

## Effective mass: discussion (2)

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## Effective mass: basic ideas

I. Free wave-packet centered around  $\vec{k}$  with dispersion relation:  $\epsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m}$

$$\Rightarrow d\epsilon_k/d\vec{k} = \nabla_{\vec{k}} \epsilon_k = \hbar \vec{V}_G = \frac{\hbar^2 \vec{k}}{m} \Leftrightarrow \text{density of states in the Fermi sea} \propto m$$

II. Infinite nuclear matter in medium-effects  $\Rightarrow \epsilon_k \neq \frac{\hbar^2 k^2}{2m}$

\*Effective mass  $m^*(k)$  defined by analogy:  $\frac{d\epsilon_k}{dk} = \frac{\hbar^2 k}{m_k^*}$

\*Considering the on-shell energy-dependence of the self-energy:  $\epsilon_k = \hbar^2 k^2/2m + \Re\Sigma(k; \epsilon_k)$

$$\frac{m}{m_k^*} = 1 + \frac{m}{\hbar^2 k} \frac{d\bar{V}(k; \omega)}{dk} \Big|_{\omega=\epsilon_k} = \underbrace{\left[ 1 + \frac{m}{\hbar^2 k} \frac{\partial \bar{V}(k; \omega)}{\partial k} \right]}_{1/m_k^{k*}} \underbrace{\left[ 1 - \frac{\partial \bar{V}(k; \omega)}{\partial \omega} \right]^{-1}}_{m/m_k^{e*}} \Big|_{\omega=\epsilon_k}$$

\* $m^*$  = concept to characterize the propagation of quasi-particles in a medium

III.  $m_k^*$  originates from non-locality ( $k$ -mass) and energy-dependence ( $e$ -mass) of  $V(k; \omega)$

$$\begin{aligned}
 -\frac{\hbar^2 \nabla^2}{2m} e^{i\vec{k} \cdot \vec{r}} + \int d\vec{r}' V(\vec{r}, \vec{r}') e^{i\vec{k} \cdot \vec{r}'} &= \epsilon_k e^{i\vec{k} \cdot \vec{r}} \quad \Rightarrow \quad \frac{\hbar^2 k^2}{2m} + \underbrace{\int d\vec{s} V(\vec{s}) e^{i\vec{k} \cdot \vec{s}}}_{\bar{V}(k) \rightarrow \text{constant if } V \text{ local}} = \epsilon_k \\
 &\Rightarrow \quad \frac{1}{m_k^*} = \frac{1}{m} + \frac{1}{\hbar^2 k} \frac{d\bar{V}(k)}{dk} \longrightarrow = \frac{1}{m} \text{ if } V \text{ is local}
 \end{aligned}$$

\*Due to finite-range and non-locality of force in space as well as its non-locality in time

# $m_{iq}^*$ in finite nuclei: Skyrme Hartree-Fock calculations

## I. Skyrme effective "nucleon-nucleon interaction" in the nucleus

$$\begin{aligned}
 v_{\text{Skyrme}}(\vec{R}, \vec{r}, \vec{k}, \overleftarrow{k}') &= t_0 (1 + x_0 P_\sigma) \delta(\vec{r}) + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^\alpha(\vec{R}) \delta(\vec{r}) \\
 &+ \frac{1}{2} t_1 (1 + x_1 P_\sigma) (\delta(\vec{r}) \vec{k}^2 + \overleftarrow{k}'^2 \delta(\vec{r})) \\
 &+ t_2 (1 + x_2 P_\sigma) \overleftarrow{k}' \cdot \delta(\vec{r}) \vec{k} \\
 &+ iW_0 (\hat{\sigma}_1 + \hat{\sigma}_2) \overleftarrow{k}' \wedge \delta(\vec{r}) \vec{k} ,
 \end{aligned}$$

\*Non-locality/finite-range: second order in initial and final relative momenta  $\vec{k}$  and  $\overleftarrow{k}'$

\*Relative momentum operator in coordinate space:  $\vec{k} = \vec{k}_1 - \vec{k}_2 = \hbar(\nabla_1 - \nabla_2)/2i$

