The role of nuclear radii in beta decay processes

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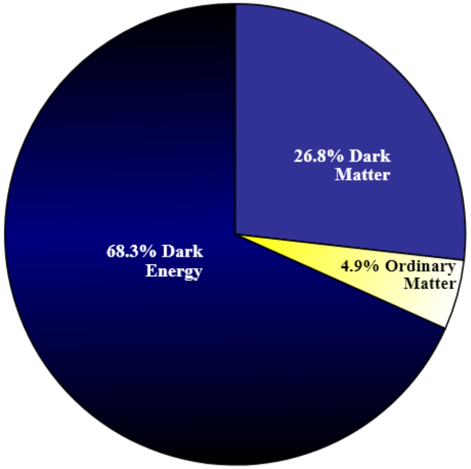
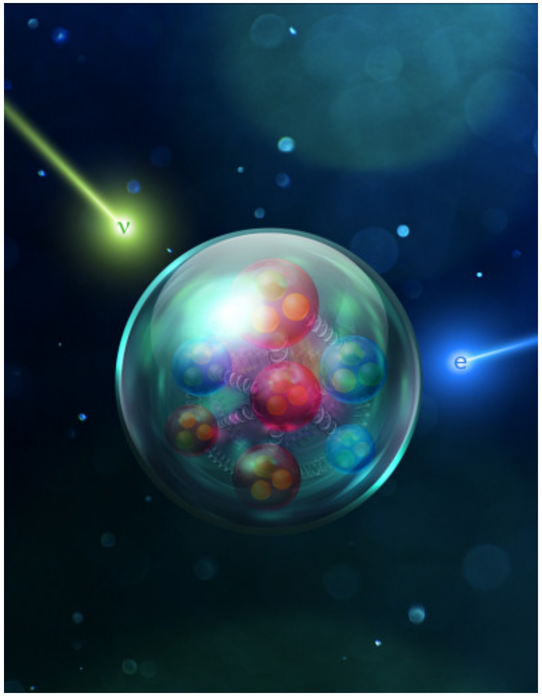
Many unresolved problems call for physics beyond the Model (BSM) !

Image credit: Wikipedia

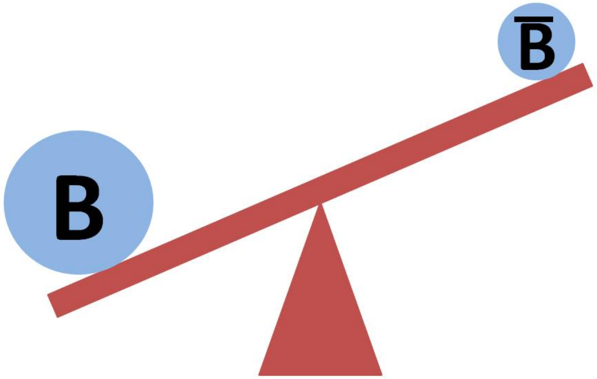


What is the origin of dark energy and dark matter?

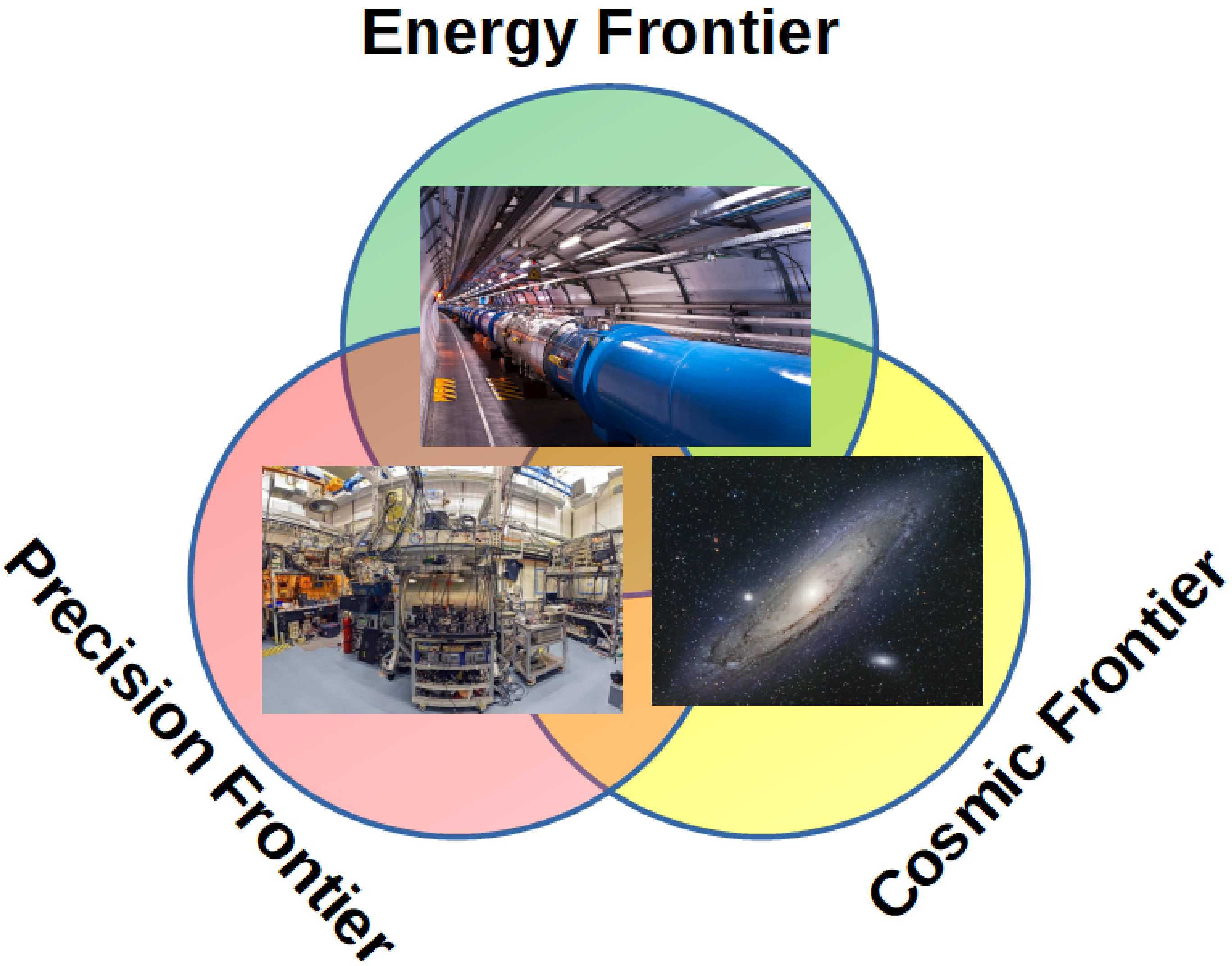
Image credit: Jefferson Lab



What is the nature of the neutrino mass?



