

Physic 231 Lecture 11

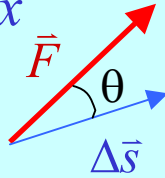
- Main points of last lecture:

- Work:

$$W = F\Delta s \cos(\theta) = F\Delta x$$

- Kinetic energy:

$$KE = \frac{1}{2}mv^2$$



- Work-energy theorem:

$$W_{total} = KE - KE_0$$

- Potential energy:

$$\Delta PE = PE_f - PE_0 = mg\Delta h$$

- Main points of today's lecture:

- Potential energy

$$\Delta PE = PE_f - PE_0 = mg(y_f - y_0)$$

- Conservative forces and the Conservation of energy:

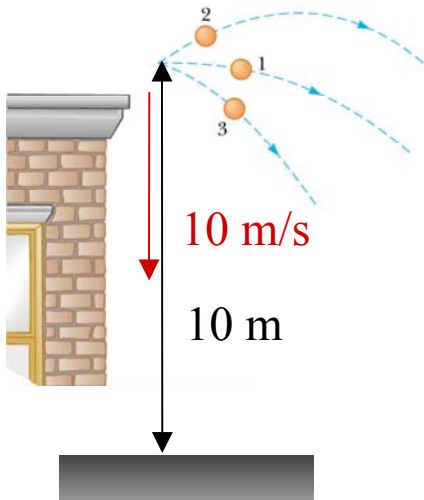
$$E = KE_f + PE_f = KE_0 + PE_0$$

- Varying forces – potential energy of a spring

$$PE = \frac{1}{2}kx^2$$

Example

- A 1 kg block is thrown downward with an initial velocity of 10 m/s. What is the kinetic energy and speed of the block when it strikes the ground 10 m below. Would it matter whether the block were thrown horizontally as in trajectory 1 or at other angles such as trajectories 2 or 3?



$$KE_f - KE_0 = Work_g = (-mg)(-10m)$$

$$KE_f = KE_0 + (mg)(10m)$$

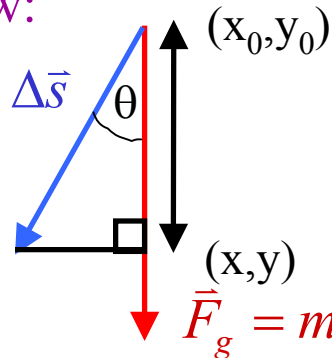
$$KE_f = \frac{1}{2}(1kg)(10m/s)^2 + (1kg)(9.8N)(10m) = 148J$$

$$\frac{1}{2}mv^2 = 148J; v = \sqrt{\frac{2(148J)}{1kg}} = 17.2m/s$$

The final speed and kinetic energy are independent of the initial angle.

Potential energy

- For certain forces, the work done by the force in going from position (x_0, y_0) to position (x, y) depends only the displacement and not on the path taken. Such a force is called conservative.
 - gravity is such a force. Consider the work on a mass m under a displacement Δs at an angle θ with respect to the vertical as shown below:



$$W_{gravity} = F_{gravity} \cos(\theta) \Delta s = mg(y_0 - y)$$

From work - energy theorem : $mg(y_0 - y_f) = KE_f - KE_0$

- From this we can see that being higher initially means that you can have a higher final kinetic energy. Thus, $mg(y_0 - y)$ is the part of the stored “potential” energy, which was changed into kinetic energy as the object moves from its initial to its final position.
- The potential energy only depends on the difference in height between the initial and final positions, i.e. on the vertical component of the displacement.

Example

- What difference between the PE of a 2000 kg car raised 10 m in the air and that of the same car on the ground? How much work would it require to lift it to that height?