## II. Skyrme-HF functional = binding energy minimized for nucleus $|\phi_0^A\rangle$

$$\begin{aligned}
 \mathcal{E}_{\text{Skyrme}} &= \int d\vec{r} \left\{ \frac{b_0}{2} \rho^2 - \frac{b'_0}{2} \sum_q \rho_q^2 + b_1 (\rho\tau - \vec{j}^2) - b'_1 \sum_q (\rho_q \tau_q - \vec{j}_q^2) - \frac{b_2}{2} \rho \Delta \rho + \frac{b'_2}{2} \sum_q \rho_q \Delta \rho_q \right. \\
 &+ \frac{b_3}{3} \rho^{\alpha+2} - \frac{b'_3}{3} \rho^\alpha \sum_q \rho_q^2 - b_4 \left[ \rho \nabla \cdot \vec{J} + \vec{s} \cdot \nabla \times \vec{j} + \sum_q (\rho_q \nabla \cdot \vec{J}_q + \vec{s}_q \cdot \nabla \times \vec{j}_q \right. \\
 &\left. \left. - \frac{1}{16} (t_1 x_1 + t_2 x_2) (\vec{J}^2 - 2 \vec{s} \cdot \vec{\tau}) + \frac{1}{16} (t_1 - t_2) \sum_q (\vec{J}_q^2 - 2 \vec{s}_q \cdot \vec{\tau}_q) \right\} , \tag{1}
 \end{aligned}$$

III. One-body local densities involved (from the full one-body density matrix  $\rho(\vec{r}sq, \vec{r}'s'q')$ )

$$\rho_q(\vec{r}) = \rho_q(\vec{r}, \vec{r}) = \sum_{k \in q} |\varphi_k(\vec{r})|^2$$

$$\tau_q(\vec{r}) = \nabla \cdot \nabla' \rho_q(\vec{r}, \vec{r}') \Big|_{\vec{r}=\vec{r}'} = \sum_{k \in q} |\nabla \varphi_k(\vec{r})|^2$$

$$\vec{s}_q(\vec{r}) = \vec{s}_q(\vec{r}, \vec{r}) = \sum_{k \in q} \varphi_k^\dagger(\vec{r}) \hat{\sigma} \varphi_k(\vec{r})$$

$$\vec{j}_q(\vec{r}) = -\frac{\imath}{2} (\nabla - \nabla') \rho_q(\vec{r}, \vec{r}') \Big|_{\vec{r}=\vec{r}'} = -\frac{\imath}{2} \sum_{k \in q} \left\{ \varphi_k^\dagger(\vec{r}) \nabla \varphi_k(\vec{r}) - \left[ \nabla \varphi_k^\dagger(\vec{r}) \right] \varphi_k(\vec{r}) \right\}$$

$$\vec{J}_q(\vec{r}) = -\frac{\imath}{2} (\nabla - \nabla') \times \vec{s}_q(\vec{r}, \vec{r}') \Big|_{\vec{r}=\vec{r}'} = -\frac{\imath}{2} \sum_{k \in q} \left\{ \varphi_k^\dagger(\vec{r}) \nabla \times \hat{\sigma} \varphi_k(\vec{r}) - \left[ \nabla \times \hat{\sigma} \varphi_k^\dagger(\vec{r}) \right] \varphi_k(\vec{r}) \right\}$$

$$\vec{\tau}_q(\vec{r}) = \nabla \cdot \nabla' \vec{s}_q(\vec{r}, \vec{r}') \Big|_{\vec{r}=\vec{r}'} = \sum_{k \in q} \left[ \nabla \varphi_k^\dagger(\vec{r}) \right] \cdot \nabla \left[ \hat{\sigma} \varphi_k(\vec{r}) \right]$$

$$\nabla \cdot \vec{J}_q(\vec{r}) = -\frac{\imath}{2} \sum_{k \in q} \left\{ \left[ \nabla \varphi_k^\dagger(\vec{r}) \right] \cdot \nabla \times \hat{\sigma} \varphi_k(\vec{r}) - \left[ \nabla \times \hat{\sigma} \varphi_k^\dagger(\vec{r}) \right] \cdot \nabla \varphi_k(\vec{r}) \right\}$$

