

# The Gamow shell model

## A new approach to exotic nuclei

Nicolas Michel

Witek Nazarewicz

Marek Płoszajczak

Jimmy Rotureau

Gamow states : definition and calculation.

Normalization and matrix elements.

Completeness relations with Gamow states.

Matrix diagonalization and physical states search.

Applications : Chains of He, Li and O isotopes.

## Calculation of the Gamow states

- Standard Schrödinger equation :  $\frac{\hbar^2}{2m}u''(r) = \left[ \frac{l(l+1)}{r^2} + V_{lj}(r) - e \right] \cdot u(r).$
- $V_{lj}(r)$  : WS or HF potential, usually non local.
- $u(0) = 0$
- $u(r) \rightarrow H_{l,\eta}^+(kr) \mid r \rightarrow +\infty$  (bound, resonant states)
- $u(r) \rightarrow H_{l,\eta}^-(kr) + S(k).H_{l,\eta}^+(kr) \mid r \rightarrow +\infty$  (scattering states)

## Normalization and matrix elements

- Radial integrals diverging with unbound states.
- Solution : complex scaling integration method.

$$\begin{aligned}\langle \widetilde{u}_f | O | u_i \rangle &= \int_0^R u_f(r) O(r) u_i(r) dr \\ &+ e^{i\theta} \int_0^{+\infty} u_f(R + x.e^{i\theta}) O(R + x.e^{i\theta}) u_i(R + x.e^{i\theta}) dx\end{aligned}$$

- Use of time reversed states :  $\langle r | \widetilde{u} \rangle = u(r)^*$
- Integrals independent of  $R$  and  $\theta$ .
- Normalization of continuum states :  $\langle \widetilde{u}(k') | u(k) \rangle = \delta(k - k')$

## Completeness relations

- One body spherical case : the Berggren relation.

T. Berggren, Nucl. Phys. **A 109** (1967) 265.

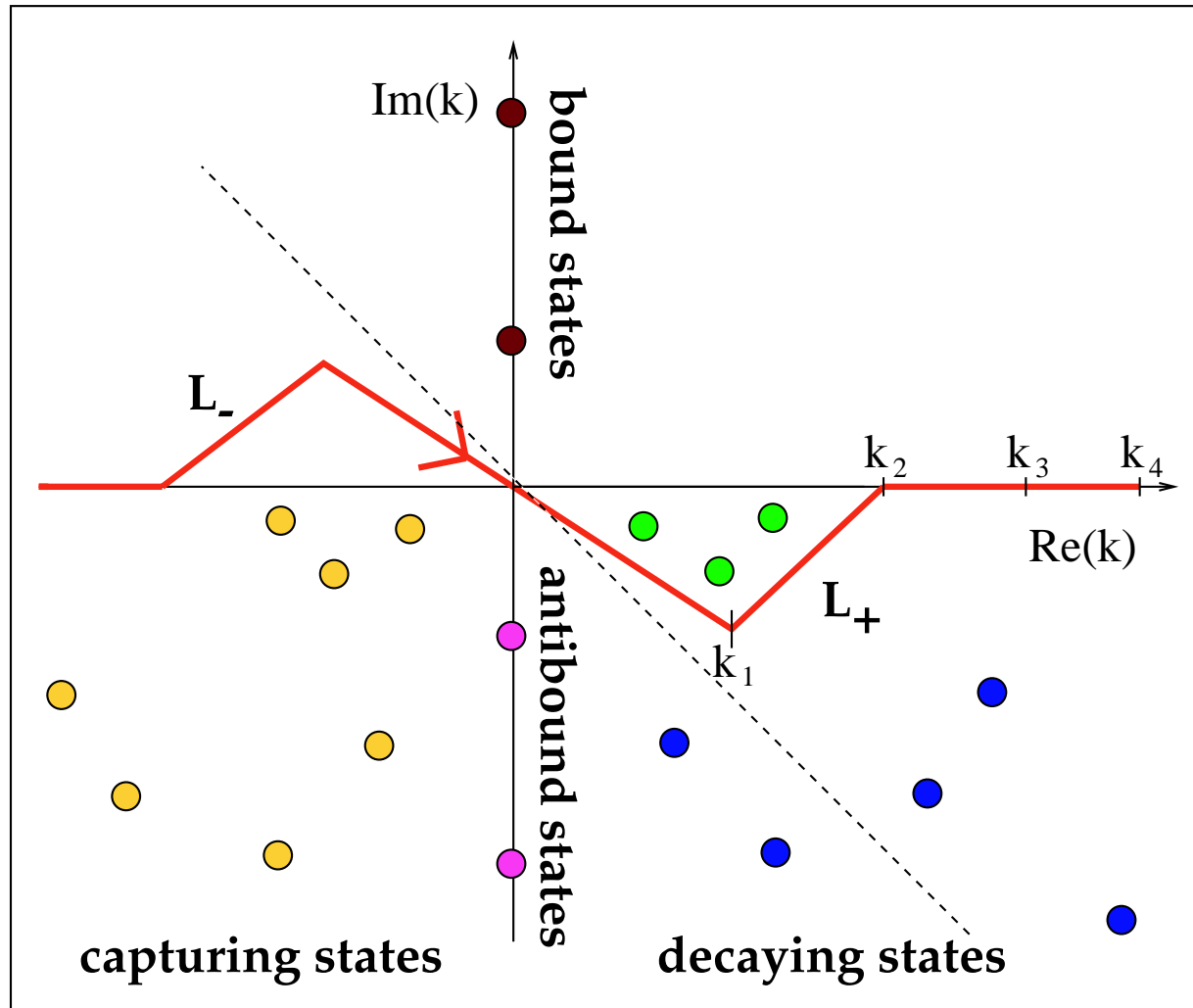
$$\sum_{b,d} |u_n\rangle \langle \widetilde{u}_n| + \int_{L^+} |u(k)\rangle \langle \widetilde{u}(k)| dk = 1$$