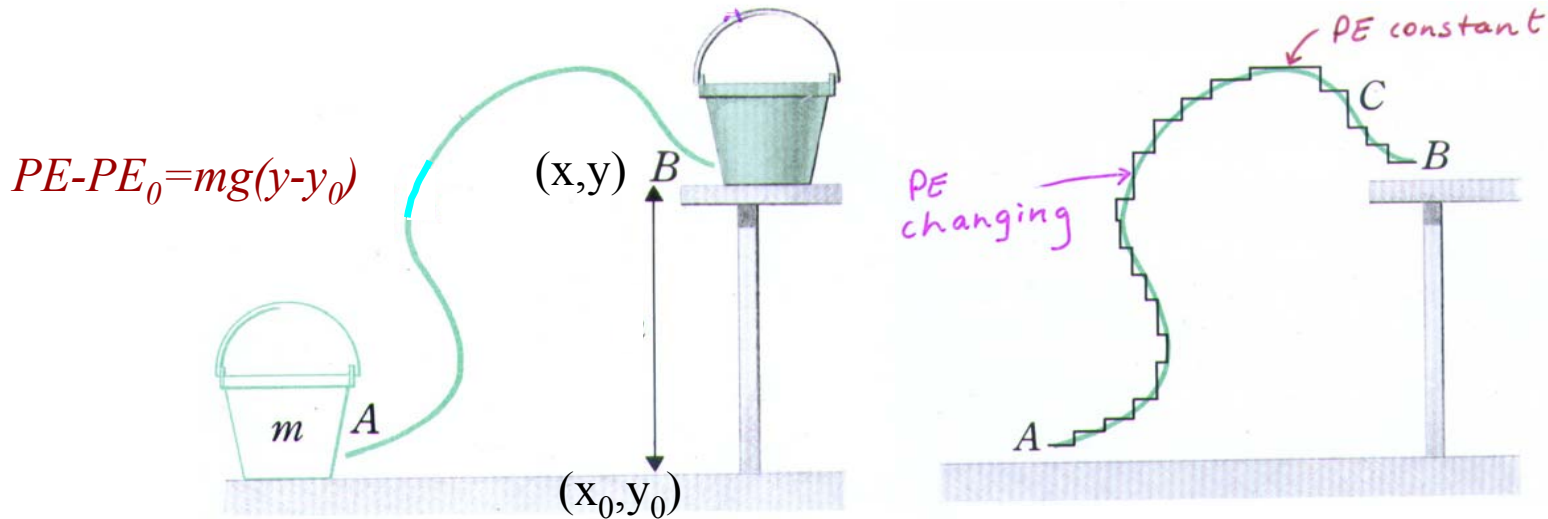
$$\Delta PE = mgh = (2000\text{kg})(9.8\text{m} / \text{s}^2)(10\text{m})$$

$$\Delta PE = 1.96 \times 10^5 \text{ J}$$

$$W = \Delta PE = 1.96 \times 10^5 \text{ J}$$

Gravitational potential energy

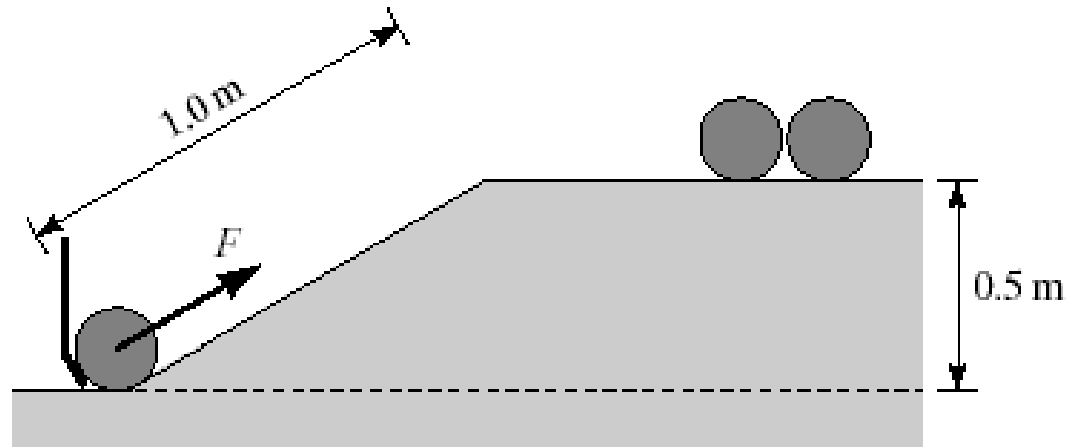
- The most important point of a potential energy is that it is only a function of the position and not of the path taken to get there.
- If we break the path of the pail into vertical sections the potential energy change is just $mg\Delta y$ and horizontal sections where the potential energy remains constant, we can see that the potential energy depends on the total vertical displacement and is independent of the path over which it is achieved.