#### IV. Action of the one-body HF field on $\varphi_{iq}$

$$\begin{aligned}
 h^q \varphi_{iq}(\vec{r}) &= \frac{\delta \mathcal{E}}{\delta \varphi_{iq}^\dagger(\vec{r})} \varphi_{iq}(\vec{r}) = \int d\vec{r}' \left\{ \frac{\delta \mathcal{E}}{\delta \tau_q(\vec{r}')} \frac{\delta \tau_q(\vec{r}')}{\delta \varphi_{iq}^\dagger(\vec{r})} + \frac{\delta \mathcal{E}}{\delta \rho_q(\vec{r}')} \frac{\delta \rho_q(\vec{r}')}{\delta \varphi_{iq}^\dagger(\vec{r})} + \dots \right\} \varphi_{iq}(\vec{r}') \\
 &= \left\{ -\nabla \cdot \left[ \frac{\delta \mathcal{E}}{\delta \tau_q(\vec{r})} + \hat{\sigma} \cdot \frac{\delta \mathcal{E}}{\delta \vec{\tau}_q(\vec{r})} \right] \nabla + \frac{\delta \mathcal{E}}{\delta \rho_q(\vec{r})} + \frac{\delta \mathcal{E}}{\delta \vec{s}_q(\vec{r})} \cdot \hat{\sigma} - \frac{i}{2} \left[ \frac{\delta \mathcal{E}}{\delta \vec{J}_q(\vec{r})} - \nabla \frac{\delta \mathcal{E}}{\delta \nabla \cdot \vec{J}_q(\vec{r})} \right] \cdot \nabla \times \hat{\sigma} \right. \\
 &\quad \left. - \frac{i}{2} \nabla \cdot \left( \frac{\delta \mathcal{E}}{\delta \vec{j}_q(\vec{r})} - \frac{\delta \mathcal{E}}{\delta \vec{J}_q(\vec{r})} \times \hat{\sigma} \right) - \frac{i}{2} \left( \frac{\delta \mathcal{E}}{\delta \vec{j}_q(\vec{r})} + \nabla \frac{\delta \mathcal{E}}{\delta \nabla \cdot \vec{J}_q(\vec{r})} \times \hat{\sigma} \right) \cdot \nabla \right\} \varphi_{iq}(\vec{r})
 \end{aligned}$$

\*s.p. spectrum  $\{\epsilon_{iq}\}$  for a given nucleus  $|\phi_0^A\rangle$ :  $h^q \varphi_{iq}(\vec{r}) = \epsilon_{iq} \varphi_{iq}(\vec{r})$

$$\Rightarrow \text{Koopmans' Theorem: } \epsilon_{iq} = E_{iq}^{A+1} - E_0^A$$

\*The non-locality of the HF field appears only through gradient terms

\*From a finite range/non-local interaction, one would obtain:

$$h^q \varphi_{iq}(\vec{r}) = -\frac{\hbar^2}{2m} \Delta \varphi_{iq}(\vec{r}) + \int d\vec{r}' u_1(-i\hbar\nabla_{\vec{r}}) v(\vec{r}, \vec{r}') u_2(-i\hbar\nabla_{\vec{r}'} ) \varphi_{iq}(\vec{r}')$$

where  $u_1(-i\hbar\nabla_{\vec{r}})$  and  $u_2(-i\hbar\nabla_{\vec{r}'})$  are functional of the gradient operator.

## VI. Effective mass in Skyrme-HF framework

\*Defined by simple "analogy" to a kinetic energy-like term (scalar part):

$$-\nabla \cdot \left[ \frac{\hbar^2}{2m_{iq}^*(\vec{r})} \right] \nabla \equiv -\nabla \cdot \left[ \frac{\delta \mathcal{E}}{\delta \tau_q(\vec{r})} + \hat{\vec{\sigma}} \cdot \frac{\delta \mathcal{E}}{\delta \vec{\tau}_q(\vec{r})} \right] \nabla \implies \frac{m}{m_q^*(\vec{r})} = 1 + \frac{2m}{\hbar^2} \left( b_1 \rho(\vec{r}) - b'_1 \rho_q(\vec{r}) \right)$$

\*Identical for all s.p. states  $\varphi_{iq}$  because expansion in the non-locality limited to second order in  $\vec{k}$

\*Note that density dependence of the force has no explicit influence.

\*However,  $m^*$ ,  $\alpha$  and  $K$  in INM strongly correlated in the fit

\*Vector part rarely discussed and never constrained is:

$$\frac{\delta \mathcal{E}}{\delta \vec{\tau}_q(\vec{r})} = \frac{1}{8} (t_1 x_1 + t_2 x_2) \vec{s}(\vec{r}) - \frac{1}{8} (t_1 - t_2) \vec{s}_q(\vec{r}) \quad (2)$$

Ok for time-reversal invariant systems (even-even ground-states)

## VI. Effective mass at the mean-field level: some facts from phenomenology

\*Isoscalar quadrupole resonance in (heavy) doubly magic nuclei (exp vs RPA); Reinhard (1999)

→ in favor of  $m^*/m = 0.8$

→ consistent with  $m_{k_F}^*(\rho_{sat}, 0)$  from (D)BHF in INM before coupling to collective modes

\*s.p. spectra in odd neighbors when including particle-vibration coupling; V. Bernard et al (1980)

→  $m^*/m \geq 1$  around the Fermi surface once particle-vibration coupling = ↗ due to ↗  $e$ -mass!