- Discretization of the continuum :  $u(k) \rightarrow \sqrt{\Delta k_i} u(k_i)$

- Discretized Berggren relation :  $\sum_{i=0}^{N-1} |u_i\rangle \langle \widetilde{u}_i| \simeq 1$

- Discretized  $N$ -body completeness relation :  $\sum_k |SD_k\rangle \langle SD_k| \simeq 1$

# Berggren basis in the complex $k$ -plane



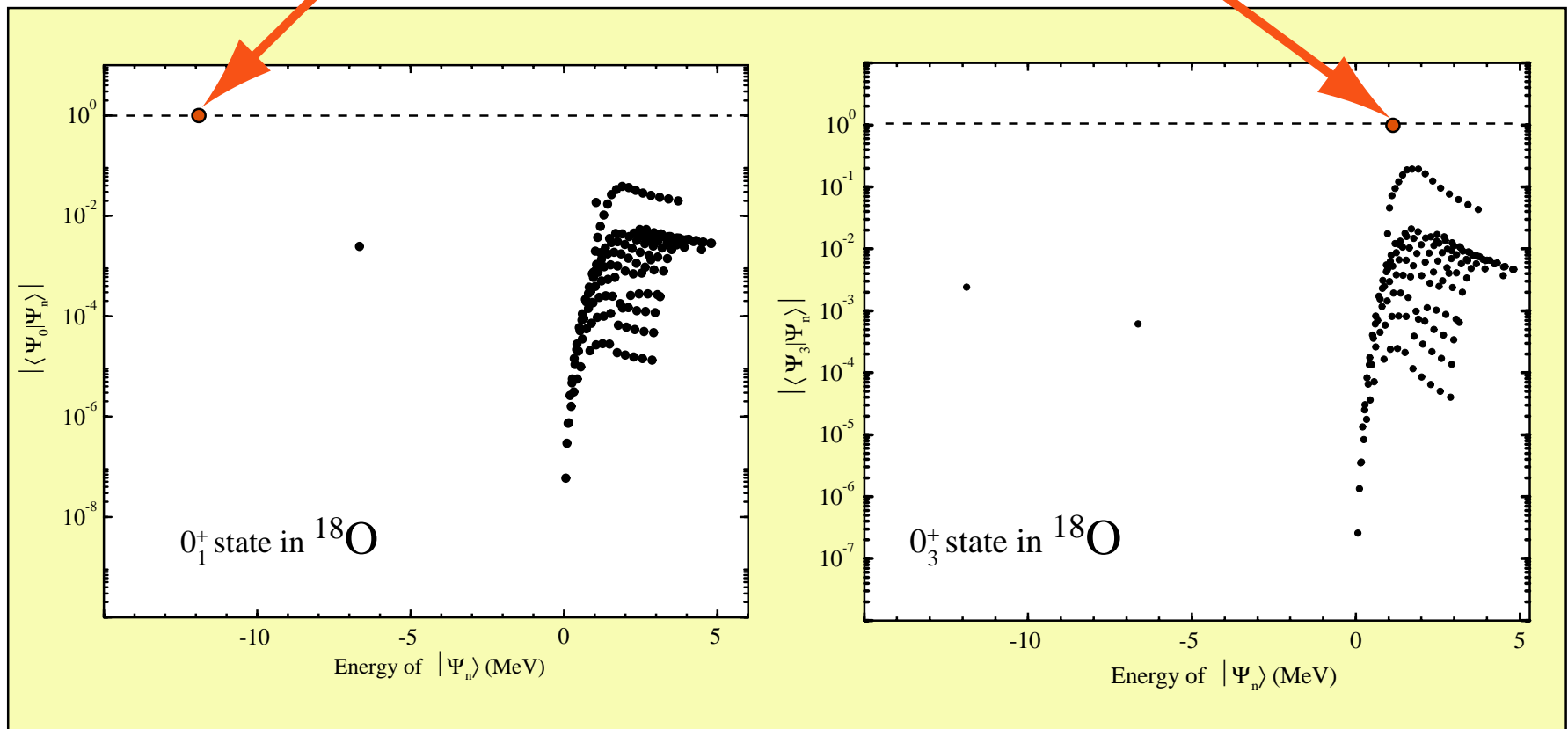
## Matrix diagonalization and physical states search

