your name____

Physics 321 Midterm #1 Practice - Monday, Oct. 3, 2022

Some integrals:

$$\begin{aligned}
\int \frac{dx}{1+x^2} &= \tan^{-1}(x), \\
\int \frac{dx}{1-x^2} &= \tanh^{-1}(x), \\
\int \frac{dx}{\sqrt{1-x^2}} &= \sin^{-1}(x), \\
\int \frac{dx}{\sqrt{1+x^2}} &= \sinh^{-1}(x), \\
\int dx\ \tan(x+\phi) &= -\ln(\cos(x+\phi)) \\
\int dx\ \tanh(x+a) &= \ln(cosh(x+a)).
\end{aligned}$$

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80 points possible

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1. (30 Brownie points) A large space shark is bearing down on the star ship Enterprise. The shark has mass M_0 and velocity v_0. The warp drive of the Enterprise is broken, and its only hope to slowing down the shark is to fire small frozen tuna pellets of mass m into the shark's angry open mouth. As the shark has an insatiable appetite, its mouth remains open as it happily swallows the pellets. The muzzle velocity of the pellets is u and the Enterprise can fire at a rate of R pellets per unit time. The first pellets are swallowed by the shark at time $t = 0$. What is the shark's velocity as a function of time?

Solution: conservation of momentum:

$$\begin{aligned} M(t)v(t) &= M_0 v_0 - (M(t) - M_0)u, \\ v(t) &= -u + M_0 \frac{v_0 + u}{M}. \end{aligned} \tag{1}$$

The number of pellets per unit length in the stream is $\rho = R/u$. The rate of mass absorption is

$$\begin{aligned} \dot{M} &= m\rho(v+u) \\ &= mR\frac{u+v}{u} \\ &= \frac{mM_0R}{M}\frac{(v_0+u)}{u} \\ M\,dM &= \frac{(v_0+u)}{u} mM_0R\,dt, \\ M^2 - M_0^2 &= \frac{(v_0+u)}{u} mM_0Rt, \\ M(t) &= \sqrt{M_0^2 + \frac{(v_0+u)}{u} mM_0Rt}, \\ &= M_0\sqrt{1 + \frac{(v_0+u)}{u}(m/M_0)Rt}, \\ v &= -u + \frac{v_0+u}{\sqrt{1+\frac{(v_0+u)}{u}(m/M_0)Rt}} \end{aligned}$$

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2. A particle of mass m experiences a potential,

$$V(x) = -\frac{1}{2}m\omega^2x^2.$$

Note that this is the opposite sign as a harmonic oscillator. The particle crosses the origin at time $t = 0$ with velocity v_0.

(a) (10 Brownie pts) Find the velocity as a function of the position x.

(b) (10 Brownie pts) Find the time as a function of x.

(c) (5 Brownie pts) Find the position as a function of time.

(d) (5 Brownie pts) Give the answer to (c) but with for a problem where the sign of $V(x)$ switches back to the usual sign, $V(x) = m\omega^2x^2/2$. You need not show your work.

Solution: a)

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}m\omega^2x^2,$$
$$v = \sqrt{v_0^2 + \omega^2x^2}$$

b)

$$\begin{aligned} t &= \int \frac{dx}{v(x)} \\ &= \int \frac{dx}{\sqrt{v_0^2 + \omega^2x^2}}, \\ &= \frac{1}{\omega_0}\sinh^{-1}(\omega x/v_0), \\ x &= \frac{v_0}{\omega_0}\sinh(\omega t). \end{aligned}$$

For usual h.o. potential,

$$x = \frac{v_0}{\omega_0}\sin(\omega t).$$

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3. A platic ball of mass m is fired straight up with initial speed v_0. The ball feels a drag force, $|F_{\text{drag}}| = bAv^2$, where b is the drag coefficient and A is the cross-sectional area.

   (a) (5 pts) What is the terminal velocity, v_t?

   (b) (10 pts) Solve for the ball's velocity as a function of time on the way up. Give your answer in terms of v_0, g and v_t.

   (c) (5 pts) How much time is required to reach the top of the trajectory?

   (d) (5 pts) Check the units of your answer.

Solution: a)

$$\begin{aligned} mg &= bAv_t^2, \\ v_t &= \sqrt{mg/bA}, \end{aligned}$$

b)

$$\begin{aligned} \dot{v} &= -g - bAv^2/m, \\ &= -g(1 + v^2/v_t^2), \\ \int \frac{dv}{(1 + v^2/v_t^2)} &= -gt, \\ v_t \tan^{-1}(v/v_t) - v_t \tan^{-1}(v_0/v_t) &= -gt, \\ v &= v_t \tan\left[\tan^{-1}(v_0/v_t) - gt/v_t\right] \end{aligned}$$

c) set $v = 0$

$$t = \frac{v_t}{g} \tan^{-1}(v_0/v_t).$$

d) arguments of tan are dimensionless v_t/g has units of time.