Why is there much more matter than antimatter in the observed universe?



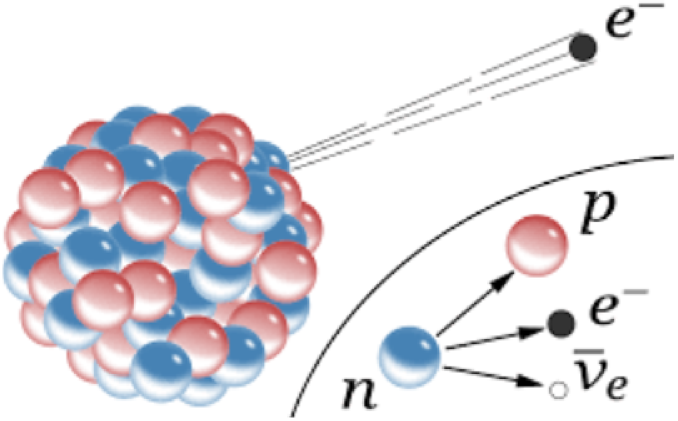
Energy Frontier

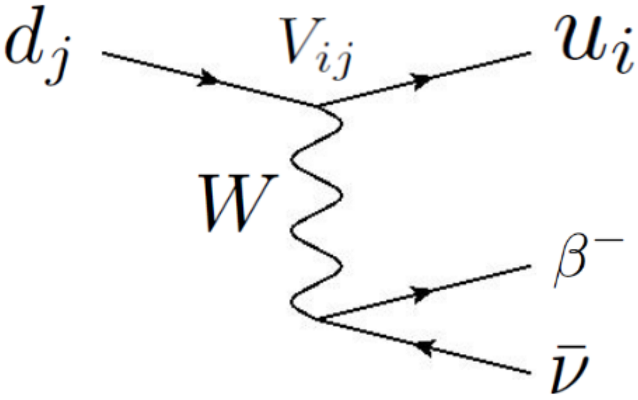
Precision Frontier

Cosmic Frontier

Precision Frontier: Measure things very precisely, and look for their **deviations** from SM prediction!

Nuclear beta decay





Observables measured at **0.01%** level! Probes new physics at the scale:

$$\left(\frac{v_{\mathrm{H}}}{\Lambda_{\mathrm{BSM}}}\right)^2 \sim 0.01\% \quad \Longrightarrow \quad \Lambda_{\mathrm{BSM}} \sim 20\ \mathrm{TeV}$$

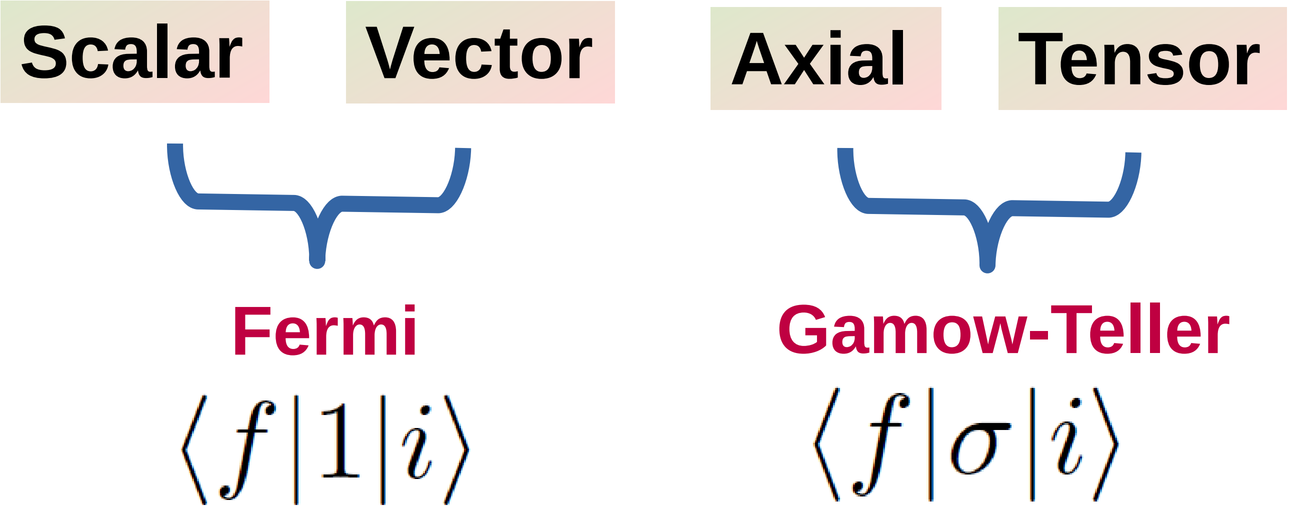
Competitive to high-energy experiments at LHC!