- Configuration space : large complex symmetric matrices.
- Lanczos method insufficient : low energy scattering nuclear states.
- Resonances hidden among these states.

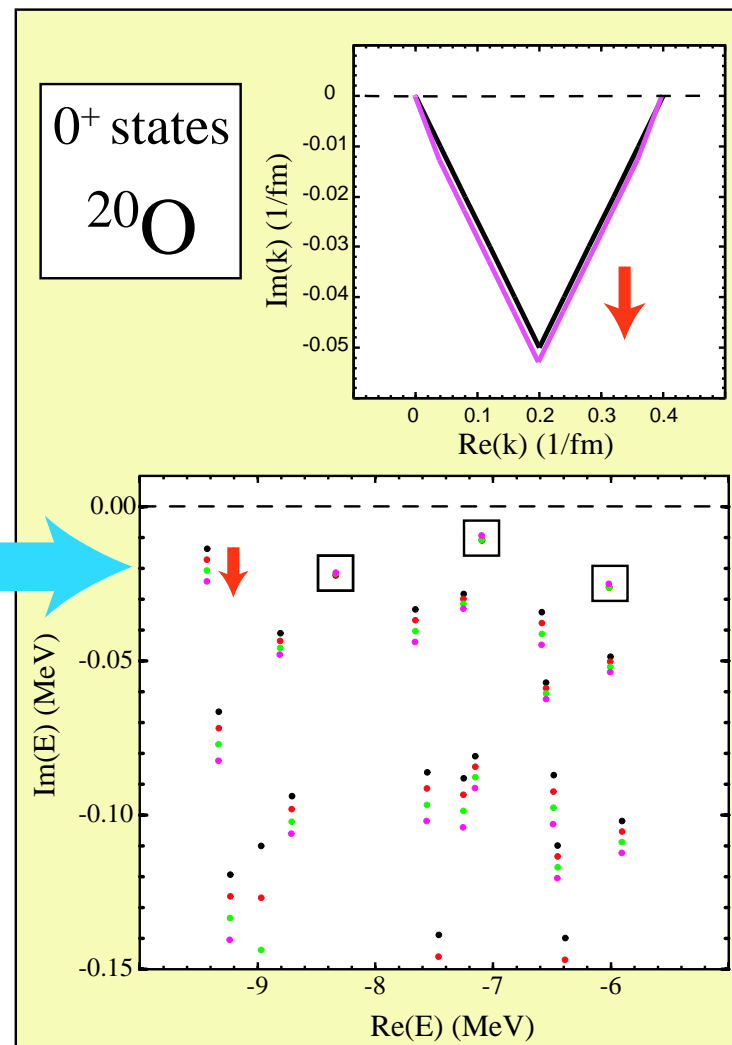
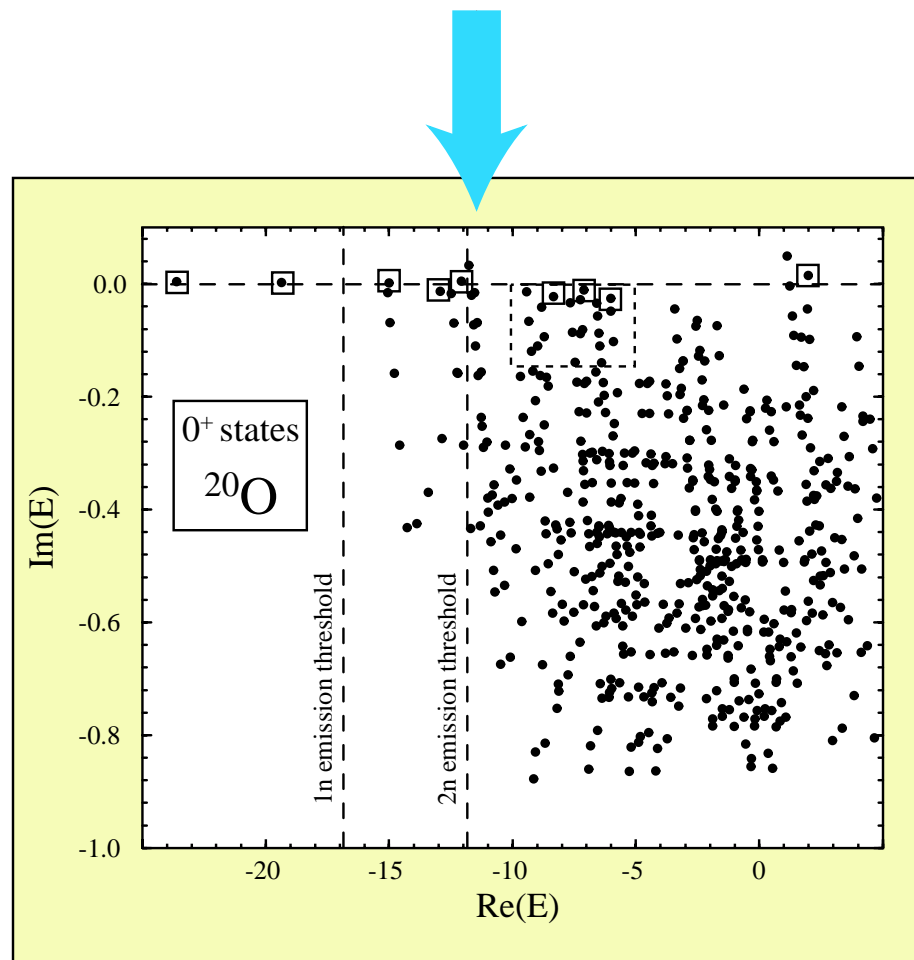
Two step search method :

