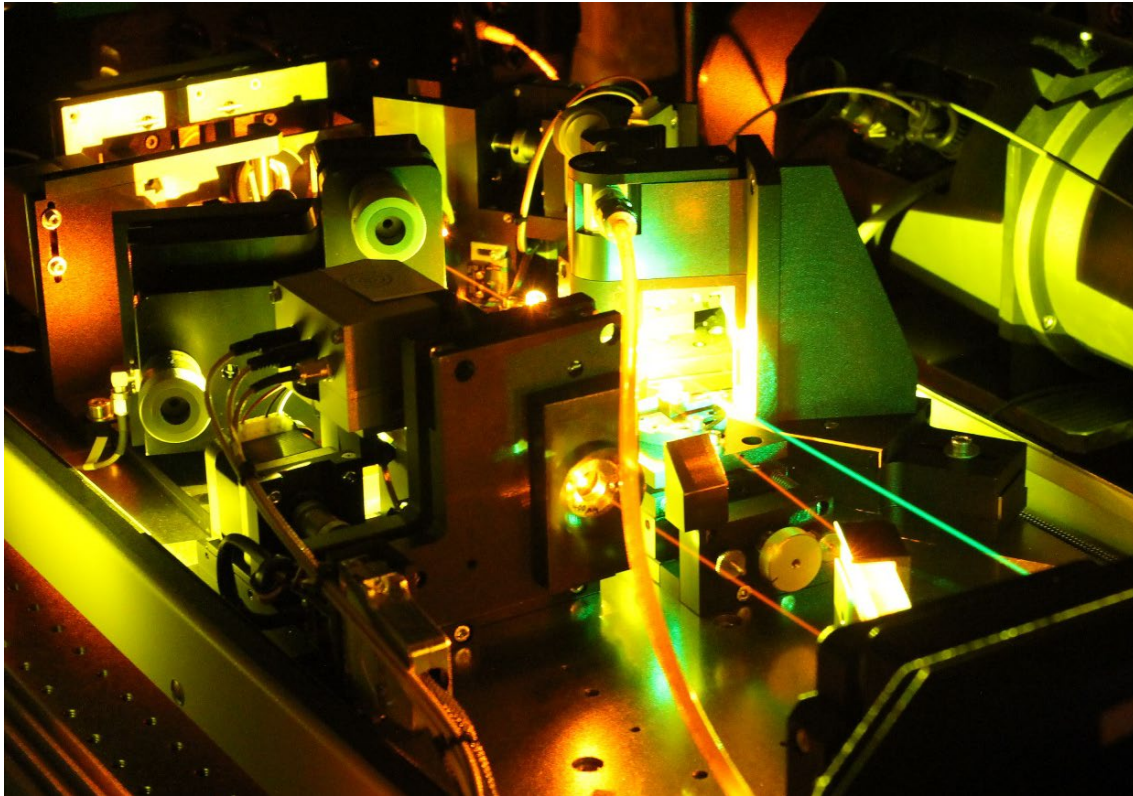


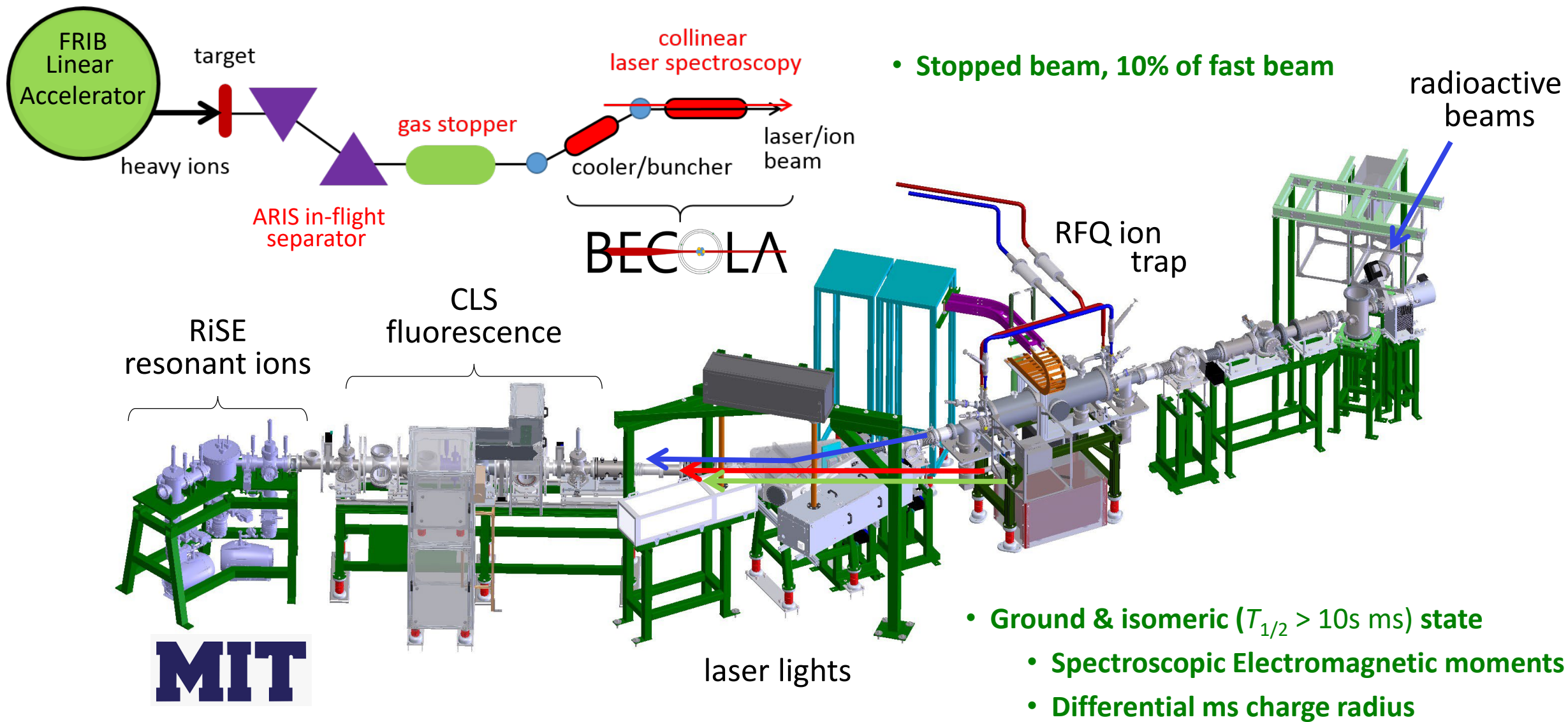
## Laser spectroscopy of rare isotopes at FRIB



**Kei Minamisono**  
**for BECOLA collaboration**

# BECOLA facility @ NSCL/FRIB/MSU

## - Bunched beam collinear laser spectroscopy -



# Electromagnetic moments

$$H_{\text{EM}} = \int \rho(\mathbf{r})\phi(\mathbf{r})d\mathbf{r} - \int \mathbf{j}(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r})d\mathbf{r}$$

$$= \underbrace{q\phi(0)}_{\text{e-monopole}} - \underbrace{\mathbf{P} \cdot \mathbf{E}(0)}_{\text{e-dipole}} - \underbrace{\boldsymbol{\mu} \cdot \mathbf{H}(0)}_{\text{m-dipole}} - \frac{1}{6} \sum_{ij} \underbrace{Q_{ij}}_{\text{e-quadrupole}} \left( \frac{\partial E_j}{\partial x_i} \right)_0 + \dots$$

$$q = \int \rho(\mathbf{r})d\mathbf{r} : \text{total charge}$$

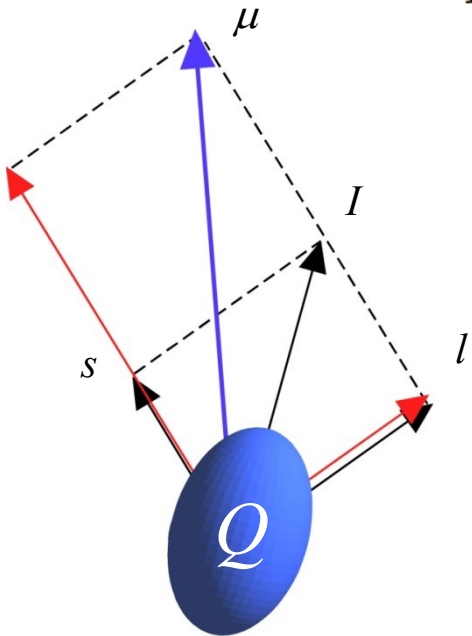
$$\mathbf{P} = \int \rho(\mathbf{r})\mathbf{r}d\mathbf{r} : \text{electric dipole moment} \rightarrow 0 \text{ (time reversal)}$$

$$\text{magnetic dipole moment : } \boldsymbol{\mu} = \int \mathbf{r} \times \mathbf{j}(\mathbf{r}) = \mu_{\text{N}} (\langle \mathbf{l} \rangle + g_{\text{p}} \langle \mathbf{s} \rangle + g_{\text{n}} \langle \mathbf{s} \rangle)$$

$$\text{electric quadrupole moment : } Q_{ij} = \int \rho(\mathbf{r}) (3x_i x_j - \delta_{ij} r^2) d\mathbf{r}$$

$\boldsymbol{\mu}$  : spin, angular momentum, configuration of nucleons  $\leftrightarrow B(\text{M1})$

$Q$  : deviation of proton distribution from spherical symmetry, static deformation  $\leftrightarrow B(\text{E2})$

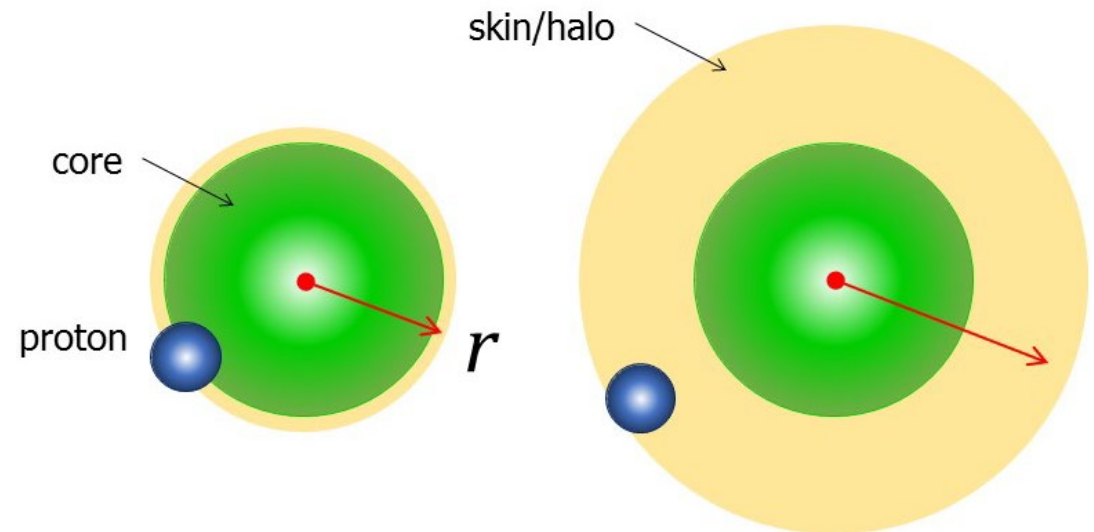
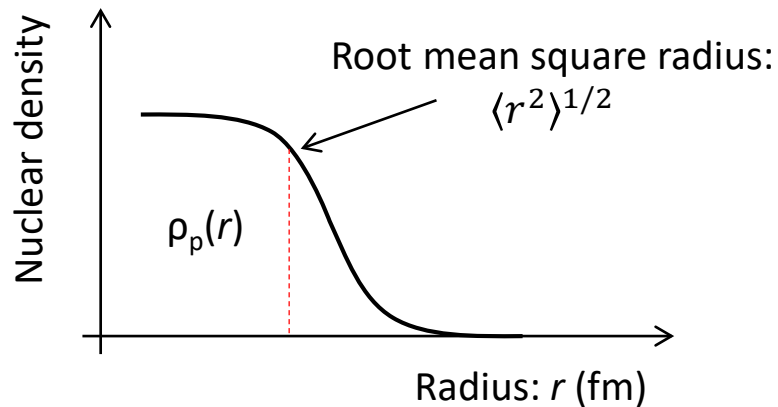


