{aligned}$$

$$\begin{aligned}
F(t) &= \frac{f_0}{2} + \sum_n f_n\cos(n\omega t) + g_n\sin(n\omega t),\\
f_n &= \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} dt\ F(t)\cos(2n\pi t/\tau),\\
g_n &= \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} dt\ F(t)\sin(2n\pi t/\tau).
\end{aligned}$$

For the equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

The solutions are:

$$\begin{aligned}
x &= Ae^{i\omega t},\\
\omega &= i\beta \pm \sqrt{\omega_0^2-\beta^2},\\
x(t) &= Ae^{-\beta t}e^{\pm i\sqrt{\omega_0^2-\beta^2}t},\ \text{underdamped},\\
&= Ae^{-\beta t\pm\sqrt{\beta^2-\omega_0^2}t},\ \text{overdamped}.
\end{aligned}$$

Write your name on EVERY page!

60 points possible

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1. A particle of mass m is attached to a harmonic oscillator of of spring constant $k = m\omega_0^2$ with a vanishingly small damping. After being at rest for a long time, the particle experiences an impulse delivered over a very small time, with a force depening on time t as

$$F(t) = P_0\delta(t).$$

   (a) (10 pts) Find the displacement $x(t)$.

   (b) (10 pts) Repeat (a) but assume the impulse is not instantaneous, and instead has the form,

$$F(t) = \begin{cases} \frac{P_0}{\tau}, & -\tau < t < \tau, \\ 0, & \text{otherwise} \end{cases}$$

   Find the displacement $x(t)$ for $t > \tau$.

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Extra Space for No.1

Solution: Particle has initial position $x = 0$ and initial momentum P_0 due to impulse. Solution is of form

$$x(t) = A\cos\omega_0 t + B\sin\omega_0 t. \tag{1}$$

$A = 0$ to make $x(t = 0) = 0$ and

$$v(t) = B\omega_0\cos(\omega_0 t), \tag{2}$$

so $B = P_0/(m\omega_0)$,

$$x = \frac{P_0}{m\omega_0}\sin(\omega_0 t). \tag{3}$$

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2. A particle of mass m experiences the potential

$$V(r) = kr,$$

where k is a positive constant.

(a) (10 pts) What is the angular frequency, ω_0 of a circular orbit of radius r_0?

(b) (10 pts) If the particle is given a small radial perturbation, what is the angular frequency, ω, with which the radius oscillates about r_0? Give your answer in terms of ω_0.

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Extra Space for No.2

Solution: From No. 1, you can see that for impulse $F(t')dt'$, the contribution to the displacement is

$$\frac{F(t)dt}{m\omega_0}\sin(\omega_0(t-t')). \tag{4}$$

Summing over all the contributions (principle of superposition,

$$\begin{aligned}
x(t) &= \int_{-\tau}^{\tau} dt' \frac{P_0}{m\omega_0\tau}\sin(\omega_0(t-t')) &(5)\\
&= \frac{P_0}{m\omega_0\tau}\int_{-\tau}^{\tau} dt'[\sin(\omega_0 t)\cos(\omega_0 t') - \cos(\omega_0 t)\sin(\omega_0 t')] &(6)\\
&= \frac{P_0}{m\omega_0\tau}\int_{-\tau}^{\tau} dt'\sin(\omega_0 t)\cos(\omega_0 t') &(7)\\
&= \frac{P_0}{m\omega_0\tau}\int_{-\tau}^{\tau} dt'\sin(\omega_0 t)\cos(\omega_0 t') &(8)\\
&= \sin(\omega_0 t)\frac{2P_0}{m\omega_0^2\tau}\sin(\omega_0\tau). &(9)
\end{aligned}$$

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3. A particle of mass m and energy E scatters off a repulsive potential

$$V = \frac{V_0}{r},$$

where $V_0 > 0$. If the particle reaches a position R it will be absorbed by the target.

(a) (5 pts) Sketch the effective potential (real plus centrifugal) for a particle with angular momentum L. Plot for a few different values of L to show how it depends on L.

(b)

(c) (5 pts) What is the angular momentum of a trajectory that barely grazes the target?

(d) (5 pts) What is the cross section for absorption?

(e) (5 pts) What is the minimum value of E for which there is absorption?

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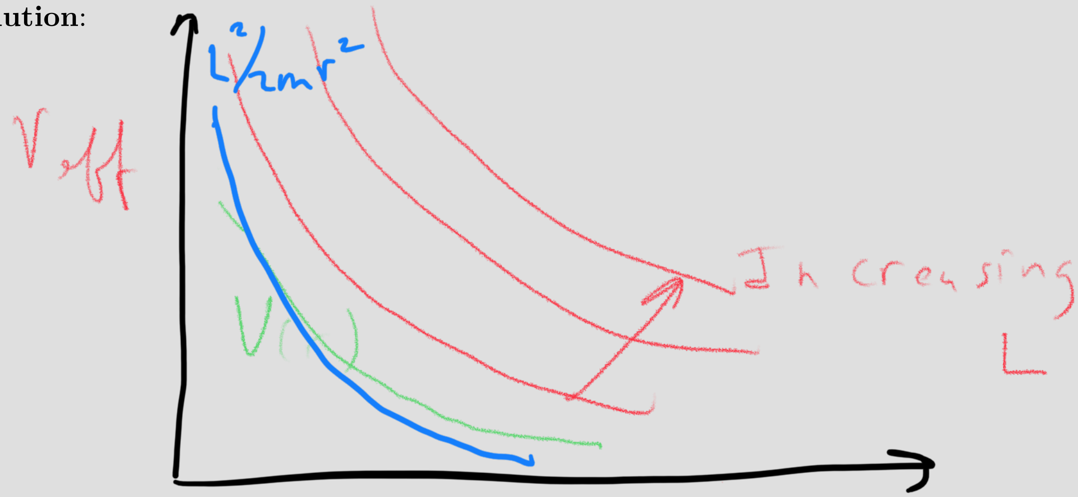
Extra Space for No.3

Solution:

a)



b) For such a trajectory, momentum is transverse at R,

$$L = Rp(r = R) = R\sqrt{2m(E - V_0/r)}. \quad (10)$$

c)

$$bp(r = \infty) = L \quad (11)$$
$$b\sqrt{2mE} = R\sqrt{2m(E - V_0/r)}, \quad (12)$$
$$b = R\sqrt{\frac{E - V_0/r}{E}}, \quad (13)$$
$$\sigma = \pi b^2 = \pi R^2 \frac{E - V_0/r}{E}. \quad (14)$$

d) This is when $b = 0$ and

$$E = \frac{V_0}{r}. \quad (15)$$