- Diagonalization without continuum : Lanczos method.  
Zeroth order eigenvectors.
- Diagonalization with continuum.  
Eigenvector refinement within a Lanczos subspace.  
Largest overlap method ( $\simeq$  Davidson method).

# Determination of many-body bound and resonant states



# Stability of the 'physical' states



## Introduction of a new interaction for He and Li

- Surface Gaussian Interaction (SGI):

$$V_{SDI}(\vec{r}_1, \vec{r}_2) = V_0 \cdot \delta(\vec{r}_1 - \vec{r}_2) \cdot \delta(|\vec{r}_1| - R_0)$$

$$V_{SGI}(\vec{r}_1, \vec{r}_2) = V_0(J, T) \cdot \exp \left[ - \left( \frac{|\vec{r}_1 - \vec{r}_2|}{\mu} \right)^2 \right] \cdot \delta(|\vec{r}_1| + |\vec{r}_2| - 2 \cdot R_0)$$

- Finite range : more realistic, no cut-off needed.
- Smooth and localized HF potential : better basis.
- Radial matrix elements : finite one dimensional integrals.
- Easy to fit in a shell model calculation.

## Hamiltonians for the He, Li and O chains

- Valence particles outside a core :  ${}^4\text{He}$  for He and Li,  ${}^{16}\text{O}$  for O.
- Core given by a Woods-Saxon potential : fitted with single particle states.
- Residual interaction : SGI for the He and Li chains, SDI for the O chain.  
 $V_0(J, T = 1)$  fixed by He ground states.  
 $V_0(J, T = 0)$  linear function of  $N$  for the Li chain.