Allowed beta decay:

$$\begin{aligned}\Delta J &= 0, \pm 1\\ \Delta\pi &= 0\end{aligned}$$

Lee-Yang Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{LY}} = & -\bar{p}\gamma^{\mu}n\left(C_V^{+}\bar{e}\gamma_{\mu}\nu_L + C_V^{-}\bar{e}\gamma_{\mu}\nu_R\right) - \bar{p}\gamma^{\mu}\gamma_5 n\left(C_A^{+}\bar{e}\gamma_{\mu}\nu_L - C_A^{-}\bar{e}\gamma_{\mu}\nu_R\right)\\ & -\bar{p}n\left(C_S^{+}\bar{e}\nu_L + C_S^{-}\bar{e}\nu_R\right) - \frac{1}{2}\bar{p}\sigma^{\mu\nu}n\left(C_T^{+}\bar{e}\sigma_{\mu\nu}\nu_L + C_T^{-}\bar{e}\sigma_{\mu\nu}\nu_R\right)\\ & +\bar{p}\gamma_5 n\left(C_P^{+}\bar{e}\nu_L - C_P^{-}\bar{e}\nu_R\right) + \mathrm{h.c.}\end{aligned}$$



Scalar **Vector** **Axial** **Tensor**

Fermi

$$\langle f|1|i\rangle$$

Gamow-Teller

$$\langle f|\sigma|i\rangle$$

SM: $C_V^{+} = \sqrt{2}G_F V_{ud} g_V \;,\; C_A^{+} = -\sqrt{2}G_F V_{ud} g_A \;,\; \mathrm{other} = 0$

Total decay rate:

$$\Gamma_i = (1+\delta_i)\,\frac{M_F^2 m_e^5}{4\pi^3}\, f_V^i\, \hat{\xi}_i \left[1 + \gamma_i b_i \left\langle \frac{m_e}{E_e} \right\rangle_i \right]$$

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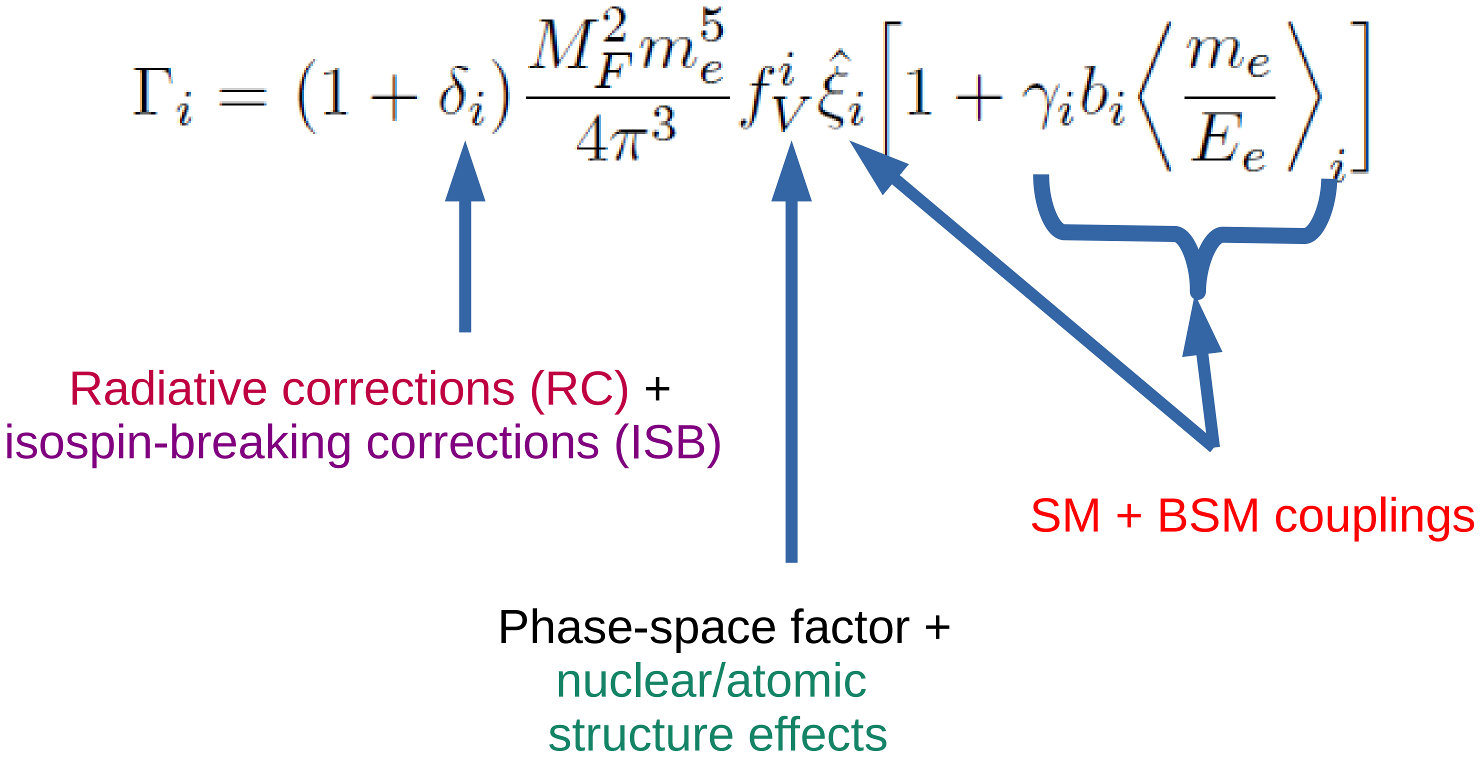
Radiative corrections (RC) +
isospin-breaking corrections (ISB)

Phase-space factor +
nuclear/atomic
structure effects

Total decay rate:



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SM + BSM couplings

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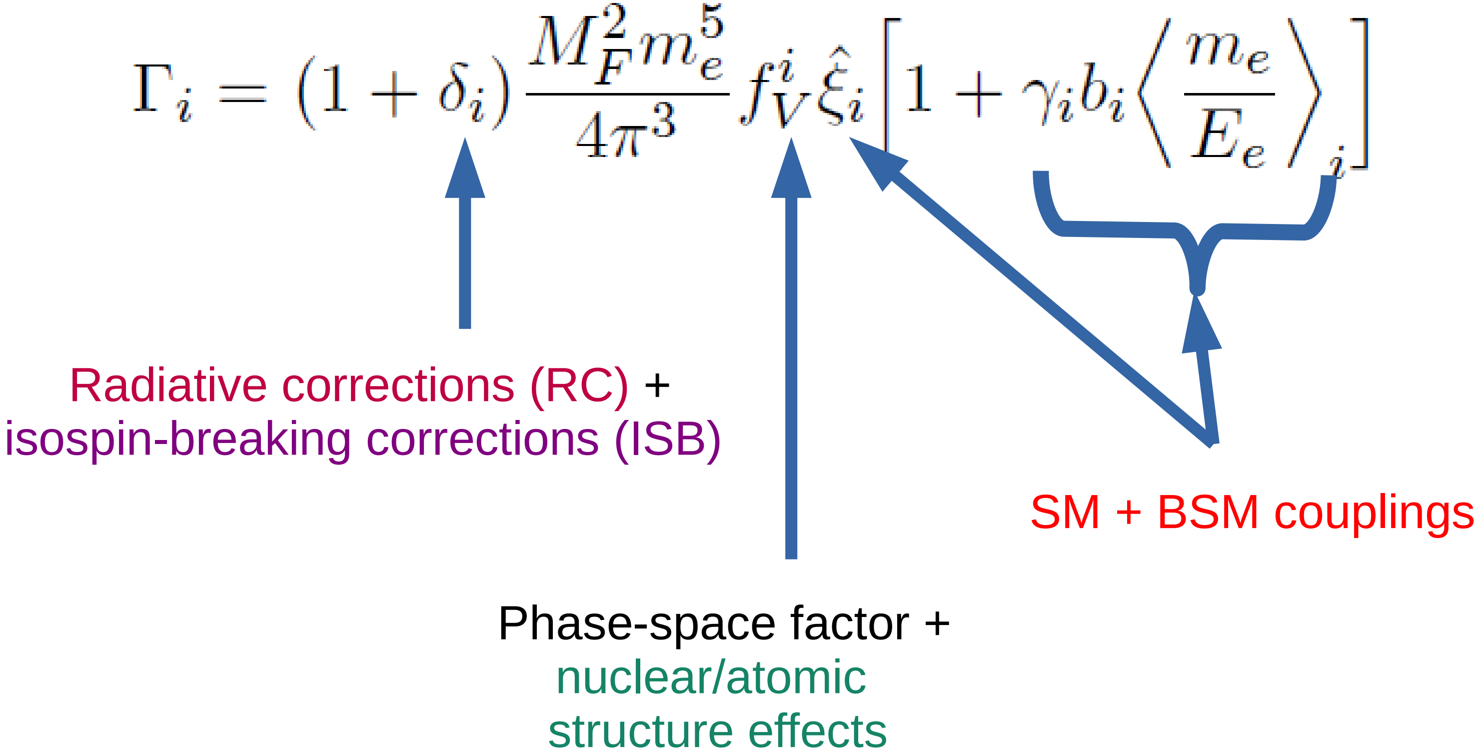

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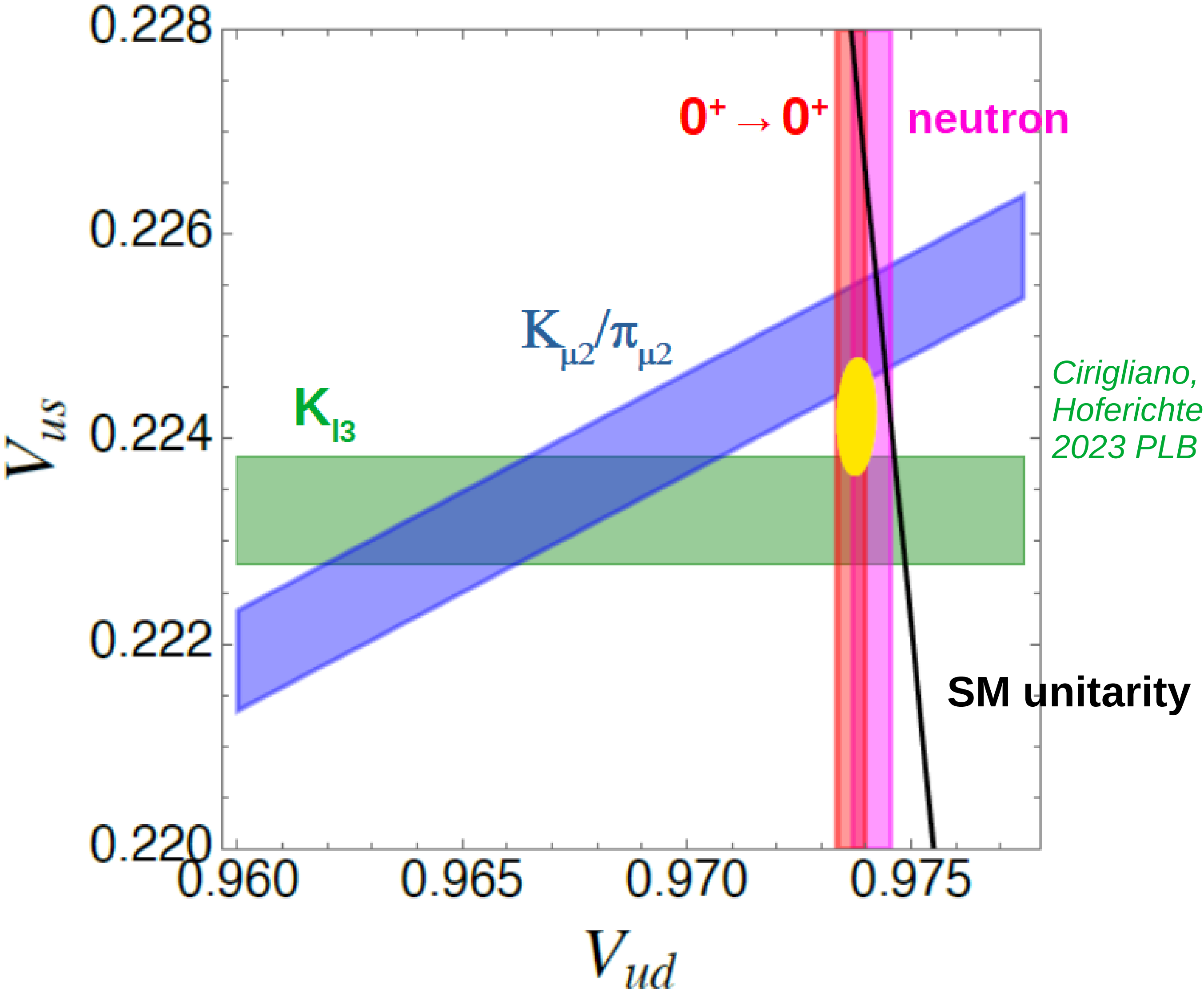
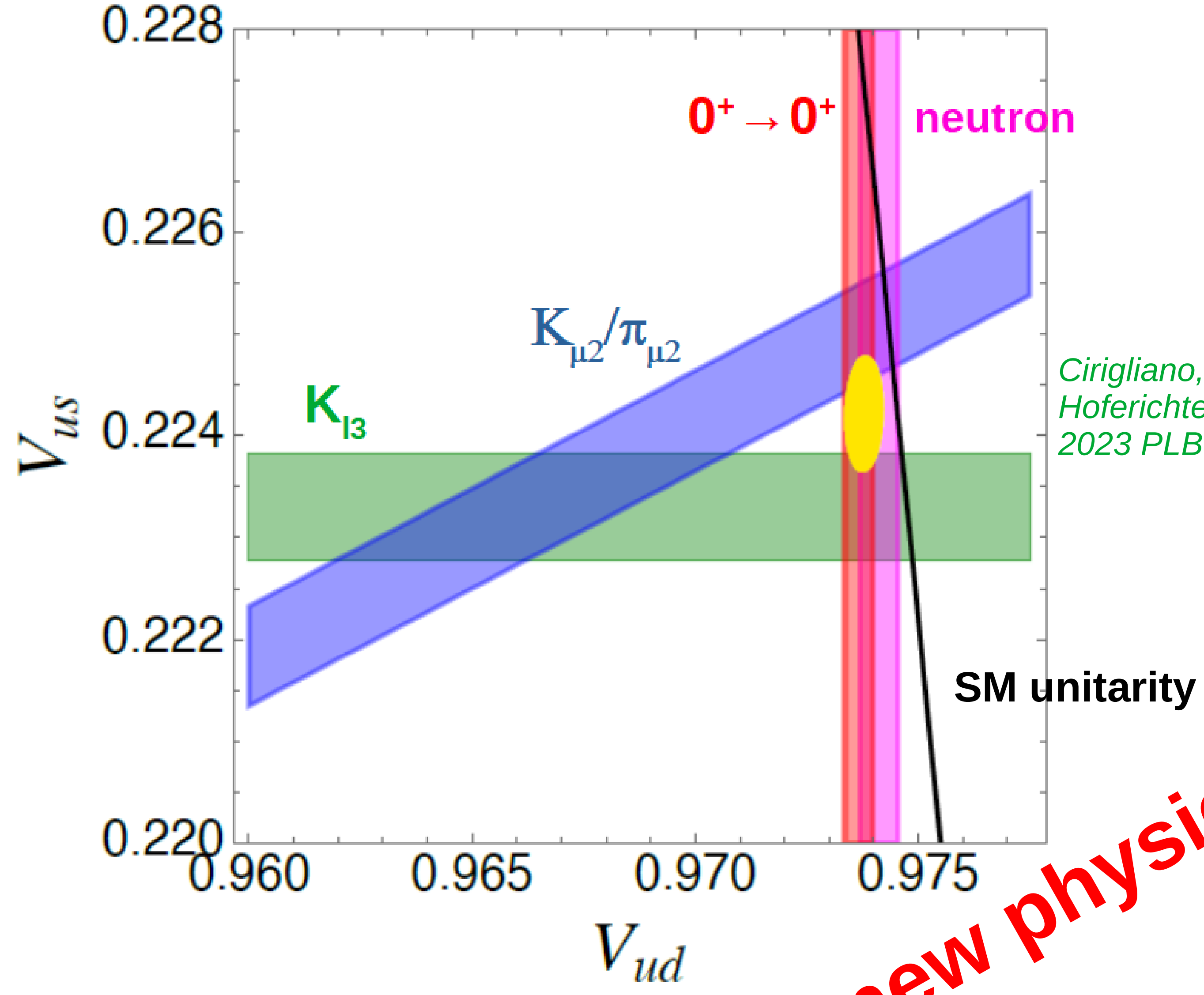
- Assuming SM couplings: Deduce V_{ud}
- Variation of lifetime over nuclei: Constrain BSM couplings



Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

$$|V_{ud}|_{0^+} = 0.97367(11)_{\mathrm{exp}}(30)_{\mathrm{th}}$$

$$|V_{ud}|^2_{0^+} + |V_{us}|^2_{K_{\ell 3}} - 1 = -0.0021(6)_{V_{ud}}(2)_{V_{us}}$$



Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

Sign of new physics??

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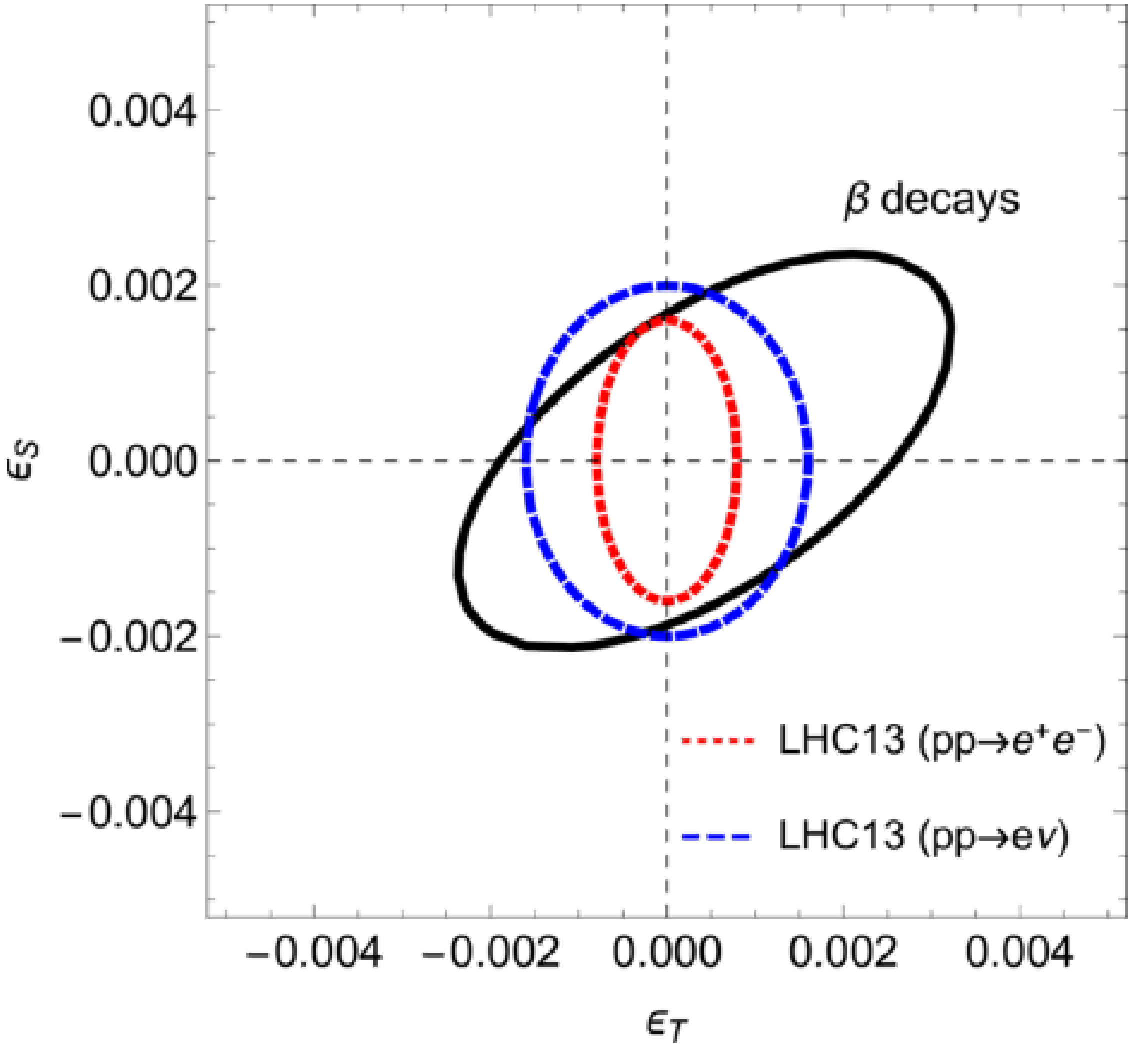
Interplay with LHC experiments:



Falkowski, Gonzalez-Alonso and Naviliat-Cuncic, 2021 JHEP

$$C^{+}_{S,T} = \sqrt{2} G_F V_{ud} g_S \epsilon_{S,T}$$

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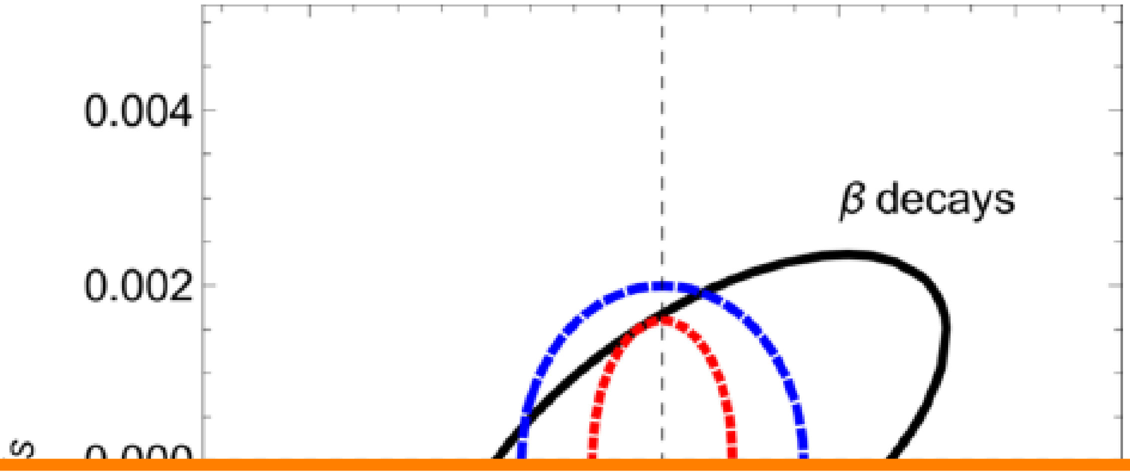

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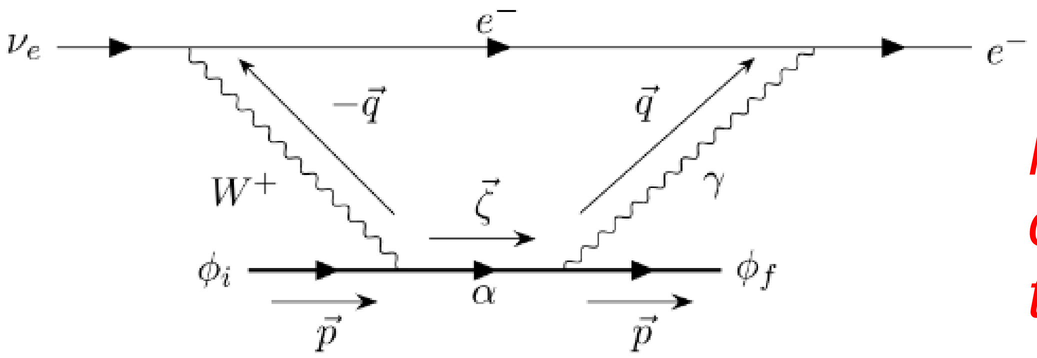

New radiative correction calculation could change the game!

ϵ_T

See Petr Navratil's talk

$$C^{+}_{S,T} = \sqrt{2} G_F V_{ud} g_S \epsilon_{S,T}$$

Role of charge radii measurements:

1. Beta decay form factors
2. Isospin-symmetry-breaking corrections

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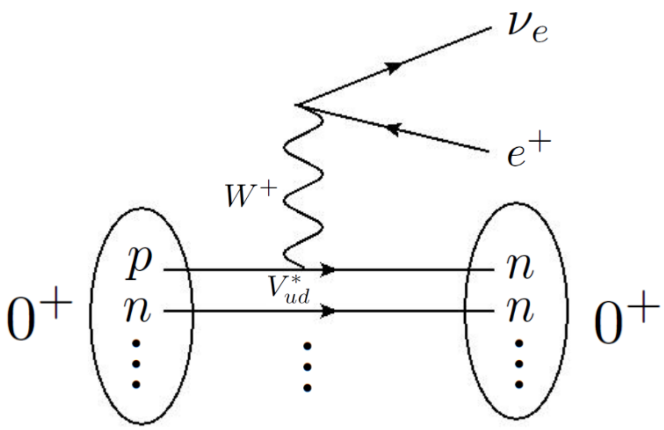
“Superallowed” beta decays of T=1, $J^p=0^+$ nuclei

$$\boxed{i(0^+) \rightarrow f(0^+) + e^+ + \nu_e}$$



$T_z = -1$
${}^{10}_{6}\mathrm{C} \rightarrow {}^{10}_{5}\mathrm{B}$
${}^{14}_{8}\mathrm{O} \rightarrow {}^{14}_{7}\mathrm{N}$
${}^{18}_{10}\mathrm{Ne} \rightarrow {}^{18}_{9}\mathrm{F}$
${}^{22}_{12}\mathrm{Mg} \rightarrow {}^{22}_{11}\mathrm{Na}$
${}^{26}_{14}\mathrm{Si} \rightarrow {}^{26}_{13}\mathrm{Al}$
${}^{30}_{16}\mathrm{S} \rightarrow {}^{30}_{15}\mathrm{P}$
${}^{34}_{18}\mathrm{Ar} \rightarrow {}^{34}_{17}\mathrm{Cl}$
${}^{38}_{20}\mathrm{Ca} \rightarrow {}^{38}_{19}\mathrm{K}$
${}^{42}_{22}\mathrm{Ti} \rightarrow {}^{42}_{21}\mathrm{Sc}$
${}^{46}_{24}\mathrm{Cr} \rightarrow {}^{46}_{23}\mathrm{V}$
${}^{50}_{26}\mathrm{Fe} \rightarrow {}^{50}_{25}\mathrm{Mn}$
${}^{54}_{28}\mathrm{Ni} \rightarrow {}^{54}_{27}\mathrm{Co}$

$T_z = 0$
${}^{26m}_{13}\mathrm{Al} \rightarrow {}^{26}_{12}\mathrm{Mg}$
${}^{34}_{17}\mathrm{Cl} \rightarrow {}^{34}_{16}\mathrm{S}$
${}^{38m}_{19}\mathrm{K} \rightarrow {}^{38}_{18}\mathrm{Ar}$
${}^{42}_{21}\mathrm{Sc} \rightarrow {}^{42}_{20}\mathrm{Ca}$
${}^{46}_{23}\mathrm{V} \rightarrow {}^{46}_{22}\mathrm{Ti}$
${}^{50}_{25}\mathrm{Mn} \rightarrow {}^{50}_{24}\mathrm{Cr}$
${}^{54}_{27}\mathrm{Co} \rightarrow {}^{54}_{26}\mathrm{Fe}$
${}^{62}_{31}\mathrm{Ga} \rightarrow {}^{62}_{30}\mathrm{Zn}$
${}^{66}_{33}\mathrm{As} \rightarrow {}^{66}_{32}\mathrm{Ge}$
${}^{70}_{35}\mathrm{Br} \rightarrow {}^{70}_{34}\mathrm{Se}$
${}^{74}_{37}\mathrm{Rb} \rightarrow {}^{74}_{36}\mathrm{Kr}$

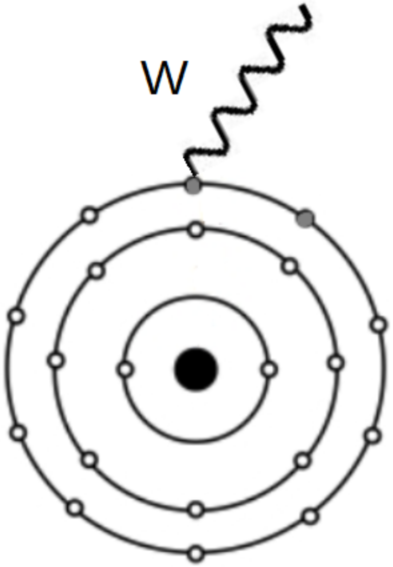
Provides the **best measurement of V_{ud} :**

- **23** measured transitions
- **15** with lifetime precision better than **0.23%**

Hardy and Towner, 2020 PRC

Nuclear (charged) weak distribution

$\rho_w(r)$: Distribution of “active” nucleons eligible for weak transitions in a nucleus



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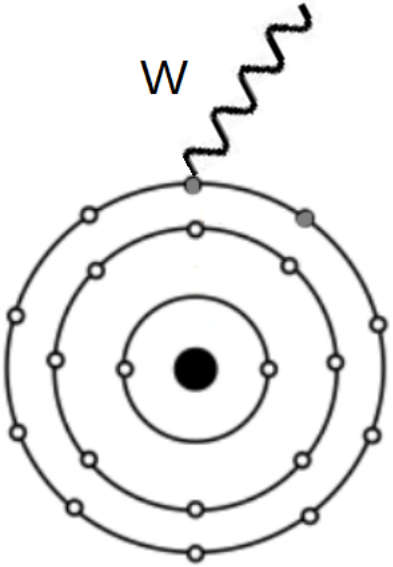

Old : Impulse approximation in **non-relativistic (NR) nuclear models**.

$$\langle f|O|i\rangle = \sum_{\alpha\beta}\langle\alpha|O|\beta\rangle\langle f|a^\dagger_\alpha a_\beta|i\rangle$$

α,β : single-nucleon states

Hardy and Towner, 2005 PRC

How reliable is such approximation at **0.01% level**?

