your name\_

Physics 321 Midterm #1 - Wednesday, Oct. 5, 2022

Some integrals:

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x),$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1}(x),$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x),$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1}(x),$$

$$\int dx \tan(x+\phi) = -\ln(\cos(x+\phi))$$

$$\int dx \tanh(x+a) = \ln(\cosh(x+a)).$$

## Write your name on EVERY page!

80 points possible

- 1. Aliyah and her iceboat have a mass  $M_0$  and are gliding on a frictionless lake straight toward a long line penguins with initial speed  $v_0$ . Each penguin has a mass m, and there are  $\lambda$ penguins per unit length. The penguins jump straight up (the only direction they can jump due to the frictionless surface) as the boat approaches and land on the boat.
  - (a) (5 pts) What is the momentum of the boat, including Aliyah and her penquin passengers, as a function of time t? The time is measured from the moment she first reaches the line of penguins.
  - (b) (5 pts) What is Aliyah's velocity as a function of the distance x? The distance is measured from the point she first reaches the penguins.
  - (c) (10 pts) Find Aliyah's position as a function of time.

- 2. A particle of mass m and charge q moves in the x y plane under the influence of a magnetic field  $\vec{B} = B\hat{z}$ . At t = 0 the particle is at the origin with velocity  $v_x(t = 0) = v_0$  and  $v_y(t = 0) = 0$ .
  - (a) (15 pts) Solve for the velocity, both  $v_x$  and  $v_y$ , as a function of time.
  - (b) (10 pts) Solve for the position as a function of time.

- 3. Two bicyclists of the same size and shape, with the same drag coefficient, travel down the road. Cyclist A travels at 15 mph and cyclist B travels at 20 mph. Assume all the work goes into fighting air resistance (force scales as  $v^2$ ).
  - (a) (5 pts) If cyclist A requires 150 Watts to maintain his speed, how much power does cyclist B require?
  - (b) (5 pts) If both cyclists travel the same distance, and if Cyclist A expends a net energy of 1 kilowatt-hour due to the biking, how much energy does Cyclist B expend?

your name



4. Bill the cat, who has mass m, is fired straight upward from a cannon with muzzle velocity  $v_0$ . The drag force on Bill has magnitude  $bAv^2$ , where A is the cross sectional area. Assume  $v_0 < v_t$ , where  $v_t$  is the terminal velocity.

- (a) (5 pts) What is the terminal velocity? Give answer in terms of b, A, m and the acceleration of gravity g.
- (b) (10 pts) Solve for Bill's velocity as a function of time on the way up. Give your answer in terms of  $v_0, g$  and  $v_t$ .
- (c) (5 pts) How much time is required to reach the top of the trajectory?
- (d) (5 pts) If, after landing on his feet, Bill was fed heavy marbles, then fired from the cannon again, with the same initial muzzle velocity, would Bill go higher? or lower? than on his first trip. Justify your answer.
- (e) (5 pts, extra credit) What is the maximum height of Bill's trajectory?
- (f) (5 pts, extra credit, contingent on previous answer being correct) When Bill passes the cannon on the way down, is his speed greater, or less than, or equal to  $v_0$ ? Justify your answer.