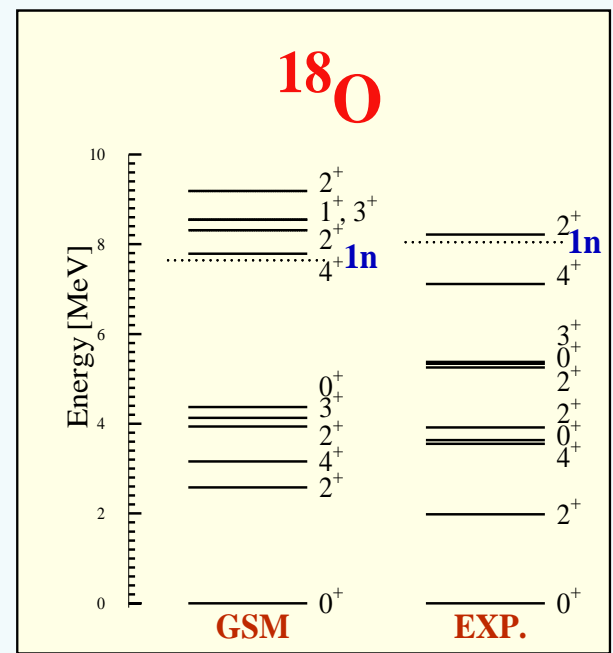
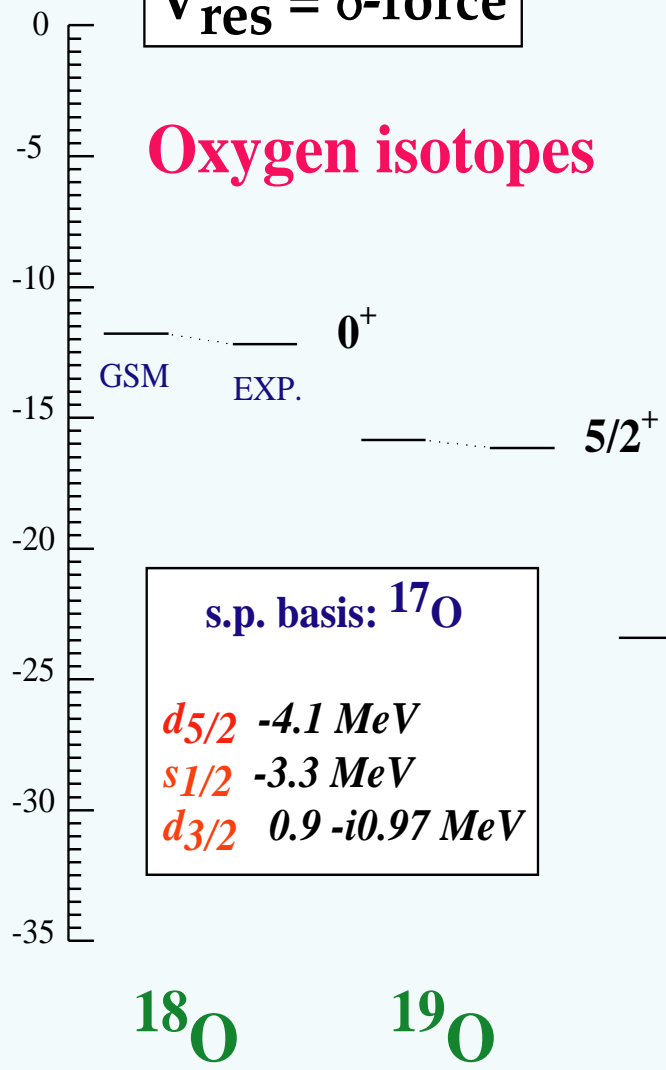
A.P. Zuker, Phys. Rev. Lett. 90, 042502 (2003).

- Valence space for the He and Li chains:  
spherical HF basis generated by the hamiltonian WS + SGI.  
 $0p_{3/2}$ ,  $0p_{1/2}$  bound or resonant.  
 $p_{3/2}$ ,  $p_{1/2}$  scattering contours : 14 points each.
- Valence space for the O chain:  
WS basis given by the WS of  ${}^{16}\text{O}$ .  
 $0d_{5/2}$ ,  $1s_{1/2}$  bound and  $0d_{3/2}$  resonant.  
 $d_{3/2}$  scattering contour : 5 to 10 points.