# Mean-square charge radius

$$\langle r^2 \rangle \sim \langle r^2 \rangle_{\text{sph}} \left( 1 + \frac{5}{4\pi} \langle \beta_2^2 \rangle \right)$$

$\langle r^2 \rangle_{\text{sph}}$  : charge radius of spherical core

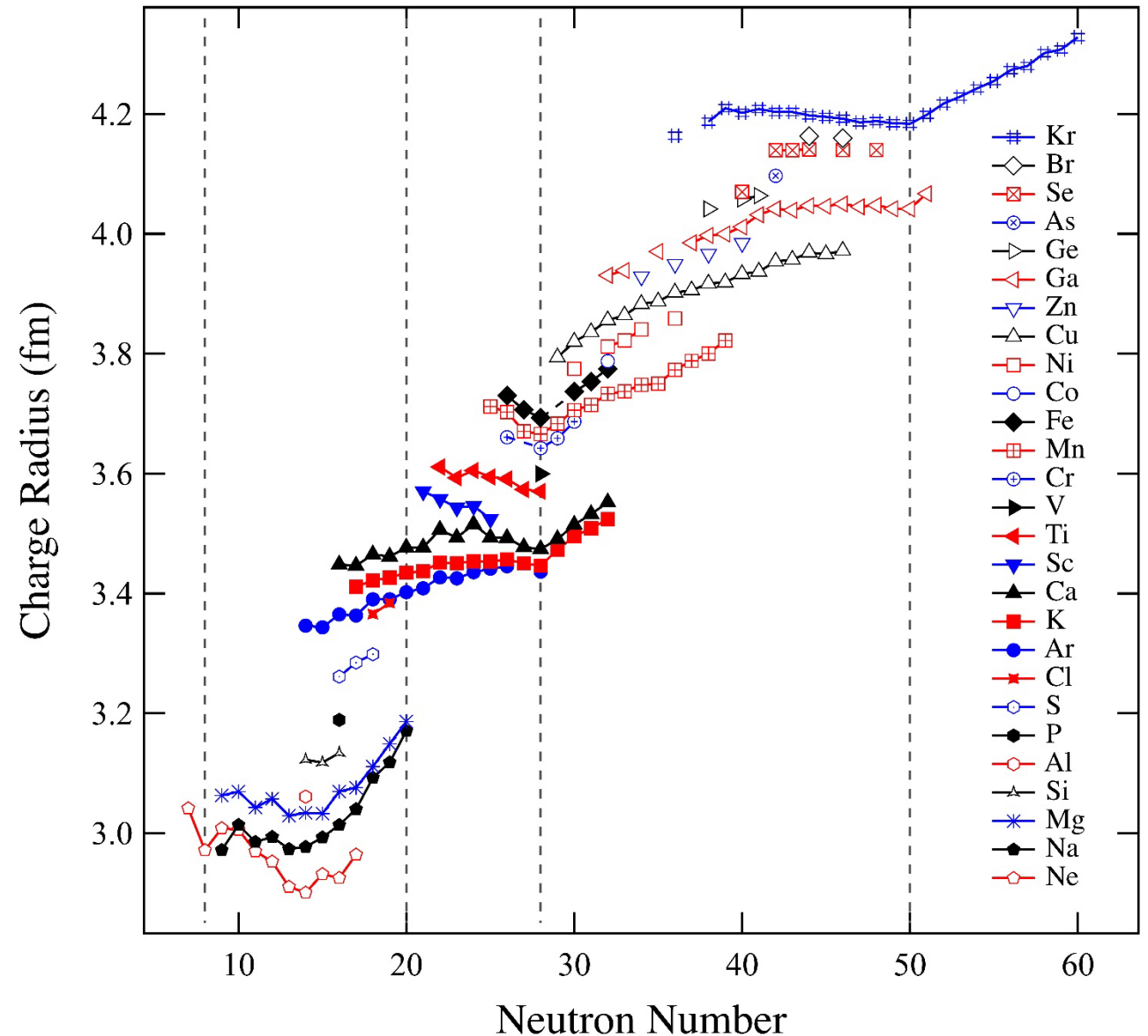
$\langle \beta_2^2 \rangle$  : quadrupole deformation



$\langle r^2 \rangle$  is sensitive to size/shape of nucleus, static and dynamic deformation (vibration)

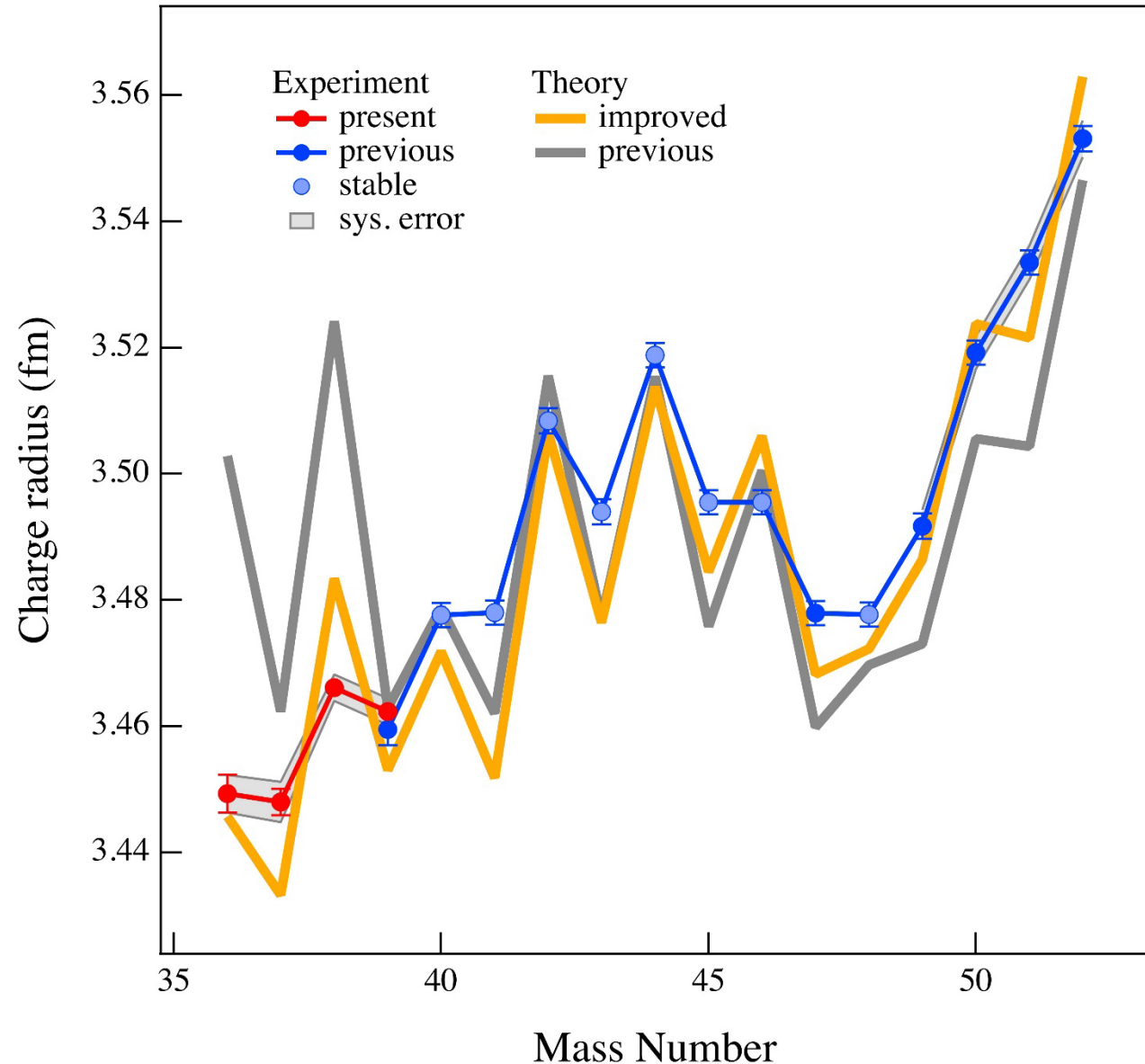
# Example: charge radii between Ne though Kr

- Radius gets larger as N and Z get bigger.
- Always satisfies:  $R(Z) < R(Z+1)$
- However not for  $R(N)$ , which shows microstructures
  - smooth increase/decrease
  - zig-zag pattern
  - parabolic change
  - sharp kinks
- All these local variations are the manifestation of underlying nuclear structures, that is what we are looking for.



# Science example: neutron-deficient Ca

- Light mass Ca isotopes are very compact.
- Maintains the zig-zag pattern (odd-even stagger)
- Our new measurements (red) deviates from the previous theory.
- The improved theory that include “superconductivity” of nuclei reproduces our data better.
- Fayans EDF reproduces Ca chain remarkably well.



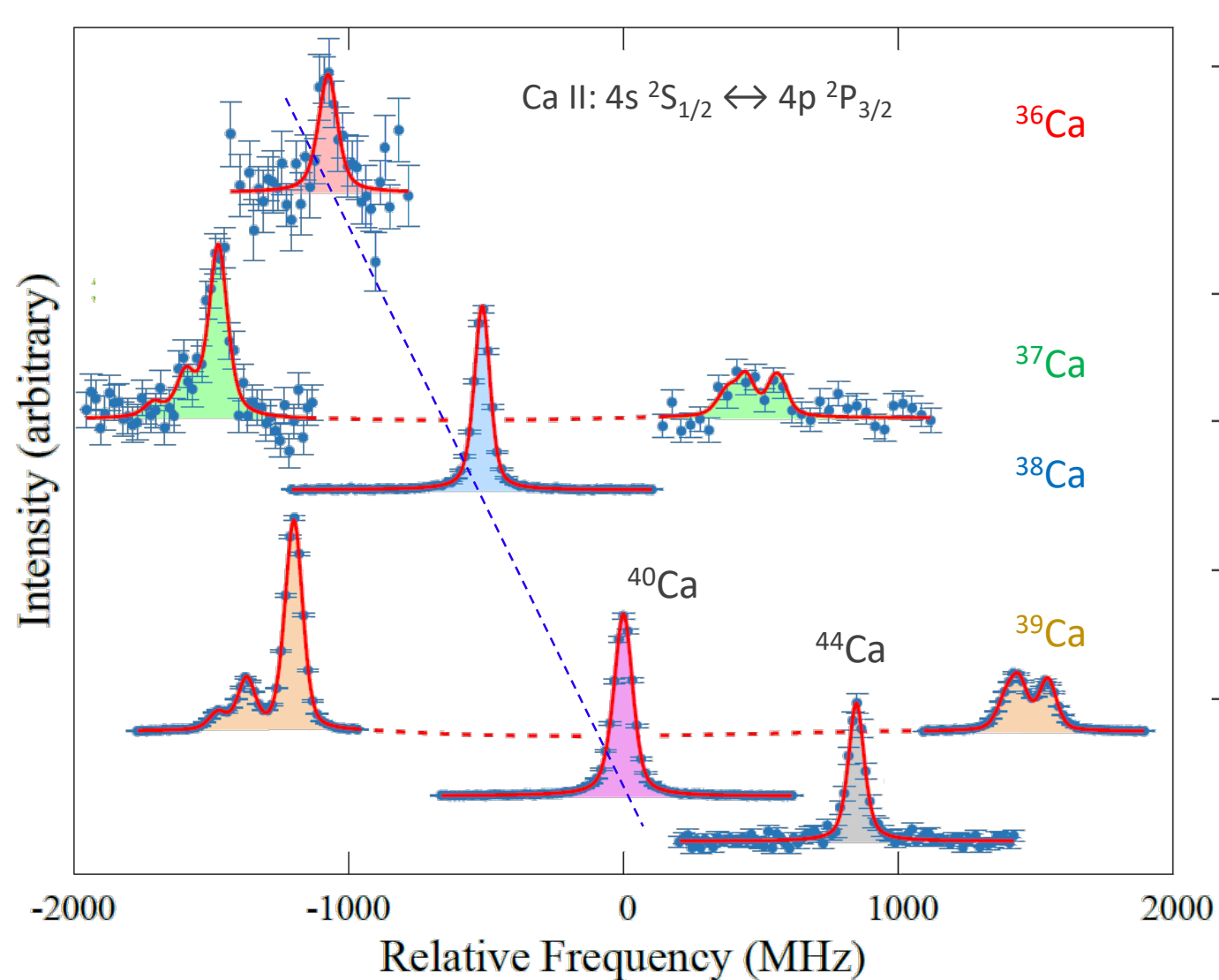
# Science example: neutron-deficient Ca

- Hyperfine spectra that shows isotope shifts. Note that
  - Even-even isotope: single peak
  - Even-odd isotope: multi peaks
- From the relative shifts drm radii can be extracted.