→ Now close to what experiment requires

\*Mass "formulas"

→ have always been in favor of  $m^*/m = 1$  to fit masses of open-shell nuclei

→ changed lately... due to effect from pairing; S. Goriely et al. (2003)

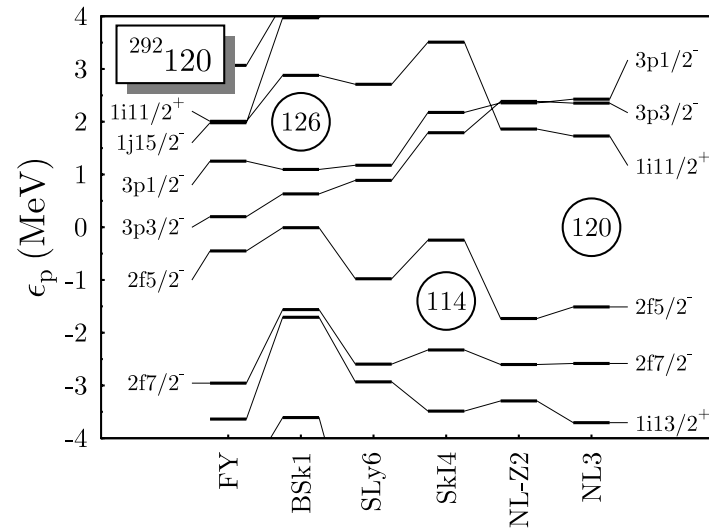
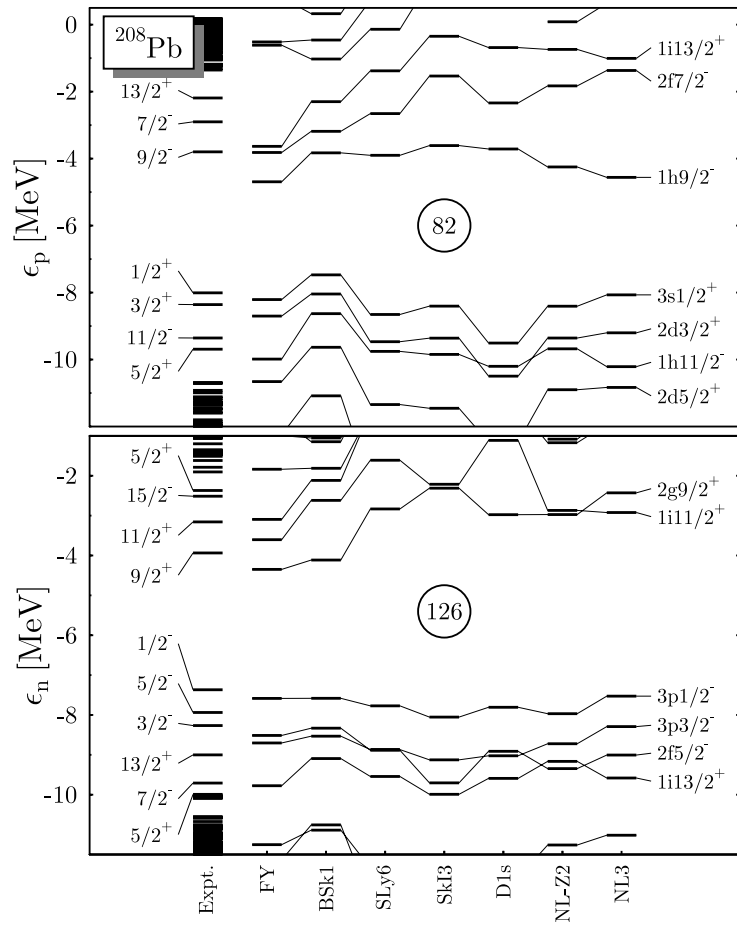
\*Skyrme effective mass is independent of  $k$  in IM

$$\frac{m}{m_q^*}(\rho, \beta) = 1 + \frac{2m}{\hbar^2}(b_1\rho - b'_1\rho_q) \equiv (1 \pm \beta) \frac{m}{m_s^*} \mp \beta \frac{m}{m_v^*}$$

where  $\rho = \rho_n + \rho_p$  and  $\beta = (\rho_n - \rho_p)/\rho$

\* $m_q^*(\rho_{sat}, 0)$  in INM always part of Skyrme forces fitting procedure

VI. Density of states: parametrisations with  $\neq m^*$  in symmetric INM; M. Bender et al. (2003)



\*Ordering of levels is correct for all forces. Less true in  $^{132}\text{Sn}$  for instance