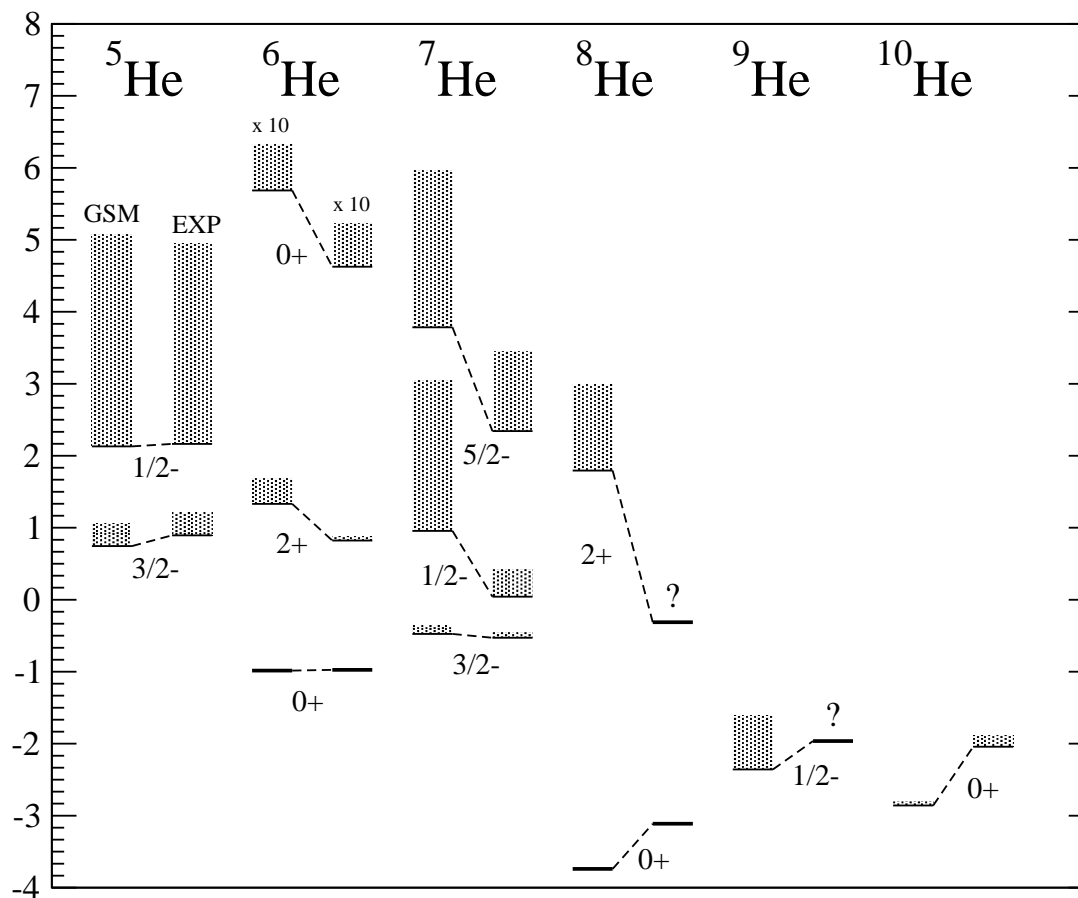
$V_{\text{res}} = \delta\text{-force}$

# Oxygen isotopes

energy [MeV]



# The He chain



# $^5\text{He}$ , $^6\text{He}$ and $^7\text{He}$ ground states configuration mixings

$^5\text{He}$ configuration	$c^2$
$0p_{3/2}^1$	1
$L_{+}^{(1)}$	0

$^6\text{He}$ configuration	$c^2$
$0p_{3/2}^2$	$0.656 - i0.566$
$0p_{1/2}^2$	$6.06 \cdot 10^{-3} - i0.0516$
$L_{+}^{(1)}$	$0.363 + i0.509$
$L_{+}^{(2)}$	$-0.0245 + i0.108$

$^7\text{He}$ configuration	$c^2$
$0p_{3/2}^3$	$0.331 - i0.0973$
$0p_{3/2}^1 0p_{1/2}^2$	$0.0111 - i0.0406$
$0p_{3/2}^2 0p_{1/2}^1$	$5.380 \cdot 10^{-4} - i6.950 \cdot 10^{-4}$
$L_{+}^{(1)}$	$0.507 + i0.0714$
$L_{+}^{(2)}$	$0.150 + i0.0672$