Atomic factors well determined:

$$\begin{cases} k = 409.35(42) \text{ GHz amu} \\ F = -284.7(82) \text{ MHz fm}^{-2} \end{cases}$$

Isotope shifts of hyperfine spectra of Ca

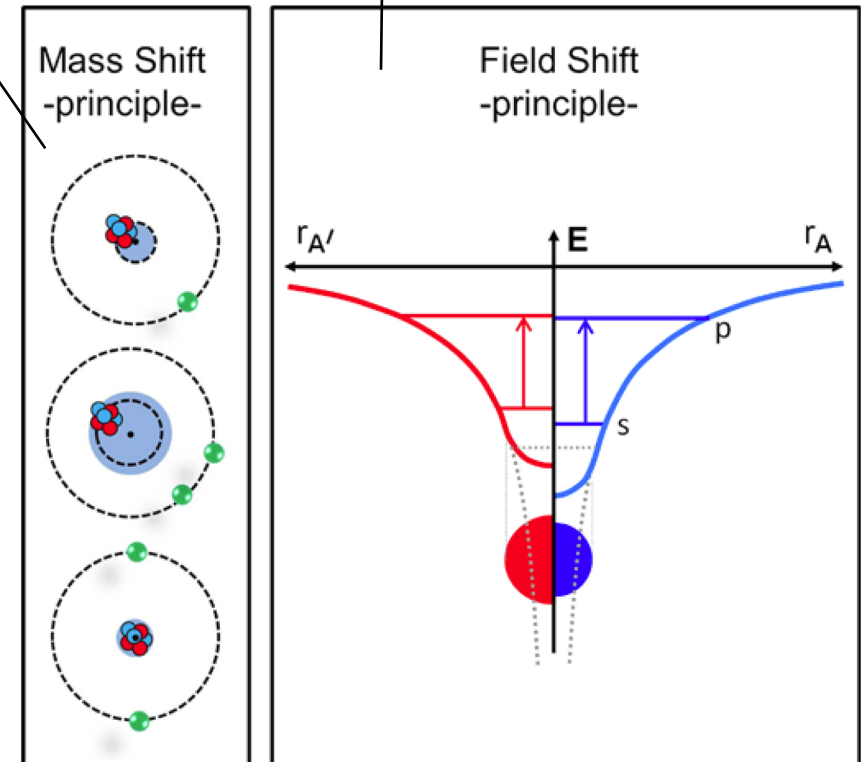


rate (1/s)	$T_{1/2}$ (ms)
50	102
960	181
13.5k	440
60k	859.6

# Charge radius: isotope shift of fine structure energies

$$\delta\nu^{A,A'} = \nu^{A'} - \nu^A = k \frac{M' - M}{M'M} + F \times \delta\langle r^2 \rangle^{A,A'}$$

- **Model independent,  $R$  can be determined with  $\sim 0.005$  fm**
- **Sensitive to  $\delta\langle r^2 \rangle$  and requires reference to deduce absolute charge radius:  $R^2 = R_{\text{ref}}^2 + \delta\langle r^2 \rangle$** 
  - $R_{\text{ref}}$  can be evaluated from e-scattering and  $\mu$ -capture experiments.
  - but  $R_{\text{ref}}$  is not always available with high enough precision we want.
- **Using King plot,  $k$  and  $F$  can be experimentally evaluated,**
  - IF there are  $\geq 3$  (stable) isotopes of the element, whose  $R$  are know.
  - otherwise need to rely on atomic theories
    - Typically with a few  $\sim 10\%$  uncertainty
    - ab-initio is feasible for 5 electron systems so far.
- In general,  $\delta\langle r^2 \rangle$  is replaced by  $\delta\langle r^2 \rangle + \tilde{c}_2 \delta\langle r^4 \rangle + \tilde{c}_3 \delta\langle r^6 \rangle + \dots$ 
  - Contribution is very small and difficult to determine

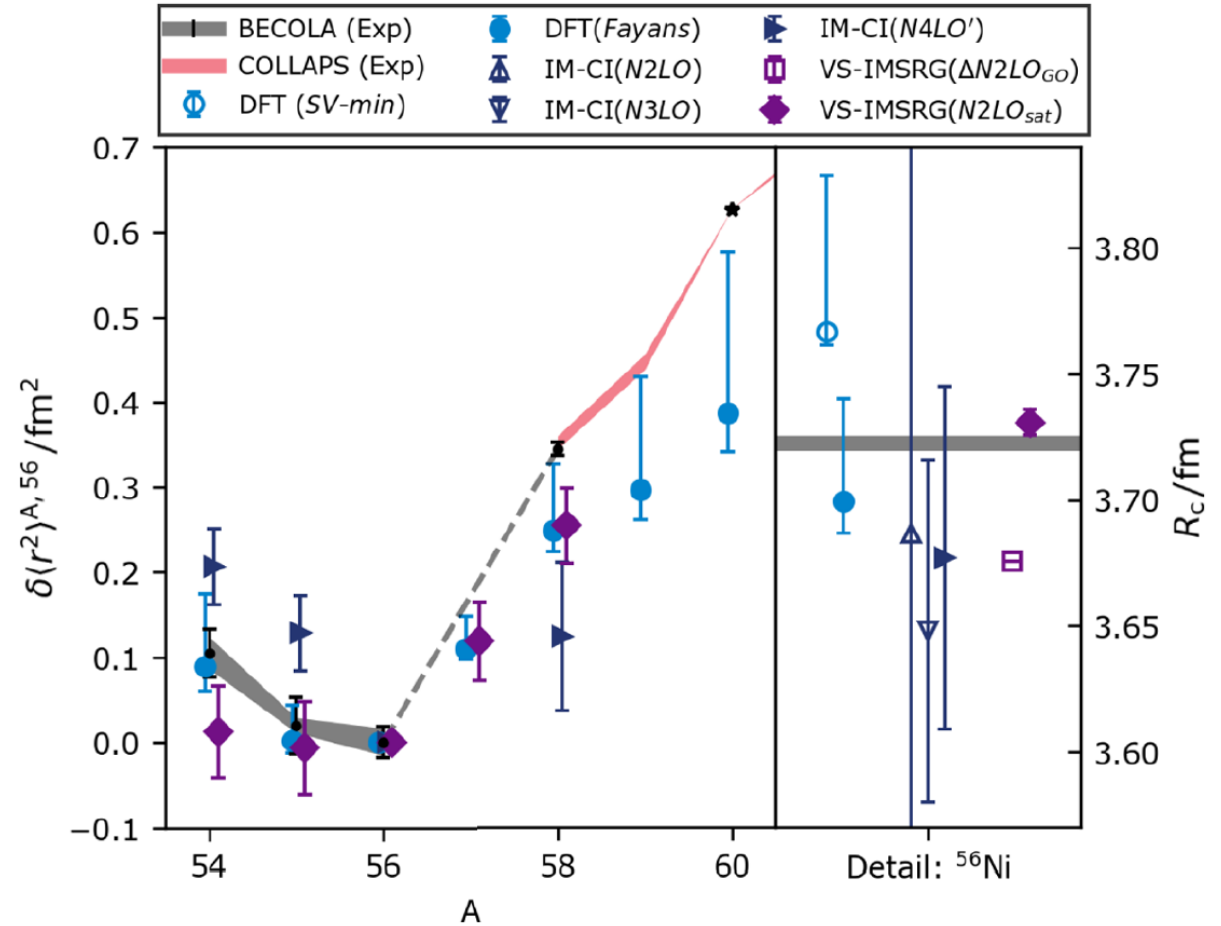
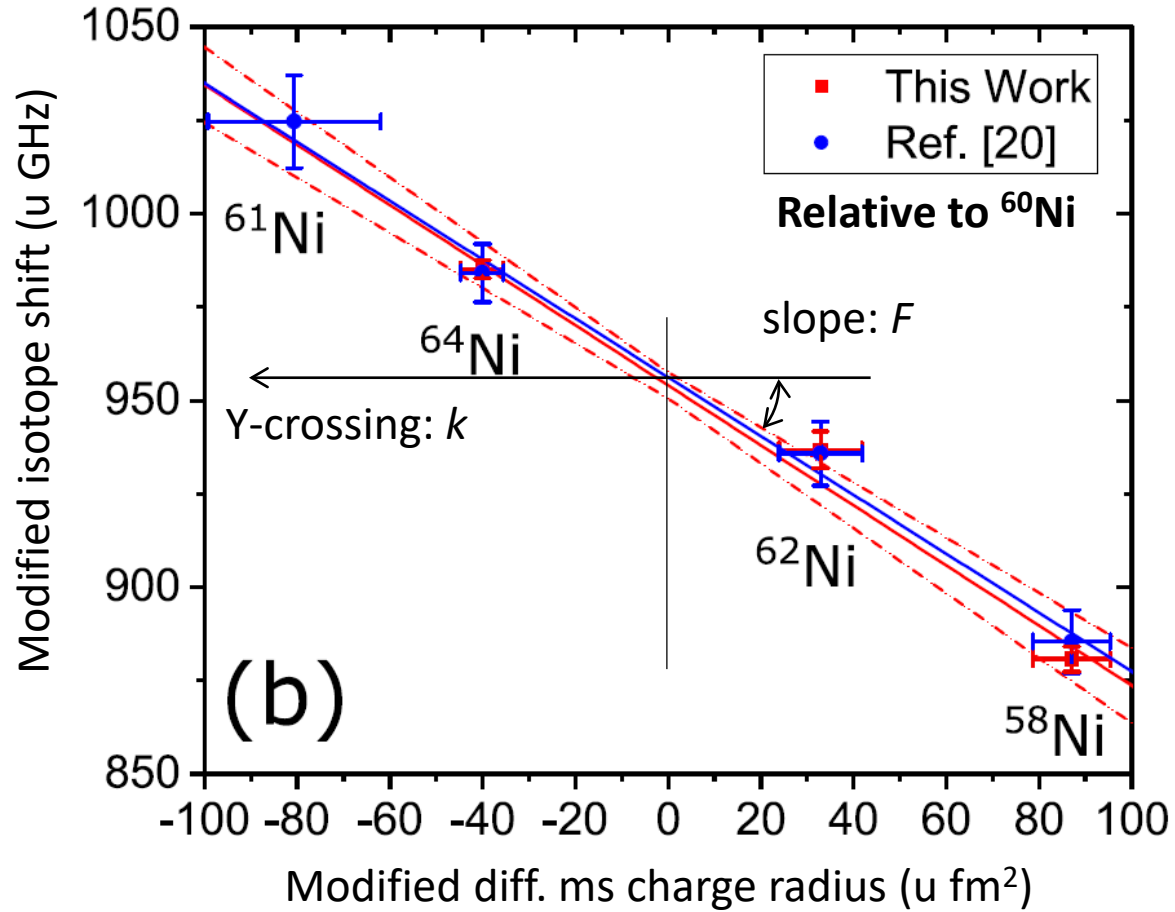