\*In  $^{208}\text{Pb}$ , strong correlations between  $m^*$  and the size of the magic gaps at the Fermi surface

\*However, no clear correlation within hole or particle states

\*No clear correlation in superheavy nuclei; mainly due to high-density of states vs spin-orbit

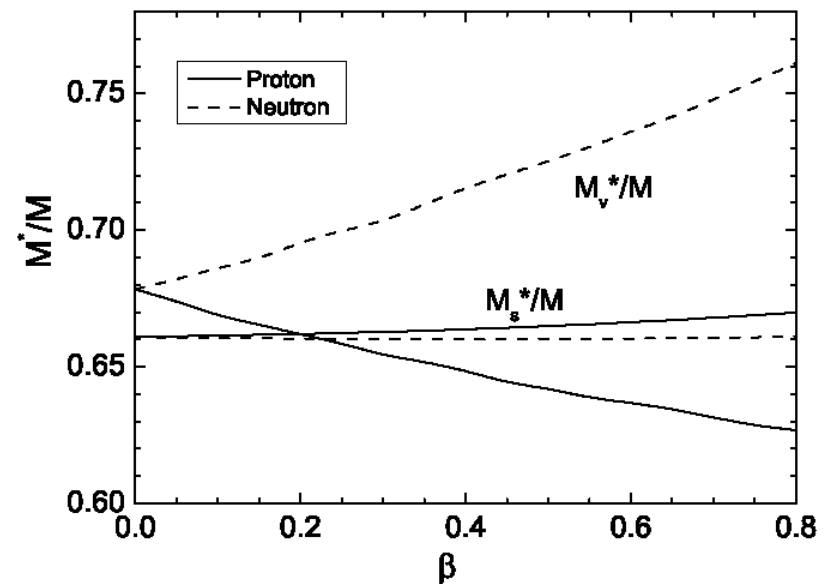
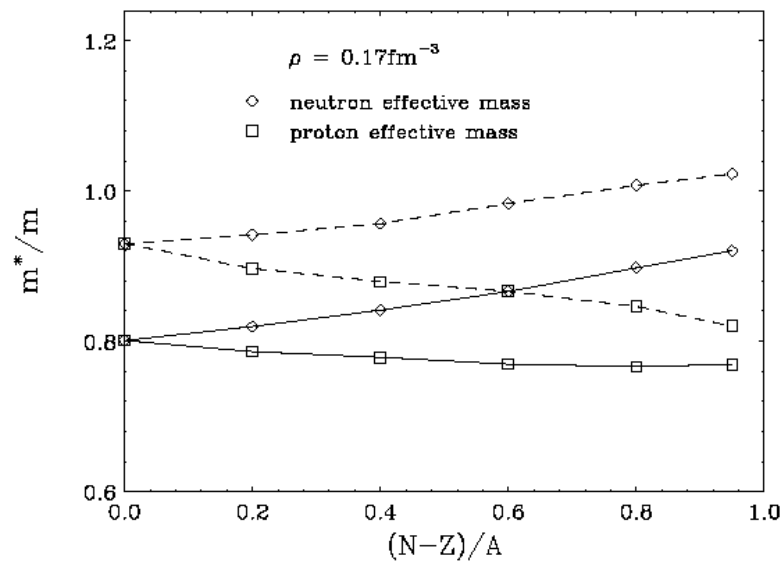
# A new component: neutron-proton effective mass splitting

\*So far, extraction of  $m_v^* \approx 0.7 - 0.9m$  from isovector dipole resonances is not very precise

\*Solid predictions for the **sign** of the splitting:  $m_n^*(\beta) \geq m_p^*(\beta)$  for  $\beta \geq 0$

→ BHF (with/without 3-body force); W. Zuo et al. (1999)

→ DBHF; Z. Ma et al. (2004), E. van Dalen et al. (2005)



\*Confirmation from the energy dependence of the Lane potential; B.-A. Li (2004)

\*SLy forces adjusted on the EOS of PNM have the opposite splitting; Chabanat et al. (1995)

\*Relativistic Mean-Field model always predict the opposite: no obvious to correct for that

# Effective-mass splitting in neutron-rich environments

I. Influence of isovector part of nucleon dependent quantity (on top of global ones; i.e.  $a_{sym}(k_F)$ )

\*Collision observables depend on the isovector/momentum dependent part of the mean-field;  
B.-A. Li et al. (2004)

\*Astrophysical processes require a precise knowledge of isoscalar and isovector effective masses;  
M. Farine et al. (2001)

\*Structure properties of **neutron-rich nuclei**:

→ masses and single-particle properties

→ shell corrections in superheavy nuclei around the island of stability ( $N = 184, Z = 120$ )

→ static pairing correlations developing onto the previous s.p. spectrum

→ dynamical correlations beyond the mean-field level (pairing, coupling to vibrations/rotations)

→ vibrational excitations of different kinds (giant resonances, low-lying states)

# First attempts with new Skyrme parametrizations

B. Cochet, K. Bennaceur, T. Duguet and J. Meyer

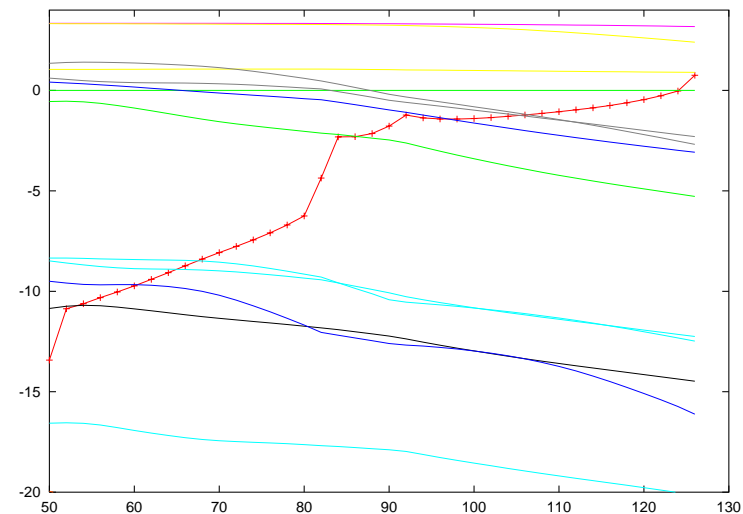
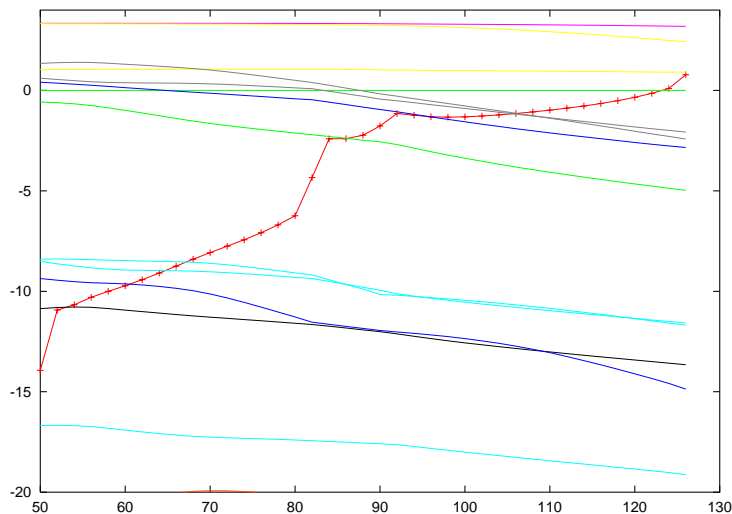
\*SLy4-like forces

- + 2 density-dependences (decorrelates  $m^*$  and  $K$ );  $m^*(\rho_{sat}, 0) = 0.7m$
- + various splitting amplitudes  $\Delta m_q^*(\rho_{sat}, \beta = 1)$  based on different BHF scenarii

\*New isospin instabilities forbid us to probe  $\Delta m_q^*(\rho_{sat}, \beta = 1)$  as much as desired

\*Forces have otherwise good "standard" properties ( $K$ ,  $a_{sym}(k_F)$ ,  $a_{surf}$ ,  $\langle r_p^2 \rangle \dots$ )

\*However, no visible effects on the density of states in neutron rich nuclei



HF neutron spectrum as a function of  $N$  in  $S_n$  isotopes: 2 forces with  $\Delta m_q^*(\rho_{sat}, \beta = 1) = \pm 0.15$

\* $\Delta m_q^*(\rho_{sat}, \beta = 1)$  mainly reabsorbed by the spin-orbit

# Better approach to effective mass in finite nuclei

## I. Assessment

\*We do not know the effective spin-orbit force in exotic nuclei

\*Also, it is obviously misleading to think about the density of states in the way it is currently done

Strong interplay between  $-\nabla \cdot \hat{A}(\vec{r})\nabla$  and spin-orbit terms

## II. Need to revisit the effective mass concept in finite nuclei

\*Non-local real part of the self-energy of the general form:  $V(\vec{r}, \vec{r}'; \epsilon_{iq}) = u_1(-i\hbar\nabla_{\vec{r}}) v(\vec{r}, \vec{r}'; \epsilon_{iq}) u_2(-i\hbar\nabla_{\vec{r}'})$