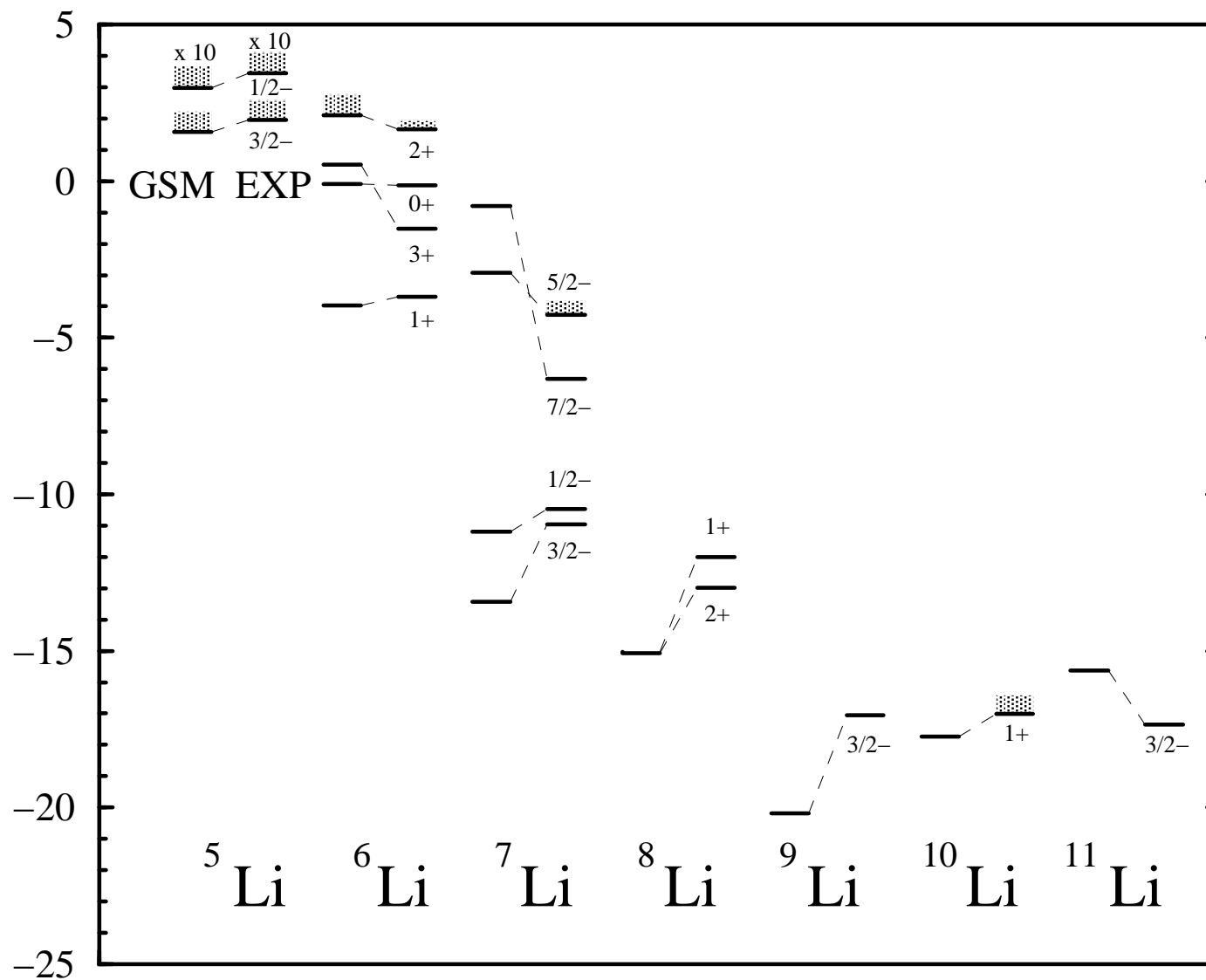
# $^8\text{He}$ , $^9\text{He}$ and $^{10}\text{He}$ ground states configuration mixings

$^8\text{He}$ configuration	$c^2$
$0p_{3/2}^4$	$0.889 - i7.826 \cdot 10^{-3}$
$0p_{3/2}^2 0p_{1/2}^2$	$0.0316 - i0.0529$
$L_{+}^{(1)}$	$0.0613 + i0.0226$
$L_{+}^{(2)}$	$0.0184 + i0.0225$

$^9\text{He}$ configuration	$c^2$
$0p_{3/2}^4 0p_{1/2}^1$	$0.937 - i0.0209$
$L_{+}^{(1)}$	$0.0473 - i1.779 \cdot 10^{-3}$
$L_{+}^{(2)}$	$0.0157 + i0.0227$

$^{10}\text{He}$ configuration	$c^2$
$0p_{3/2}^4 0p_{1/2}^2$	$0.965 - i0.0206$
$L_{+}^{(1)}$	$-5.640 \cdot 10^{-3} + i8.392 \cdot 10^{-3}$
$L_{+}^{(2)}$	$0.0409 + i0.0122$