Change of the center of the motion energy

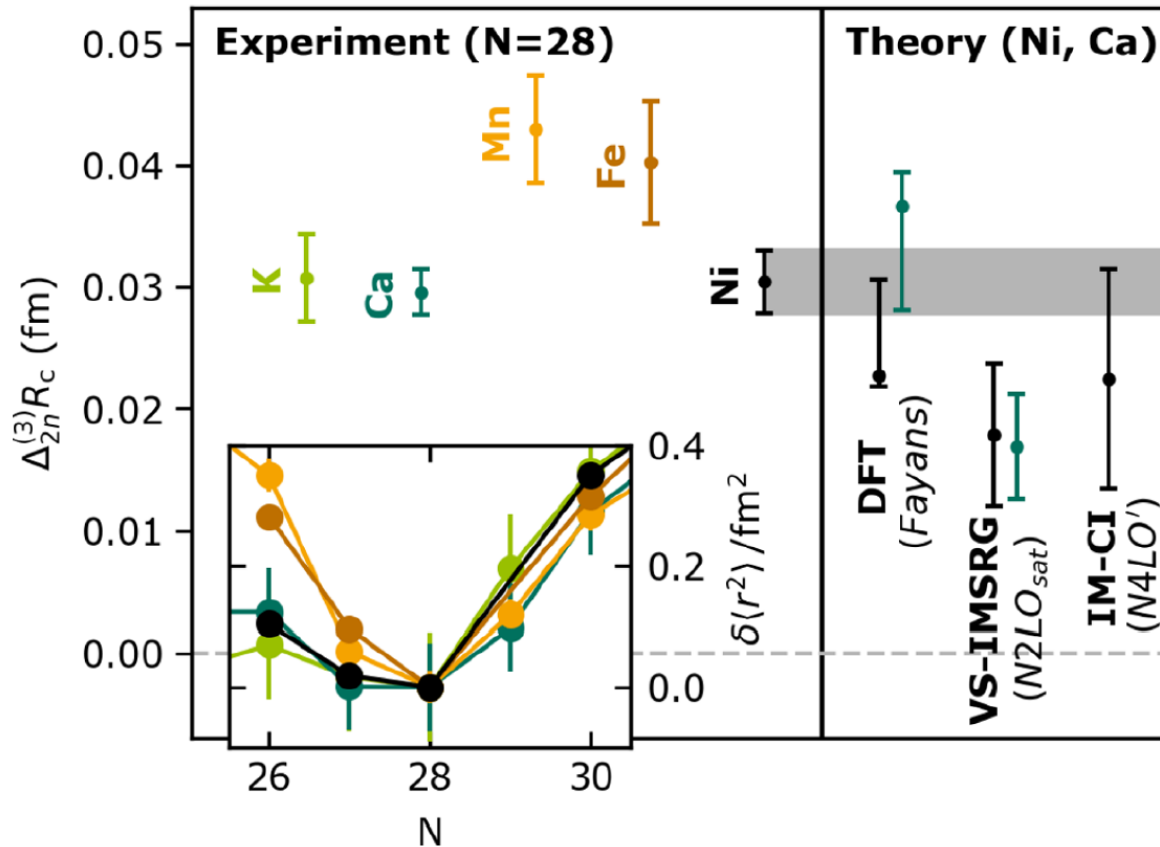
Change of the size (radius) of the nucleus



# Science example: neutron-deficient Ni



# Science example: neutron-deficient Ni

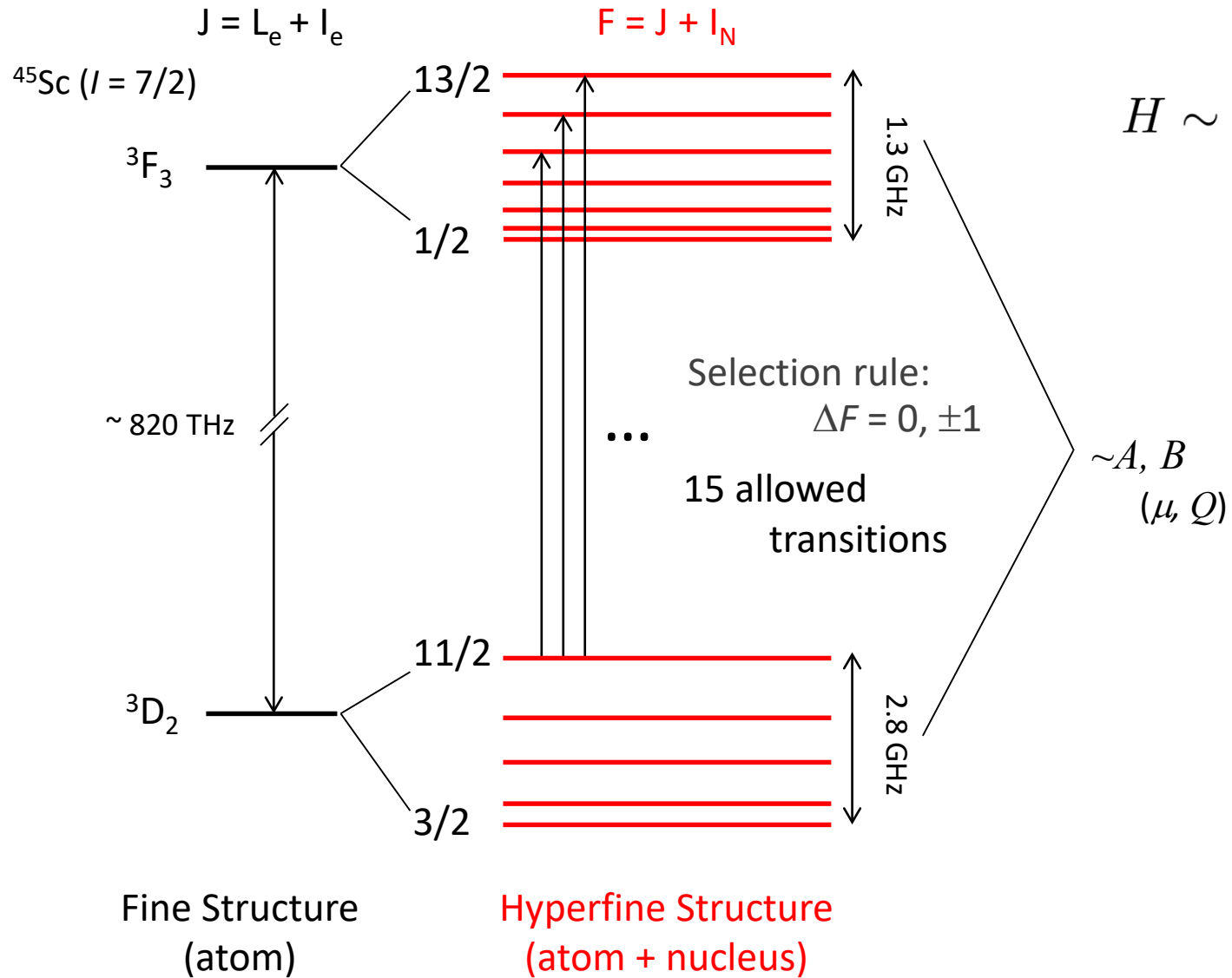


- DFT and ab-initio calculations reproduce Ni radii well.
- Ni and Ca show very similar kink structure at  $N = 28$
- **Kink does not directly reflect the strength of a shell closure.**

Three-point indicator for kink structure:

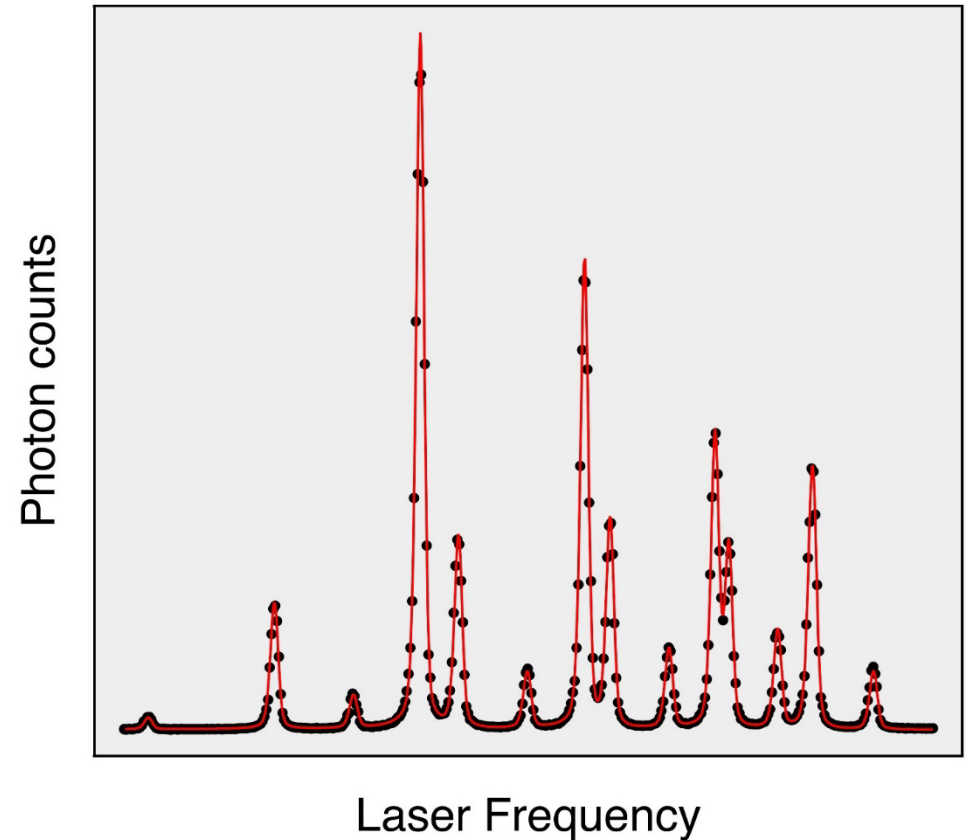
$$\Delta_{2n}^{(3)} R_c(N) = \frac{1}{2} [R_c(N + 2) - 2R_c(N) + R_c(N - 2)]$$

# Electromagnetic moments: HF structure



$$H \sim A \cos(\widehat{IJ}) + B \frac{3 \cos^2(\widehat{IJ}) - 1}{2}$$

$^{45}\text{Sc } 3d4s \ ^3D_2 \leftrightarrow 3d4p \ ^3F_3 \text{ @ } 363 \text{ nm}$



# Electromagnetic moments: HF structure

Magnetic dipole

$$\left\{ \begin{array}{l} A = \frac{\mu B_M(0)}{IJ} \\ \mu = \mu_R \frac{A}{A_R} \frac{I}{I_R} \end{array} \right.$$

Electric quadrupole

$$\left\{ \begin{array}{l} B = eQ \left\langle \frac{\partial^2 V_e}{\partial z^2} \right\rangle_0 \\ Q = Q_R \frac{B}{B_R} \end{array} \right.$$