- Thus we can define $PE=0$ for some y_0 and then $PE=mg(y-y_0)$ thereafter.
- PE is $-W_{grav}$. It is the work one would need to do to move the pail from A to B.

Conceptual problem

- At the bowling alley, the ball-feeder mechanism must exert a force to push the bowling balls up a 1.0-m long ramp. The ramp leads the balls to a chute 0.5 m above the base of the ramp. Approximately how much force must be exerted on a 5.0-kg bowling ball?



- a) 200 N
- b) 50 N
- c) 25 N
- d) 5.0 N
- e) impossible to determine

$$\text{work} = \text{PE} = F\Delta s$$

$$F = \frac{\text{PE}}{\Delta s} = \frac{mg\Delta y}{\Delta s}$$
$$= \frac{(5) \cdot (9.8) \cdot (0.5)\text{J}}{1\text{m}} \approx 25\text{N}$$

Conservation of energy

- Up to an additive constant, we can define $PE=mgy$. It is equal in magnitude but opposite in sign to the work being done by gravity.. Then

$$KE_f - KE_0 = PE_0 - PE_f$$

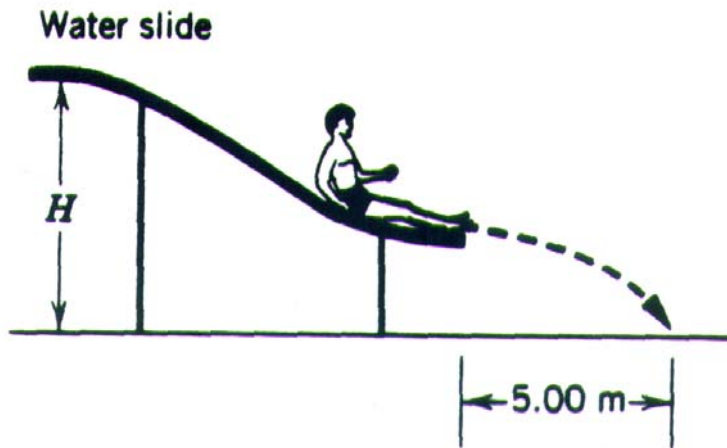
- We can define the total energy E as the sum of kinetic and potential energy. Then we have

$$E = KE_f + PE_f = KE_0 + PE_0$$

- Thus, when all work being done by forces in a problem can be expressed in terms of a potential energy, the total energy is conserved (i.e. remains constant). This is true for gravity and many other forces, but not for friction, for example.

Example

- A water slide is constructed so that swimmers, starting from rest at the top of the slide, leave the end of the slide traveling horizontally. As the drawing shows, one person is observed to hit the water 5.00 m from the end of the slide in a time of 0.500 s after leaving the slide. Ignoring friction and air resistance, find the height H in the drawing.



$$\Delta x = v_f t; \Rightarrow v_f = \frac{\Delta x}{t} = \frac{5m}{0.5s} = 10m/s$$

h = height at bottom of slide :