\*The s.p. on-shell energy is given by:

$$\begin{aligned}
 h_{iq}(\epsilon_{iq}) &= \int d\vec{r} \varphi_{iq}^*(\vec{r}) \left[ -\frac{\hbar^2}{2m} \Delta \varphi_{iq}(\vec{r}) + \int d\vec{r}' V(\vec{r}, \vec{r}'; \epsilon_{iq}) \varphi_{iq}(\vec{r}') \right] \\
 &= \iint d\vec{R} d\vec{s} \varphi_{iq}^*\left(\vec{R} + \frac{\vec{s}}{2}\right) \left[ \frac{\delta(\vec{s})}{2m} \left( -i\hbar\nabla_{\vec{R} + \frac{\vec{s}}{2}} \right)^2 + u_1(-i\hbar\nabla_{\vec{R} + \frac{\vec{s}}{2}}) v\left(\vec{R}, \vec{s}; \epsilon_i\right) u_2(-i\hbar\nabla_{\vec{R} - \frac{\vec{s}}{2}}) \right] \varphi_{iq}\left(\vec{R} - \frac{\vec{s}}{2}\right)
 \end{aligned}$$

where the  $\{\varphi_{iq}(\vec{r})\}$  are the bound quasi-particles states diagonalizing  $G(\vec{r}, \vec{r}'; \omega)$

\*The effective mass still characterizes the non-localities of the self-energy

\*The **state-dependent effective mass**  $m_{iq}^*$  is thus defined through:

$$\left. \frac{d h_{iq}(\omega)}{d - i\hbar \vec{\nabla}_{\vec{r}}} \right|_{\omega=\epsilon_{iq}} = \left[ 1 - \left. \frac{\partial h_{iq}(\omega)}{\partial \omega} \right|_{\omega=\epsilon_{iq}} \right]^{-1} \left. \frac{\partial h_{iq}(\omega)}{\partial -i\hbar \vec{\nabla}_{\vec{r}}} \right|_{\omega=\epsilon_{iq}}$$

$$\left. \frac{\partial h_{iq}(\omega)}{\partial -i\hbar \vec{\nabla}_{\vec{r}}} \right|_{\omega=\epsilon_{iq}} \Big|_{\mu} \equiv -i\hbar \varphi_{iq}^{\dagger}(\vec{r}) \left[ \frac{1}{2\hat{m}_{iq}^{k*}(\vec{r})} \right]_{\mu\nu} \vec{\nabla} \varphi_{iq}(\vec{r}) \Big|_{\nu} + c.c. + \varphi_{iq}^{\dagger}(\vec{r}) \vec{v}_q(\vec{r}) \Big|_{\mu} \varphi_{iq}(\vec{r})$$

where the last term represents a global current.

\*Simple example:

$$h(\vec{R}, \vec{s}) = \frac{\delta(\vec{s})}{2m} \left( -i\hbar \nabla_{\vec{R}-\frac{\vec{s}}{2}} \right)^2 \implies \left. \frac{d h_{iq}(\omega)}{d - i\hbar \vec{\nabla}_{\vec{r}}} \right|_{\omega=\epsilon_{iq}} = \frac{-i\hbar \varphi_i^*(\vec{r}) \vec{\nabla} \varphi_i(\vec{r}) + c.c.}{2m} \implies \hat{m}_{iq}^*(\vec{r}) = m$$

\*We finally introduce the **tensor of inverse effective  $k$  masses** /

$$\left[ \frac{1}{m_{iq}^{k*}} \right]_{\mu\nu} \equiv \int d\vec{r} \varphi_{iq}^{\dagger}(\vec{r}) \left[ \frac{1}{\hat{m}_{iq}^{k*}(\vec{r})} \right]_{\mu\nu} \varphi_{iq}(\vec{r})$$

\*The motion of the nucleon in direction  $\nu$  depends on the forces in direction  $\mu$

\*The tensor may be diagonalized to find the eigen-directions of group velocity

\*Extraction of the  $e$ -mass:

$$\frac{m_{iq}^{e*}}{m} = 1 - \iint d\vec{r}' d\vec{r} \varphi_{iq}^\dagger(\vec{r}') \frac{\partial V_q(\vec{r}', \vec{r}; \omega)}{\partial \omega} \varphi_{iq}(\vec{r}) \Big|_{\omega=\varepsilon_{iq}}$$