Higher order moments

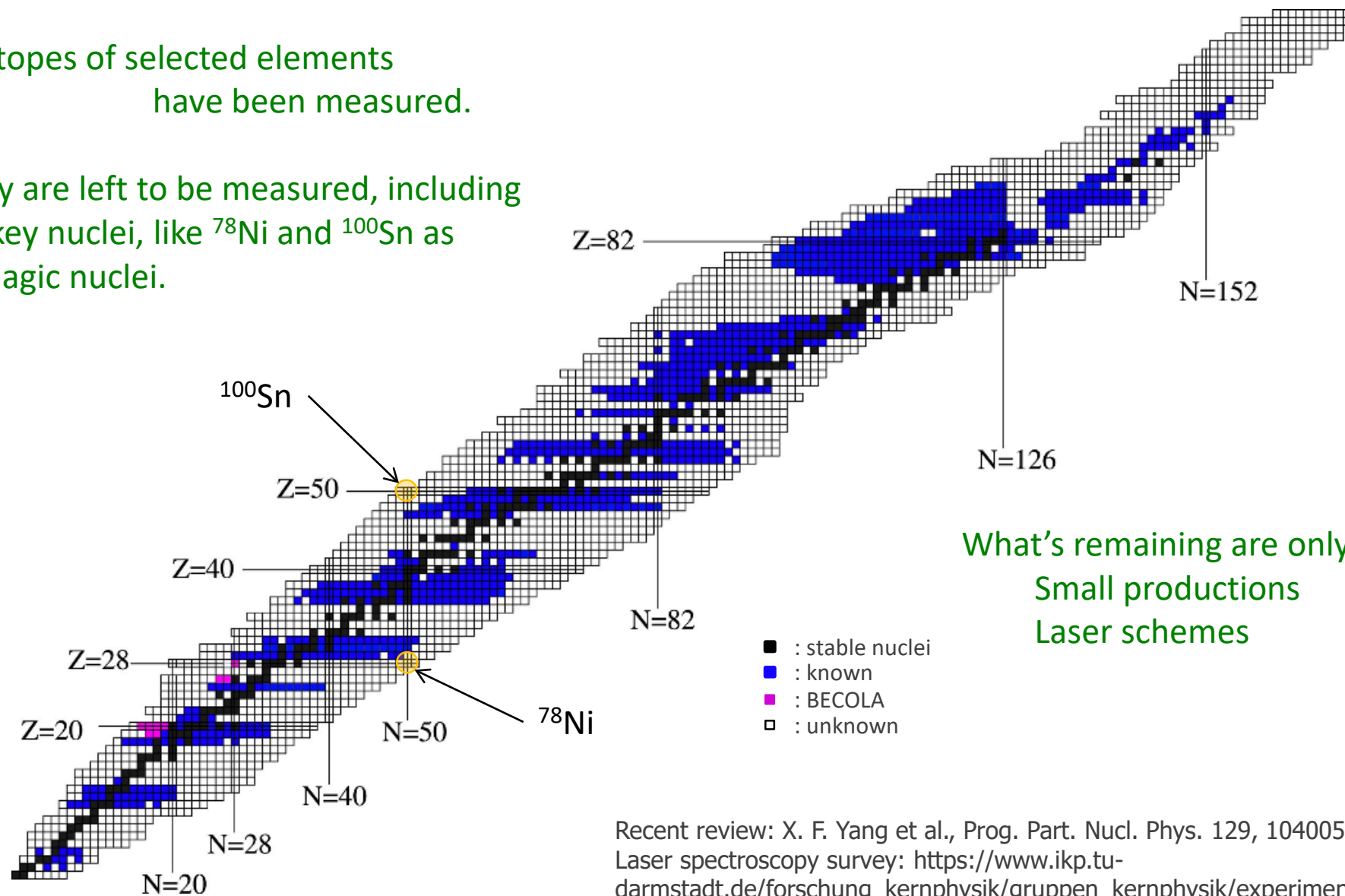
- **Dominates the pattern of hyperfine spectrum**
  - $A$  and  $\mu$  can be determined with high precision ( $\ll 1\%$ )
    - Need reference to deduce unknown  $\mu$
    - Precise  $\mu_R$  is available from NMR or  $\beta$ -NMR measurements
  - Can “measure” nuclear spin  $I$
- **Smaller contribution to the hyperfine spectrum**
  - $B$  and  $Q$  can only be determined with poorer precision (several  $\sim 10\%$ )
    - Need reference to deduce unknown  $Q$
  - Eventually need to rely on calculations of the field gradient  $d^2V/dz^2$
- **Much smaller and in general difficult to deduce from hyperfine spectra**
- Specific system
- RF and/or microwave spectroscopy

# Laser spectroscopy measurements

-  $I, \mu, Q, \delta\langle r^2 \rangle$  -

Many isotopes of selected elements have been measured.

Still, many are left to be measured, including some of key nuclei, like  $^{78}\text{Ni}$  and  $^{100}\text{Sn}$  as doubly Magic nuclei.



What's remaining are only difficult cases.  
Small productions  
Laser schemes

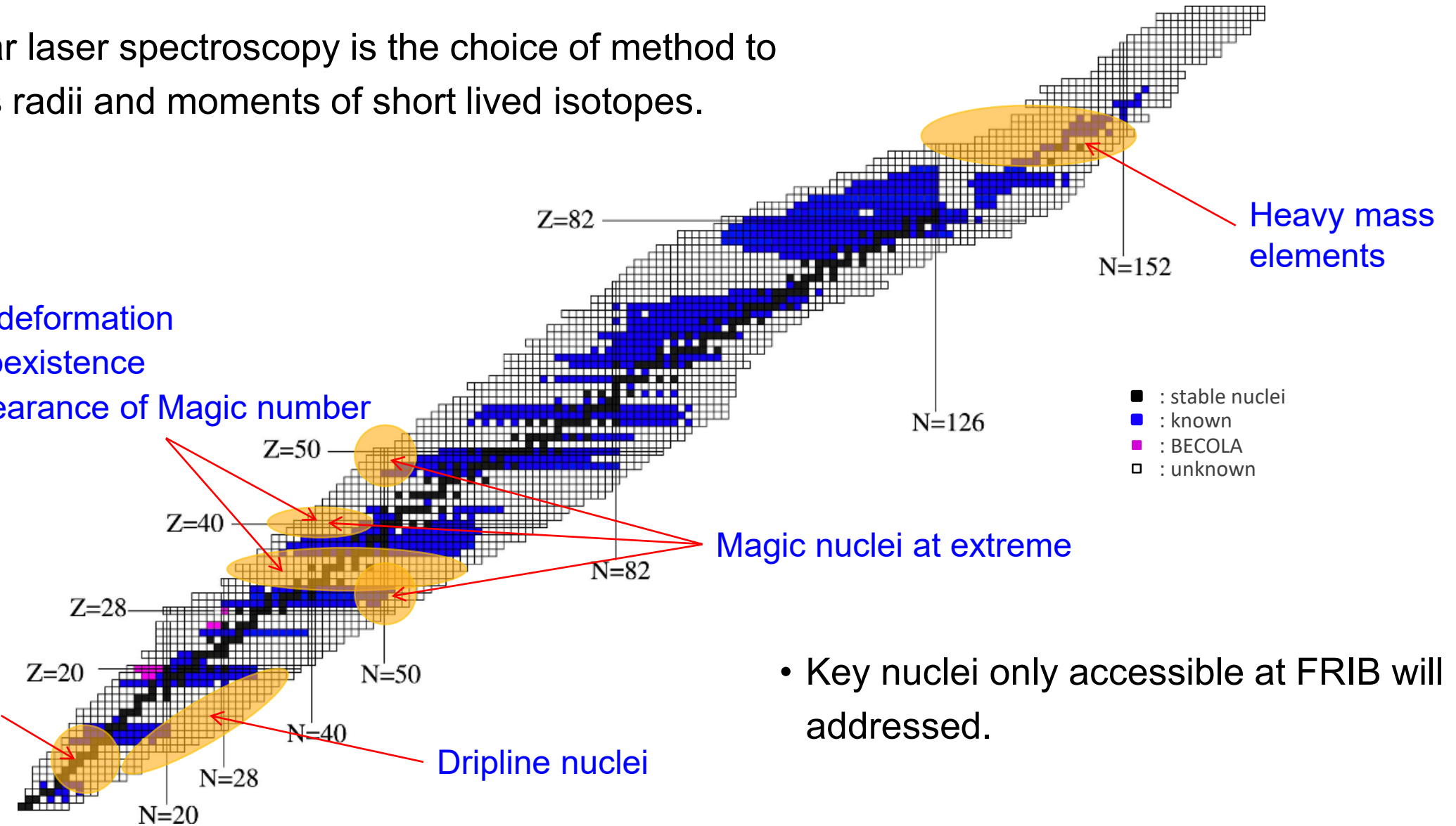
Recent review: X. F. Yang et al., Prog. Part. Nucl. Phys. 129, 104005 (2023);  
Laser spectroscopy survey: [https://www.ikp.tu-darmstadt.de/forschung\\_kernphysik/gruppen\\_kernphysik/experiment/ag\\_w\\_noertershaeuser/](https://www.ikp.tu-darmstadt.de/forschung_kernphysik/gruppen_kernphysik/experiment/ag_w_noertershaeuser/)

# Laser spectroscopy coming 5-10 years

- Collinear laser spectroscopy is the choice of method to address radii and moments of short lived isotopes.

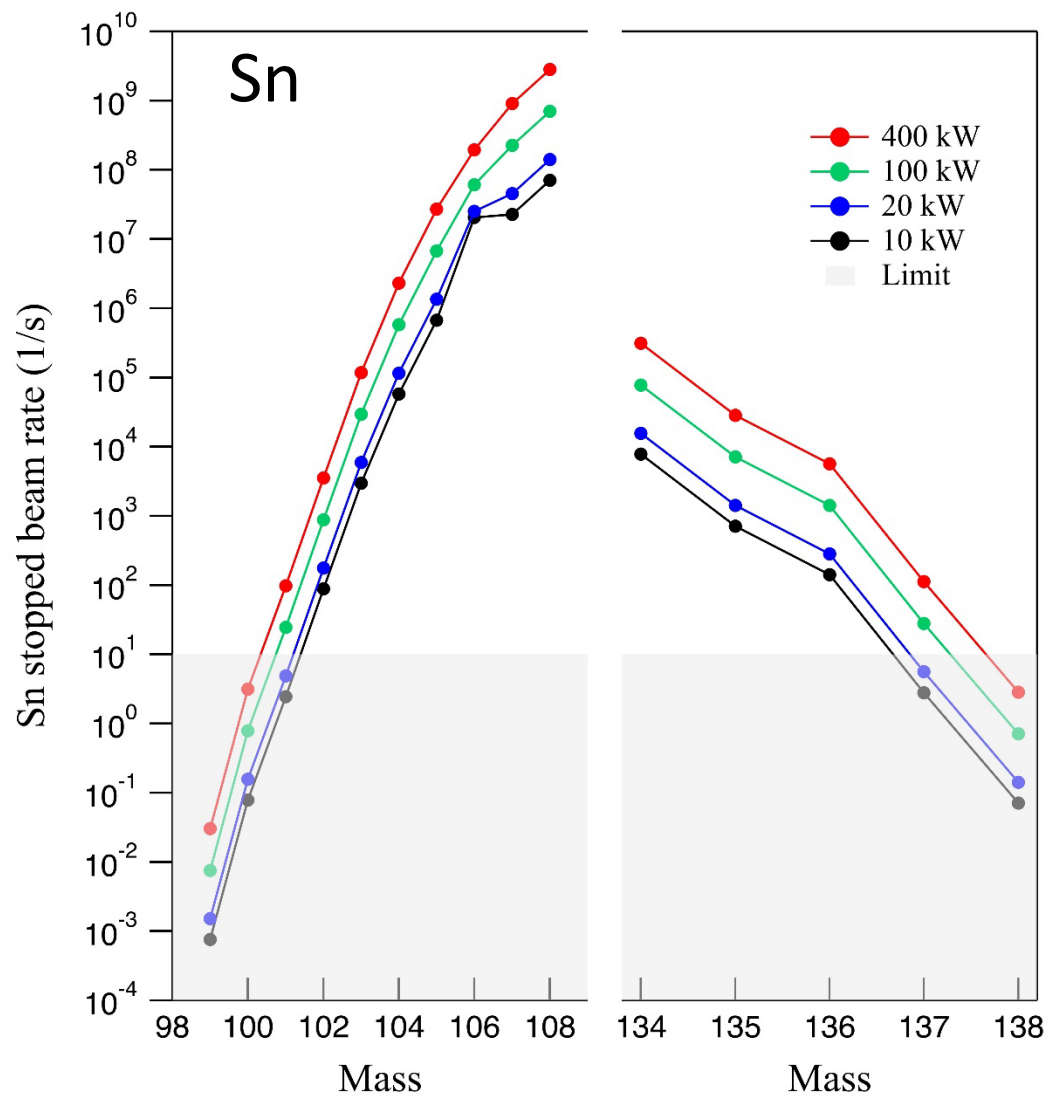
- onset of deformation
- shape coexistence
- (dis)appearance of Magic number