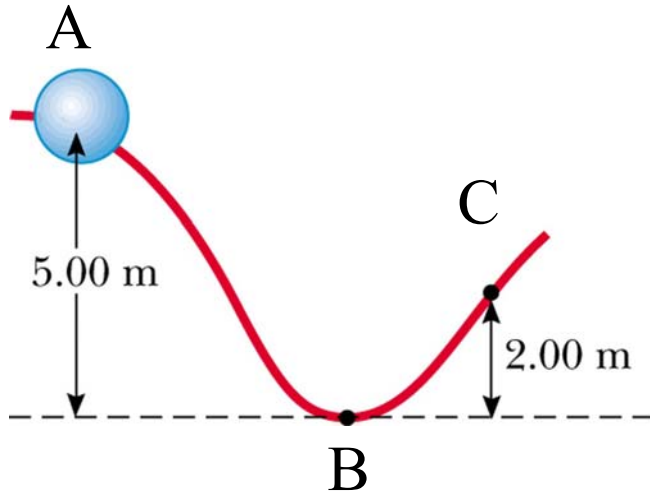
$$-h = -\frac{1}{2}gt^2 \Rightarrow h = (4.9m/s)(0.5s)^2 = 1.23m$$

Conservation of energy :

$$\frac{1}{2}mv^2 = mg(H - h); \Rightarrow H = h + \frac{v^2}{2g} = 6.33m$$

quiz

- A 0.400-kg bead slides on a curved wire, starting from rest at point A in the figure below. If the wire is frictionless, find the speed of the bead at C.



- a) 5.2 m/s
- b) 1.4 m/s
- c) 23 m/s
- d) 7.7 m/s

$$KE_f + PE_f = KE_0 + PE_0$$

$$KE_f = PE_0 - PE_f = mg(y_0 - y_f)$$

$$\frac{1}{2}mv_f^2 = mg(y_0 - y_f)$$

$$\Rightarrow v_f = \sqrt{2g(y_0 - y_f)} = 7.7 \text{ m/s}$$

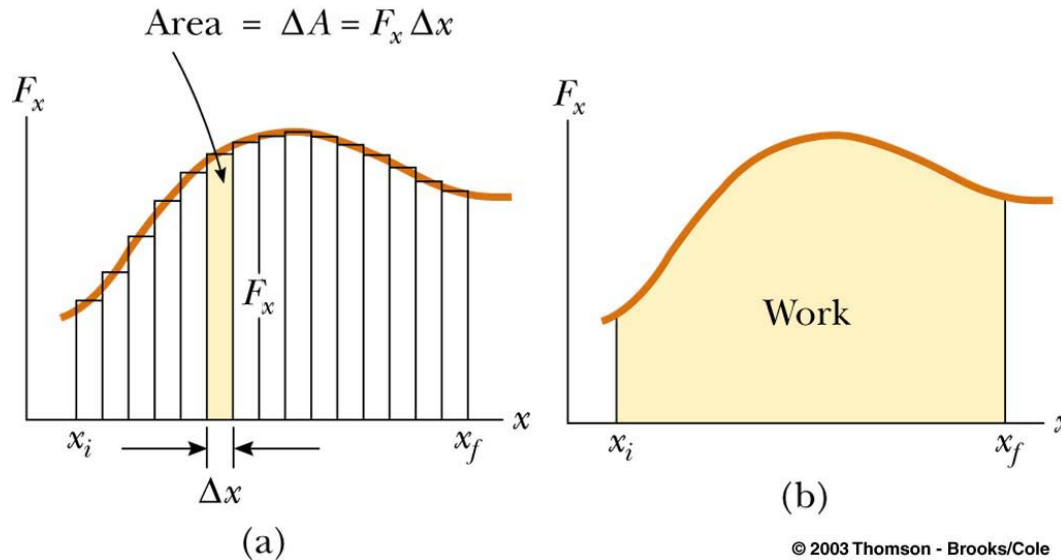
Work and PE for non-constant forces

- When the force is not constant, the work can still be computed for small displacements Δx over which force is approximately constant:

$$W = F_x \Delta x$$

- To get the total work, we add the contributions for the steps:

$$W = \sum F_x \Delta x$$



- W is just the area under the curve, if the force is conservative the change in potential energy is $-W$. It is equal to the work one would need to do against the force to move the object over the chosen path.