\*Extraction of the tensor of inverse  $k$ -mass (self-energy with no gradient operator)

$$\left[ \frac{1}{m_{iq}^{k*}} \right]_{\mu\nu} = -\frac{1}{2\hbar^2} \iint d\vec{r} d\vec{s} \left( \varphi_{iq}^\dagger(\vec{r} - \frac{\vec{s}}{2}) + \varphi_{iq}^\dagger(\vec{r}) \right) V_q(\vec{r}, \vec{s}; \varepsilon_{iq}) \frac{s_\mu s_\nu}{\vec{s} \cdot \vec{u}_r} \int_0^{\vec{s} \cdot \vec{u}_r} \varphi_{iq}(\vec{r} + \frac{\vec{s}'}{2}) d\vec{s}' \cdot \vec{u}_r$$

\*Extraction of the tensor of inverse  $k$ -mass (self-energy bilinear in gradient operators)

$$\left[ \frac{1}{m_{iq}^{k*}} \right]_{\mu\nu} = \int d\vec{r} \varphi_{iq}^\dagger(\vec{r}) \int d\vec{r}' V_q\left(\frac{\vec{r} + \vec{r}'}{2}, \vec{r} - \vec{r}'; \varepsilon_{iq}\right) \vec{\nabla}_{\vec{r}'} \Big|_\mu \frac{(\vec{r} - \vec{r}') \cdot \vec{u}_r}{(\vec{r} - \vec{r}') \cdot \vec{u}_{\vec{r}'}} \int_0^{(\vec{r} - \vec{r}') \cdot \vec{u}_r} \varphi_{iq}(\vec{r} + \vec{r}'') d\vec{r}'' \cdot \vec{u}_r$$

→  $\vec{u}_{\vec{r}}$  is the the unit vector  $\vec{u}_{\vec{r}} = \vec{\nabla}_{\vec{r}} \varphi_{iq}(\vec{r}) / \|\vec{\nabla}_{\vec{r}} \varphi_{iq}(\vec{r})\|$

\*Recover the usual expressions in infinite matter

$$\frac{m_{kq}^{e*}}{m} = 1 - \frac{\partial}{\partial \omega} \int d\vec{s} e^{-i\vec{k}\cdot\vec{s}} V_q(|\vec{s}|; \omega) \Big|_{\omega=\epsilon_{iq}} = 1 - \frac{\partial V_{kq}(\omega)}{\partial \omega} \Big|_{\omega=\epsilon_{iq}},$$

$$\frac{1}{m_{kq}^{k*}} = \frac{1}{m} + \frac{1}{\hbar^2 k} \frac{\partial V_{kq}(\omega)}{\partial k} \Big|_{\omega=\epsilon_{kq}}$$

→  $V_{kq}(\omega)$  is the Fourier transformed of  $V_q(|\vec{r} - \vec{r}'|; \omega)$  multiplied by  $(2\pi)^{3/2}$

→ The tensor of the inverse  $k$ -mass reduces to one diagonal term

\*For the Skyrme functional, one obtains (no explicit  $e$ -mass):

$$\left[ \frac{1}{m_{ipq}^{k*}} \right]_{\mu\nu} = \frac{2}{\hbar^2} \int d\vec{r} \varphi_{ipq}^\dagger(\vec{r}) \left[ \frac{\delta \mathcal{E}}{\delta \tau_q(\vec{r})} \delta_{\mu\nu} + \hat{\sigma} \cdot \frac{\delta \mathcal{E}}{\delta \vec{\tau}_q(\vec{r})} \delta_{\mu\nu} + \imath \frac{\delta \mathcal{E}}{\delta \vec{\nabla} \cdot \vec{J}_q(\vec{r})} \hat{\sigma}_\lambda \epsilon_{\lambda\mu\nu} \right] \varphi_{ipq}(\vec{r})$$

$$- \frac{1}{2Am} \iint d\vec{r} d\vec{s} \left( \varphi_{ipq}^\dagger(\vec{r} - \frac{\vec{s}}{2}) + \varphi_{iq}^\dagger(\vec{r}) \right) \tau_{-p}(\vec{r} - \frac{\vec{s}}{2}, \vec{r} + \frac{\vec{s}}{2}) \frac{s_\mu s_\nu}{\vec{s} \cdot \vec{u}_{\vec{r}}} \int_0^{\vec{s} \cdot \vec{u}_{\vec{r}}} \varphi_{ipq}(\vec{r} + \frac{\vec{s}'}{2}) d\vec{s}' \cdot \vec{u}_{\vec{r}},$$

→ new contributions from the spin-orbit field and the center of mass correction

→ numerical calculations and study of the different contributions are underway