Light mass elements



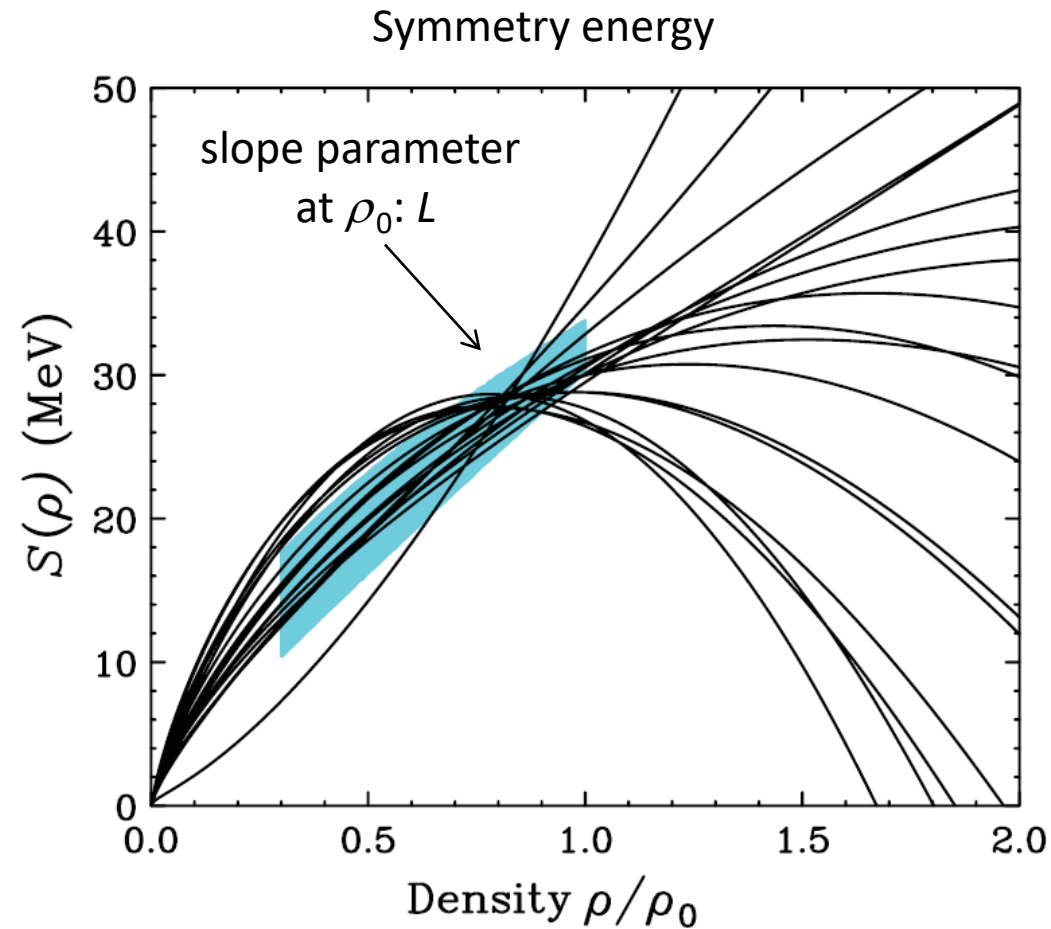
- Key nuclei only accessible at FRIB will be addressed.

# Laser spectroscopy and rate of Sn



- Significant number of isotopes are within the reach at FRIB  $\leq 20$  kW for laser spectroscopy.
- However, key nuclei like  $^{78}\text{Ni}$ ,  $^{100}\text{Sn}$  and beyond seem difficult.
- New techniques required
  - Greater sensitivity
  - Smaller background
  - More efficient use of beams

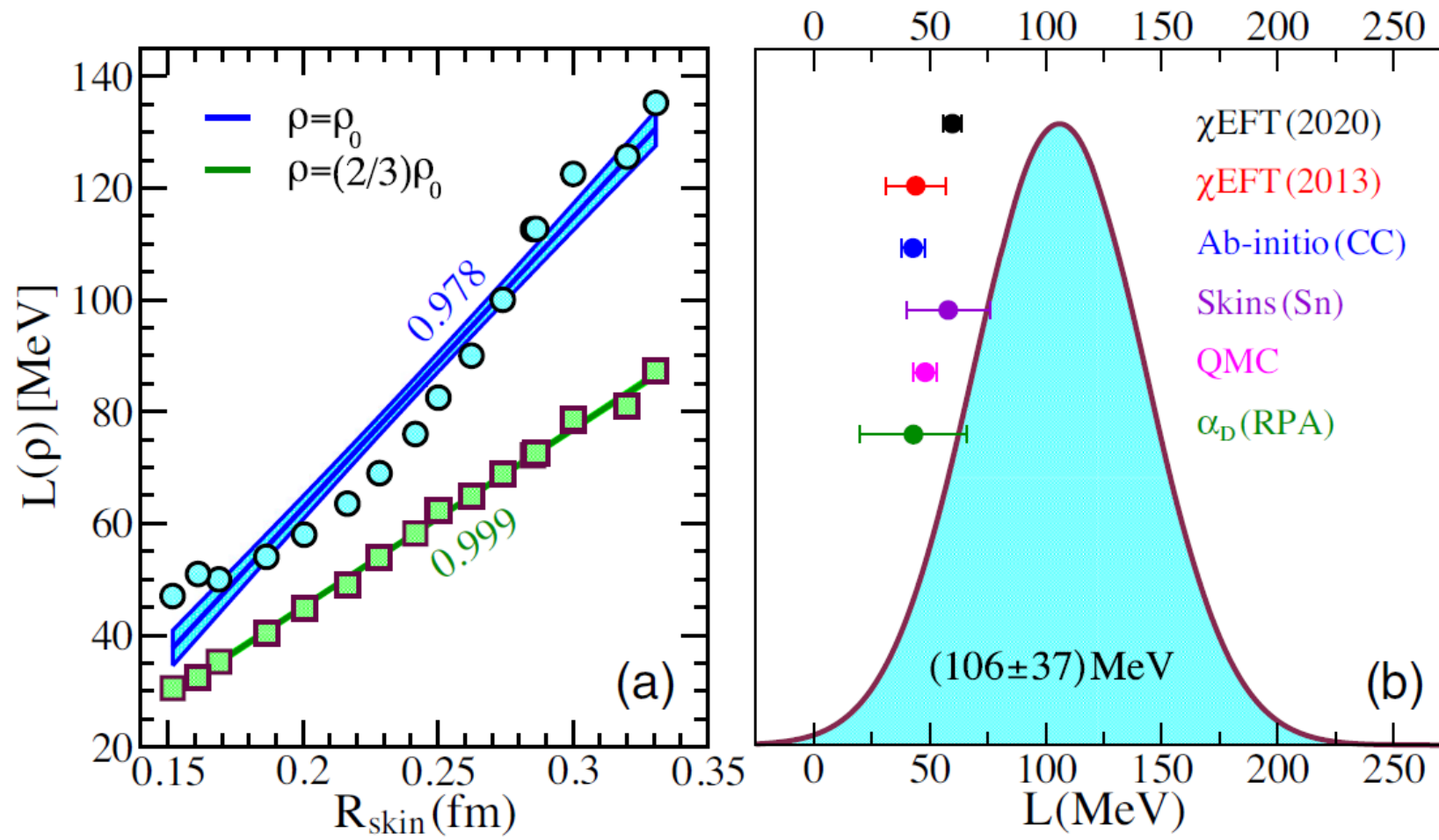
# Neutron Equation of State and Slope Parameter $L$



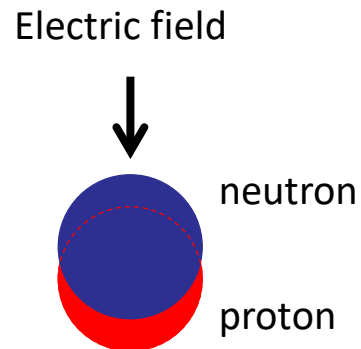
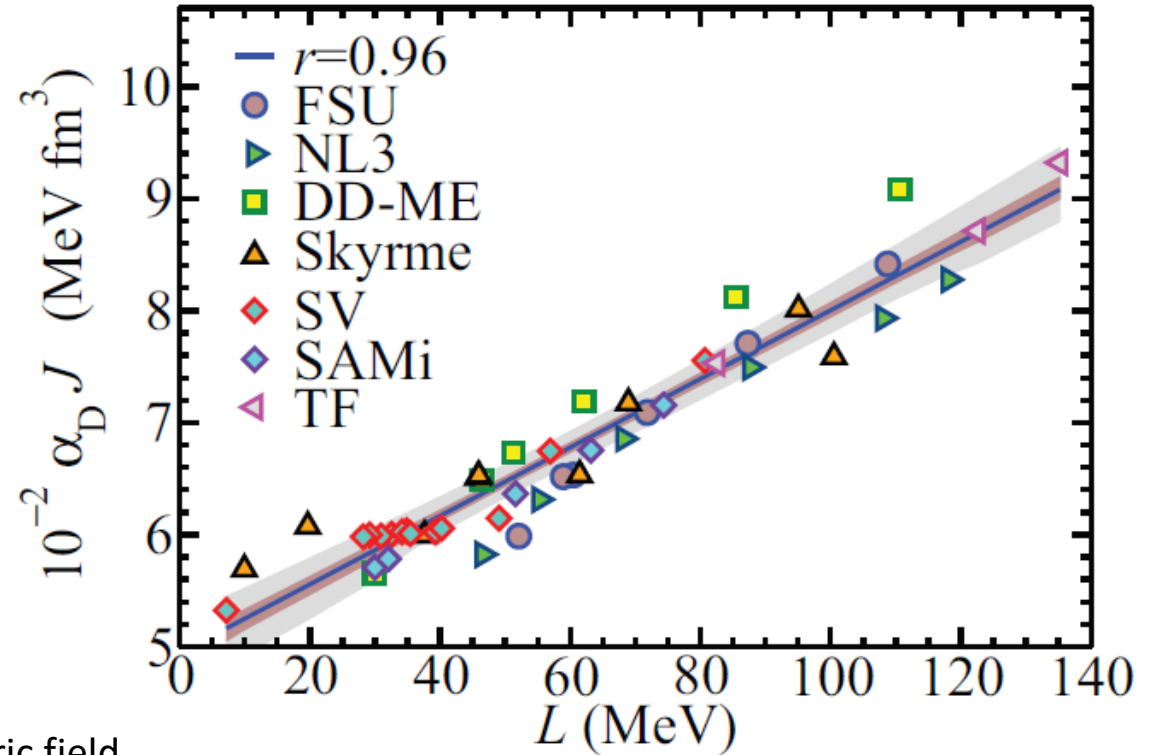
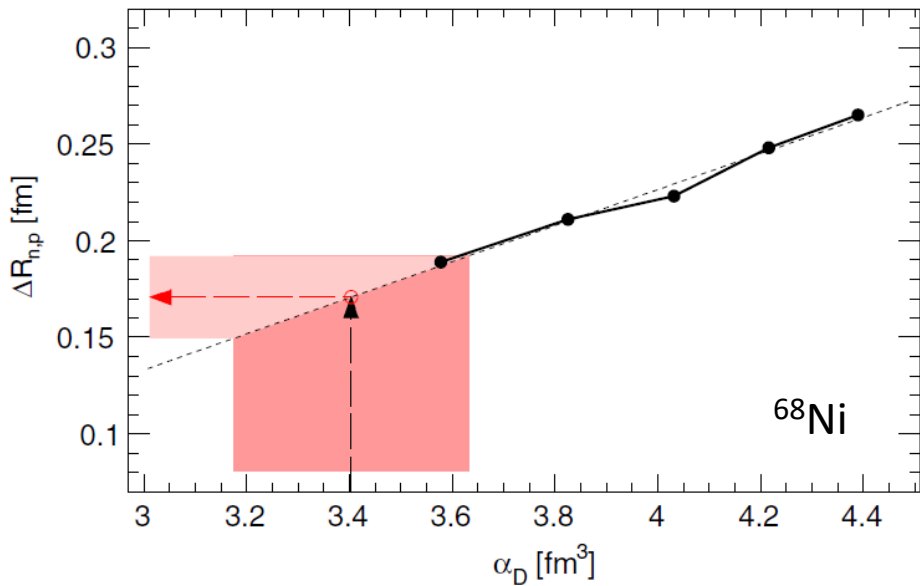
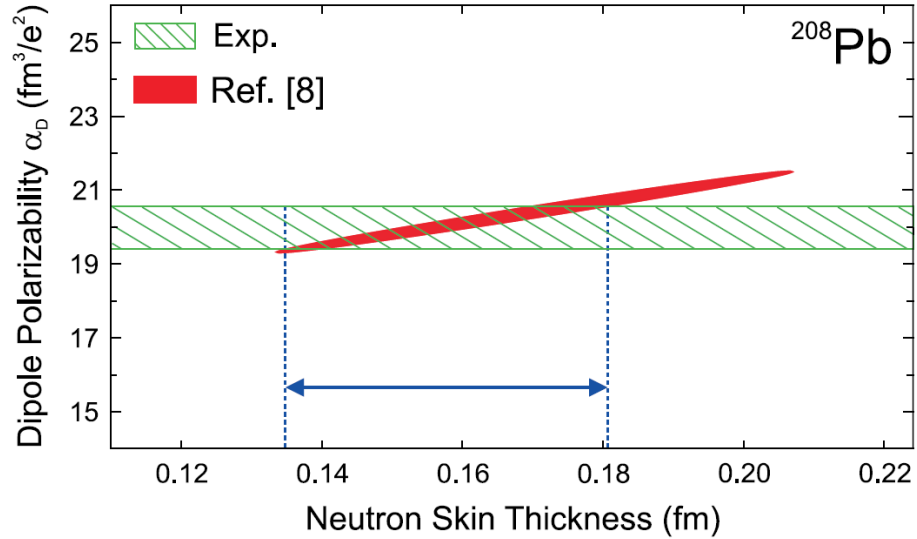
Neutrons skin/radius  $\leftrightarrow L$ : slope of the symmetry energy



# Ex. PREX Correlation between $\Delta R_{np}$ vs $L$



# ex. Electric Dipole Polarizability to Determine $L$



X. Roca-Maza et al., PRC 88, 024316 (2013);  
 P. -G. Reinhard and W. Nazarewicz, PRC 81, 051303R (2010);  
 A. Tamii et al., PRL 107, 062502 (2011);  
 D. M. Rossi et al., PRL 111, 242503 (2013).