# The Li chain



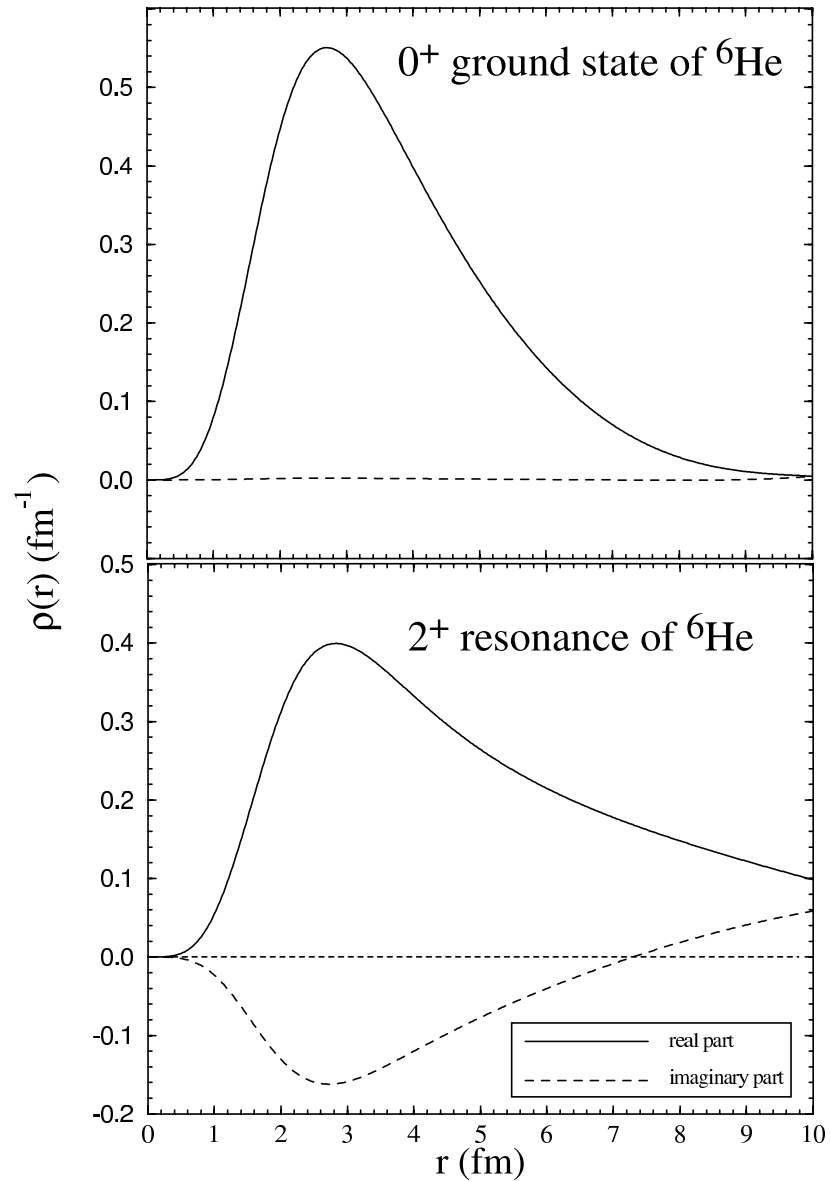
## Perspectives for GSM : total interaction

- Total interaction :  $H = T + V_{tot} + U_c + U_{sat} - E_{DC}$
- Interaction similar to the Gogny force :  $V_{tot} = V_{CE} + V_{DD} + V_{SO}$   
Finite range for all components.  
Halo nuclei and resonances possible in a HF calculation.
- spherical HF : basis and hamiltonian.
- $U_c$  : Coulomb potential (averaged Hartree approximation).  
Good asymptotic behavior and Coulomb energy.
- $U_{sat}$  : Saturation potential put in the hamiltonian.  
1p-1h vanish for closed shell nuclei.
- $E_{DC}$  : Double counting correction at the HF level.

## Perspectives for GSM : center of mass correction

- Lawson method :  $H \rightarrow H + \lambda \cdot \left( H_{cm} - \frac{3}{2} \hbar \omega \right)$
- Problem with HF states : not HO states.  
 $R^2$  incompatible with finite depth wave functions.
- Solution : HF states  $\rightarrow$  HO states for the  $H_{cm}$  matrix elements.  
Approximation :  $H_{cm}$  no longer an exact center of mass operator.
- $H_{cm} = \widetilde{\pi}^\dagger \left( \frac{P^2}{2M} + \frac{1}{2} M \omega^2 R^2 \right) \pi$  ,  $\pi = \sum_{n \in \text{res.}} |n_{HO}\rangle \langle \widetilde{n}_{res}|$
- Tests of the method :  
 $\langle \Psi | \frac{P^2}{2M} | \Psi \rangle \simeq \text{const}$  for all nuclear states.  
 $\langle \Psi | L_{CM}^2 | \Psi \rangle \simeq 0$  for all nuclear states.

# Density distribution of valence neutrons ${}^6\text{He}$



## Conclusions and outlook

- Generalization of the shell model into the complex  $k$ -plane :
  - ⇒ complete description of correlations in bound and unbound states
  - ⇒ no limitation on the number of particles in the continuum
  - ⇒ pole expansion insufficient if one aims at shell model accuracy.

Future developments :

- Better discretization approximation for the contours in the continuum.
- Density dependent interaction : HF basis and shell model matrix.
- Density Matrix Renormalization Group (DMRG) approach:  
Method to deal with the numerous scattering configurations.