# Difference of Mirror Charge Radii

ASSUMING the charge symmetry is a perfect symmetry:

**Neutrons radius of a nucleus is equal to protons radius of its mirror nucleus.**

$$\Delta R_{np} \equiv R_n \left( {}^A_Z X_N \right) - R_{ch} \left( {}^A_Z X_N \right)$$

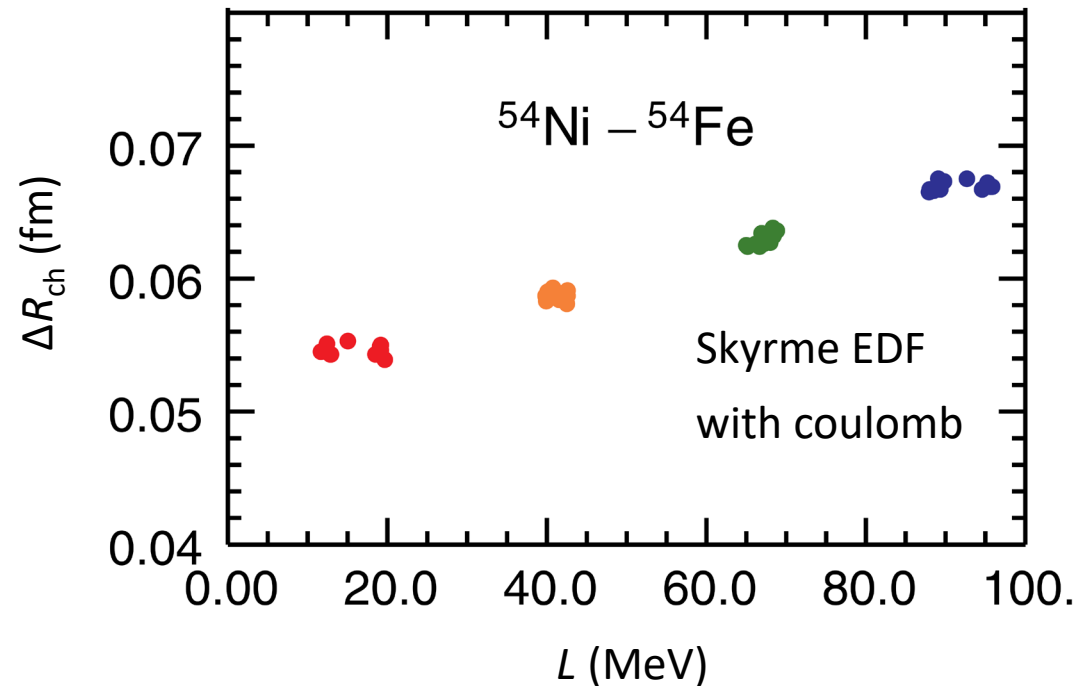
$$\longrightarrow R_{ch} \left( {}^A_N Y_Z \right) - R_{ch} \left( {}^A_Z X_N \right) \equiv \Delta R_{ch}$$

- pure electromagnetic probe
- model independent determination of  $\Delta R_{ch}$

Even with Coulomb, correlation remains, also

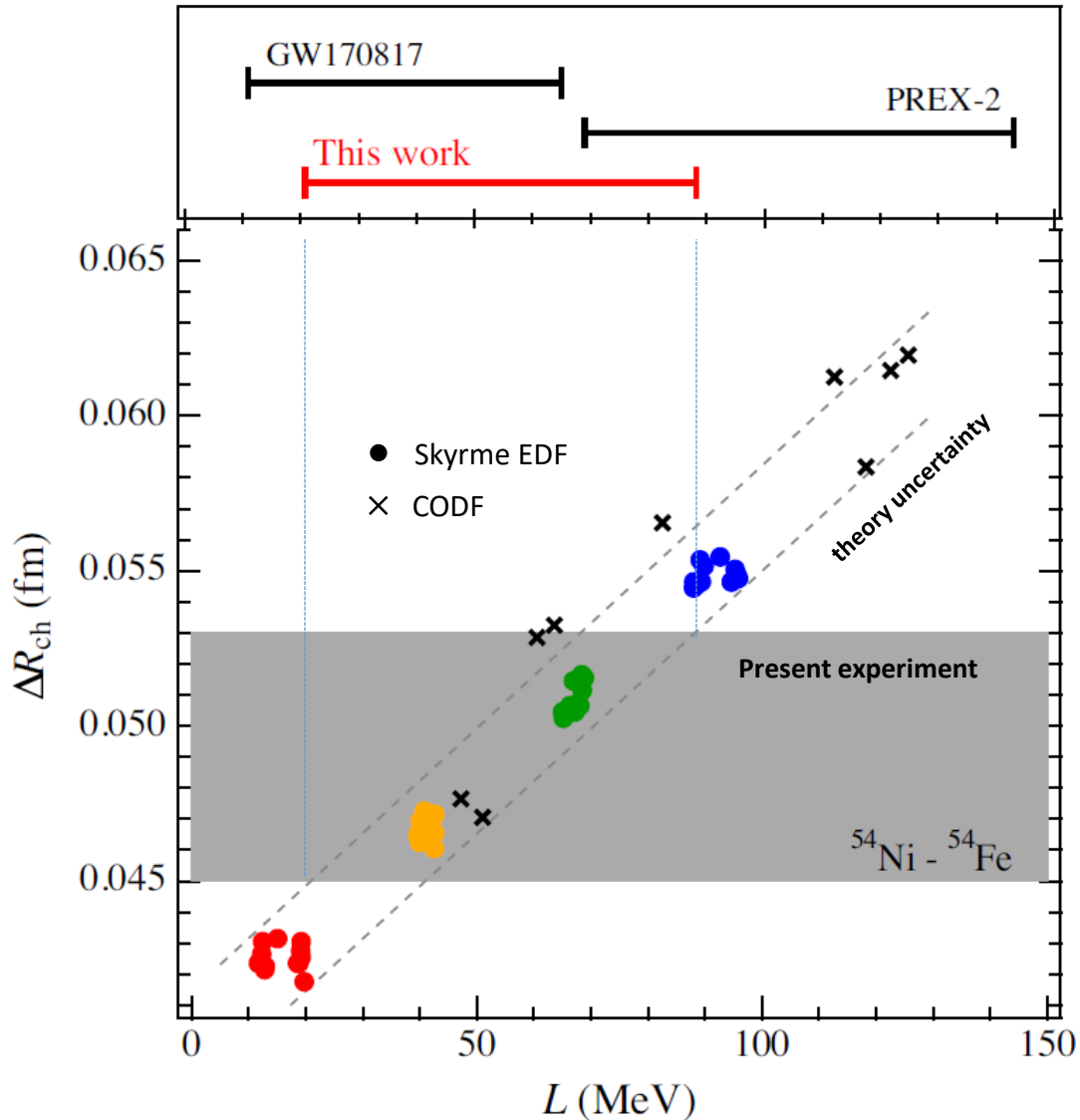
$$\Delta R_{ch} \sim |N - Z| \times L$$

$|N - Z| = 6$  is the largest ( ${}^{22}\text{Si}$ - ${}^{22}\text{O}$ ) ( $|N - Z| = 8$  for  ${}^{48}\text{Ni}$ - ${}^{48}\text{Ca}$ ).



$$\Delta R_{np}({}^{208}\text{Pb}) = \begin{cases} 0.12 \text{ fm: red} \\ 0.16 \text{ fm: orange} \\ 0.20 \text{ fm: green} \\ 0.24 \text{ fm: blue} \end{cases}$$

# Constraint on Symmetry Energy in EOS using Difference of Mirror Charge Radii $^{54}\text{Ni}$ and $^{54}\text{Fe}$



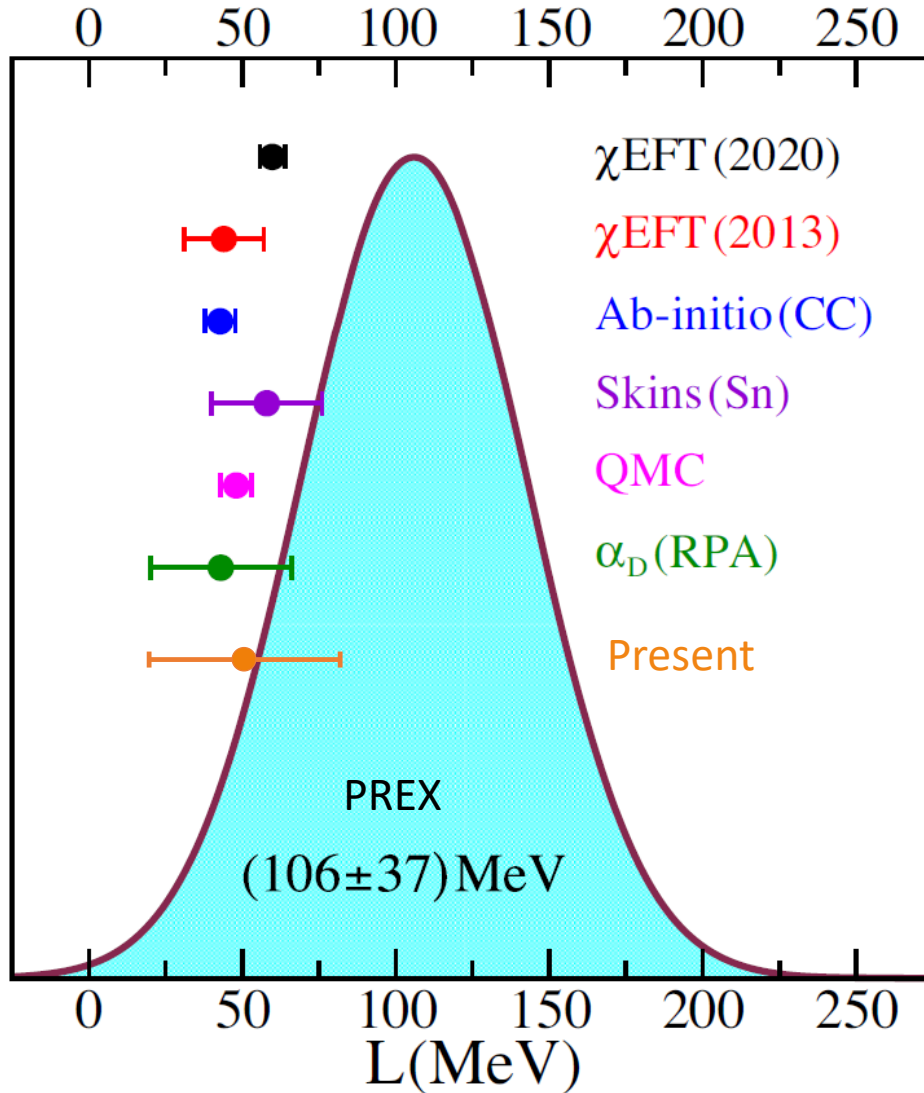
Present result:  $L = 20 \sim 90$  MeV

Our result:

- indicates soft EOS, and rather small radius of a neutron star
- is consistent with the binary neutron star merger GW170817 and PREX
- PREX, however, points to stiffer EOS and a larger neutron star radius.

$$\Delta R_{\text{np}}(^{208}\text{Pb}) = \begin{cases} 0.12 \text{ fm: red} \\ 0.16 \text{ fm: orange} \\ 0.20 \text{ fm: green} \\ 0.24 \text{ fm: blue} \end{cases}$$

# Constraint on Symmetry Energy in EOS using Difference of Mirror Charge Radii $^{54}\text{Ni}$ and $^{54}\text{Fe}$



- PREX suggests a rather stiff EOS and a larger neutron star radius.
- **All  $L$  “measurements” are model dependent.**
- **It is critical to have variety of experimental observables. Mirror charge radii is one of them.**
- **More experimental and theoretical investigations are encouraged.**
- **Assessment of model dependence is critical.**

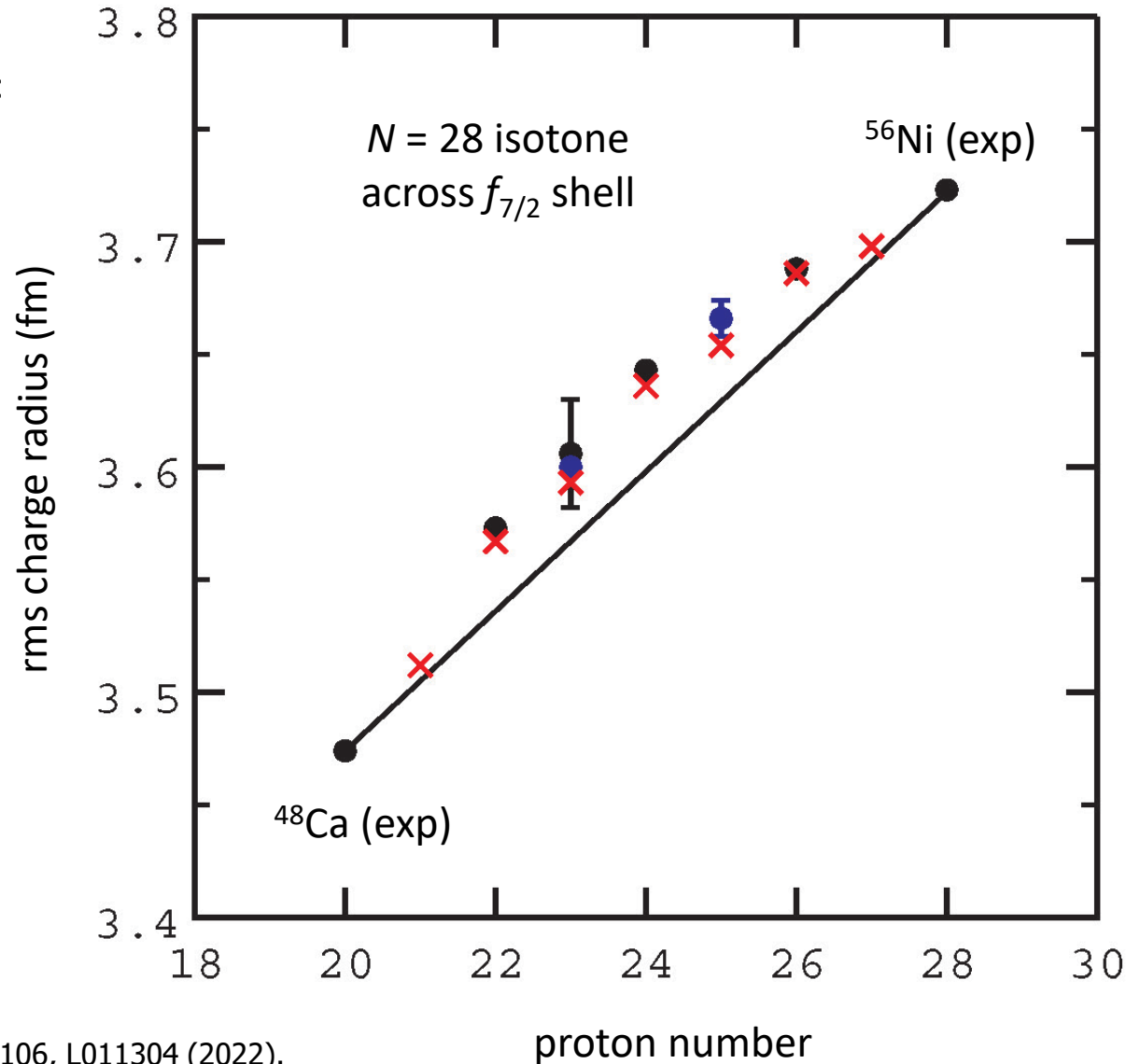
# Assessing Model Dependence: $\beta^2$ Model

According to Bohr collective model:

$$\langle r^2 \rangle = \langle r^2 \rangle_0 \left( 1 + \frac{5}{4\pi} \sum_{\lambda \geq 2} \beta_\lambda^2 \right)$$

$$\sum_{\lambda \geq 2} \beta_\lambda^2 \sim \beta_2^2$$

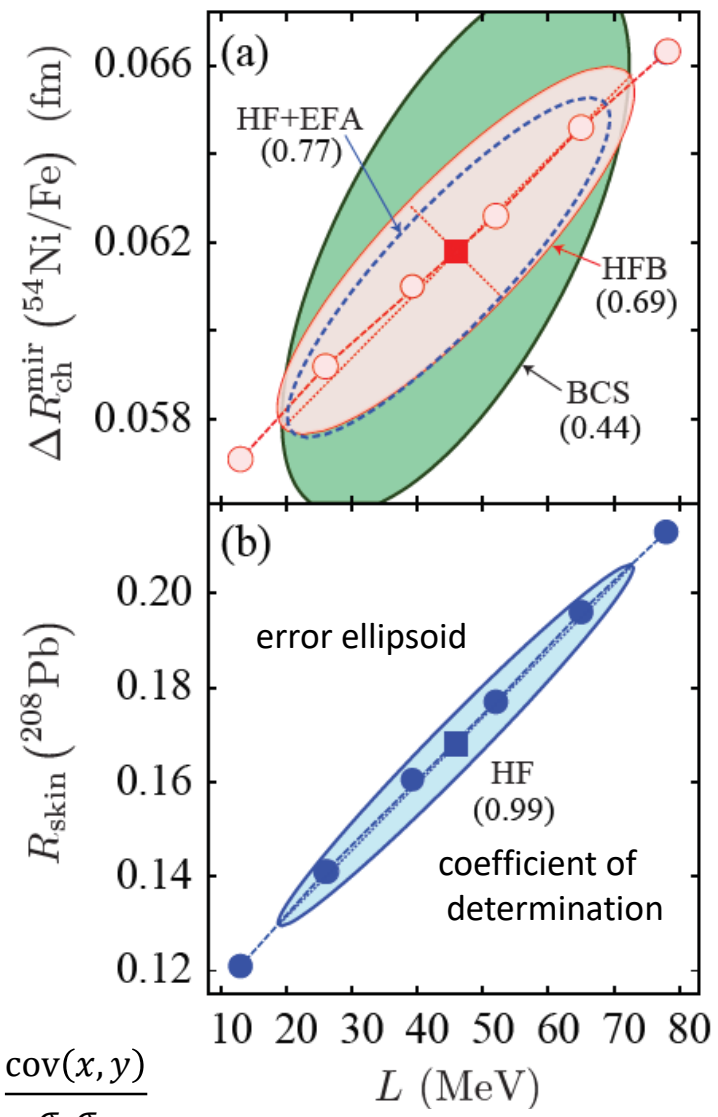
$$\beta_2^2 = \frac{B(E2, \uparrow)}{\left( \frac{3}{4\pi} Z R_0^2 \right)^2}$$



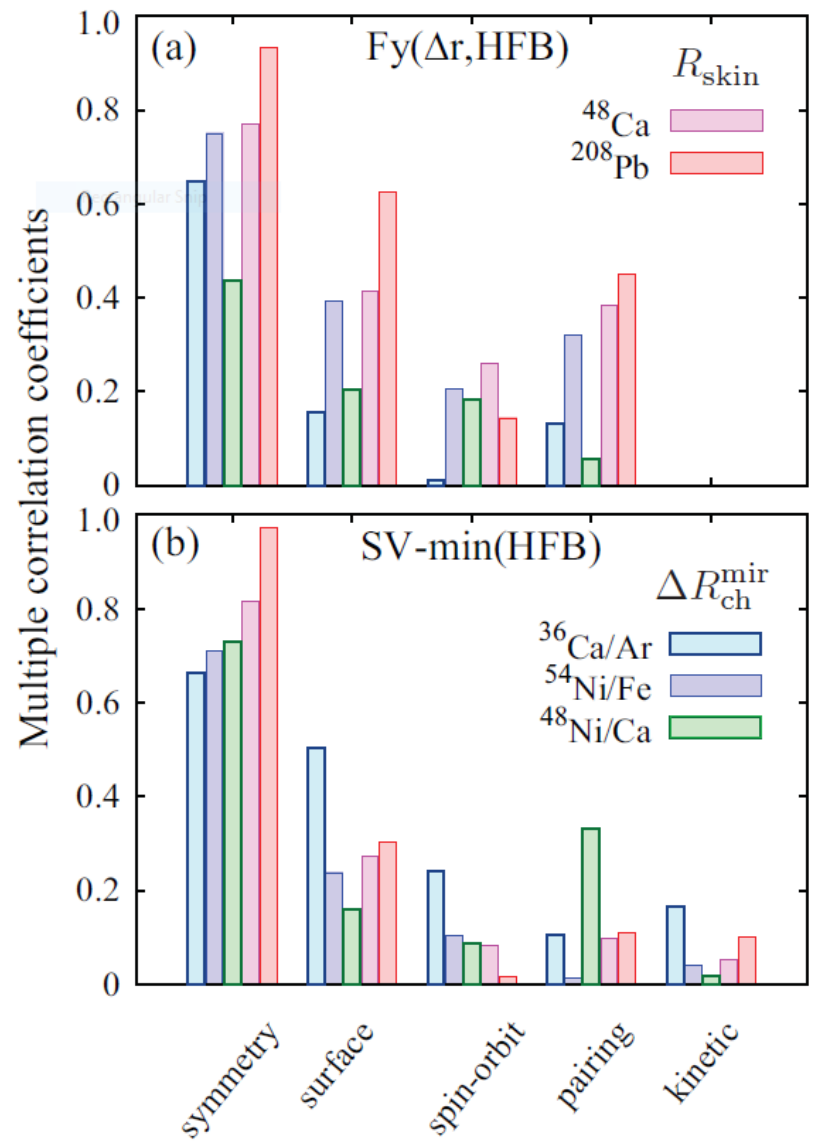
Used in the present work.

Specific model to the region of interest.

# Assessing Model Dependence: Pairing Interaction



$$CoD = \frac{cov(x, y)}{\sigma_x \sigma_y}$$



“...precise data on mirror charge radii with an error of about 0.005 fm, while extremely valuable for studying isospin effects in nuclei and model developments, cannot provide a stringent constraint on  $L$ ” in this model.

Global fit with rich pairing interactions

# Summary

- **Laser spectroscopy will provide critical inputs to benchmark theories.**
  - Spectroscopic electromagnetic moments
  - Charge radius
- **Continuing collaboration with theorists for more exotic, heavier and more deformed nuclei is critical.**
- **Tell me how we can help to develop your theories!**
  - Specific region, element, isotopes... will greatly help experimental design.
- **Mirror charge radii correlation to  $L$** 
  - $^{52}\text{Ni}$ - $^{52}\text{Cr}$  experiment approved
  - Further theoretical inputs (ab-initio calculations...)



# Acknowledgement

## BECOLA collaboration

MICHIGAN STATE  
UNIVERSITY



U.S. DEPARTMENT OF  
**ENERGY**

Office of  
Science

# Thank you!