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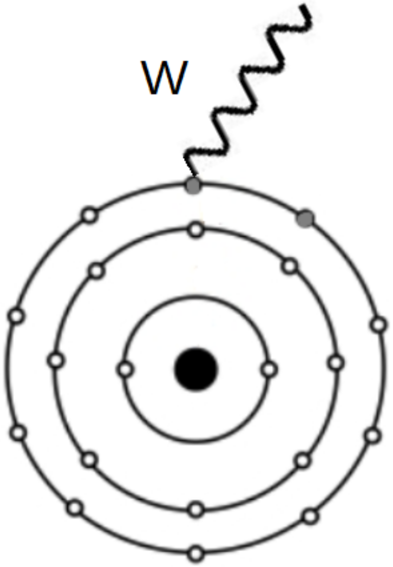

New: Utilize **CVC** to relate (charged) weak distribution to nuclear **charge distributions**

Hostein, 1974 RMP; CYS, 2023 PRL

$$\begin{aligned}\rho_w(r) &= \rho_{\mathrm{Ch},1}(r) + Z_0\left(\rho_{\mathrm{Ch},0}(r) - \rho_{\mathrm{Ch},1}(r)\right)\\ &= \rho_{\mathrm{Ch},1}(r) + \frac{Z_{-1}}{2}\left(\rho_{\mathrm{Ch},\text{-}1}(r) - \rho_{\mathrm{Ch},1}(r)\right)\end{aligned}$$

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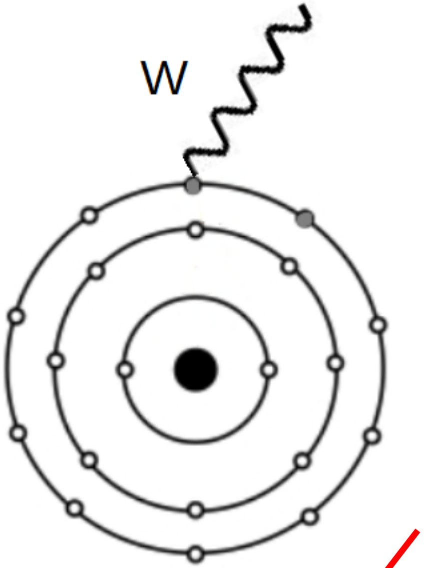
$\rho_w(r)$: Distribution of "active" nucleons eligible for weak transitions in a nucleus



enhanced by large atomic number!

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Form factor expansion: $F(q^2) = 1 + \frac{q^2}{6}\underbrace{\langle r^2 \rangle}_{\text{M.S. radius}} + \ldots$

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M.S. radius

Existing charge radii compilation:

- Fricke and Heilig, “Nuclear charge radii”, 2004
- Angeli and Marinova, Atom.Data Nucl.Data Tabl. 99 (2013) 69
- Li, Luo and Wang, Atom.Data Nucl.Data Tabl. 140 (2021) 101440
- New measurements, e.g. ^{38}Ca, ^{38m}K, ^{54}Ni...

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Data + isospin predicts **unexpectedly large (charged) weak radii**!

E.g. $$\begin{aligned}\left\langle r_{\mathrm{Ch}}^2\right\rangle_{^{38}\mathrm{Ar}}^{1/2} &= 3.4028(19)\ \mathrm{fm}\\ \left\langle r_{\mathrm{Ch}}^2\right\rangle_{^{38}\mathrm{Ca}}^{1/2} &= 3.467(1)\ \mathrm{fm}\end{aligned} \quad \Rightarrow \quad \left\langle r_w^2\right\rangle^{1/2} = 4.00(4)\ \mathrm{fm}$$

Simplified shell model prediction: $\left\langle r_w^2\right\rangle^{1/2} \approx 3.62\ \mathrm{fm}$

Wilkinson, 1993 Nucl.Instrum.Meth.A

A	$R_{\mathrm{Ch},-1}$ (fm)	$R_{\mathrm{Ch},0}$ (fm)	$R_{\mathrm{Ch},1}$ (fm)
10	${}^{10}_{6}\mathrm{C}$	${}^{10}_{5}\mathrm{B}$(ex)	${}^{10}_{4}\mathrm{Be}$: 2.3550(170)[a]
14	${}^{14}_{8}\mathrm{O}$	${}^{14}_{7}\mathrm{N}$(ex)	${}^{14}_{6}\mathrm{C}$: 2.50 25(87)[a]
18	${}^{18}_{10}\mathrm{Ne}$: 2.9714(76)[a]	${}^{18}_{9}\mathrm{F}$(ex)	${}^{18}_{8}\mathrm{O}$: 2.77 26(56)[a]
22	${}^{22}_{12}\mathrm{Mg}$: 3.0691(89)[b]	${}^{22}_{11}\mathrm{Na}$(ex)	${}^{22}_{10}\mathrm{Ne}$: 2.9525(40)[a]
26	${}^{26}_{14}\mathrm{Si}$	${}^{26m}_{13}\mathrm{Al}$	${}^{26}_{12}\mathrm{Mg}$: 3.0337(18)[a]
30	${}^{30}_{16}\mathrm{S}$	${}^{30}_{15}\mathrm{P}$(ex)	${}^{30}_{14}\mathrm{Si}$: 3.1336(40)[a]
34	${}^{34}_{18}\mathrm{Ar}$: 3.3654(40)[a]	${}^{34}_{17}\mathrm{Cl}$	${}^{34}_{16}\mathrm{S}$: 3.2847(21)[a]
38	${}^{38}_{20}\mathrm{Ca}$: 3.467(1)[c]	${}^{38m}_{19}\mathrm{K}$: 3.437(4)[d]	${}^{38}_{18}\mathrm{Ar}$: 3.4028(19)[a]
42	${}^{42}_{22}\mathrm{Ti}$	${}^{42}_{21}\mathrm{Sc}$: 3.5702(238)[a]	${}^{42}_{20}\mathrm{Ca}$: 3.5081(21)[a]
46	${}^{46}_{24}\mathrm{Cr}$	${}^{46}_{23}\mathrm{V}$	${}^{46}_{22}\mathrm{Ti}$: 3.6070(22)[a]
50	${}^{50}_{26}\mathrm{Fe}$	${}^{50}_{25}\mathrm{Mn}$: 3.7120(196)[a]	${}^{50}_{24}\mathrm{Cr}$: 3.6588(65)[a]
54	${}^{54}_{28}\mathrm{Ni}$: 3.738(4)[e]	${}^{54}_{27}\mathrm{Co}$	${}^{54}_{26}\mathrm{Fe}$: 3.6933(19)[a]
62	${}^{62}_{32}\mathrm{Ge}$	${}^{62}_{31}\mathrm{Ga}$	${}^{62}_{30}\mathrm{Zn}$: 3.9031(69)[b]
66	${}^{66}_{34}\mathrm{Se}$	${}^{66}_{33}\mathrm{As}$	${}^{66}_{32}\mathrm{Ge}$
70	${}^{70}_{36}\mathrm{Kr}$	${}^{70}_{35}\mathrm{Br}$	${}^{70}_{34}\mathrm{Se}$
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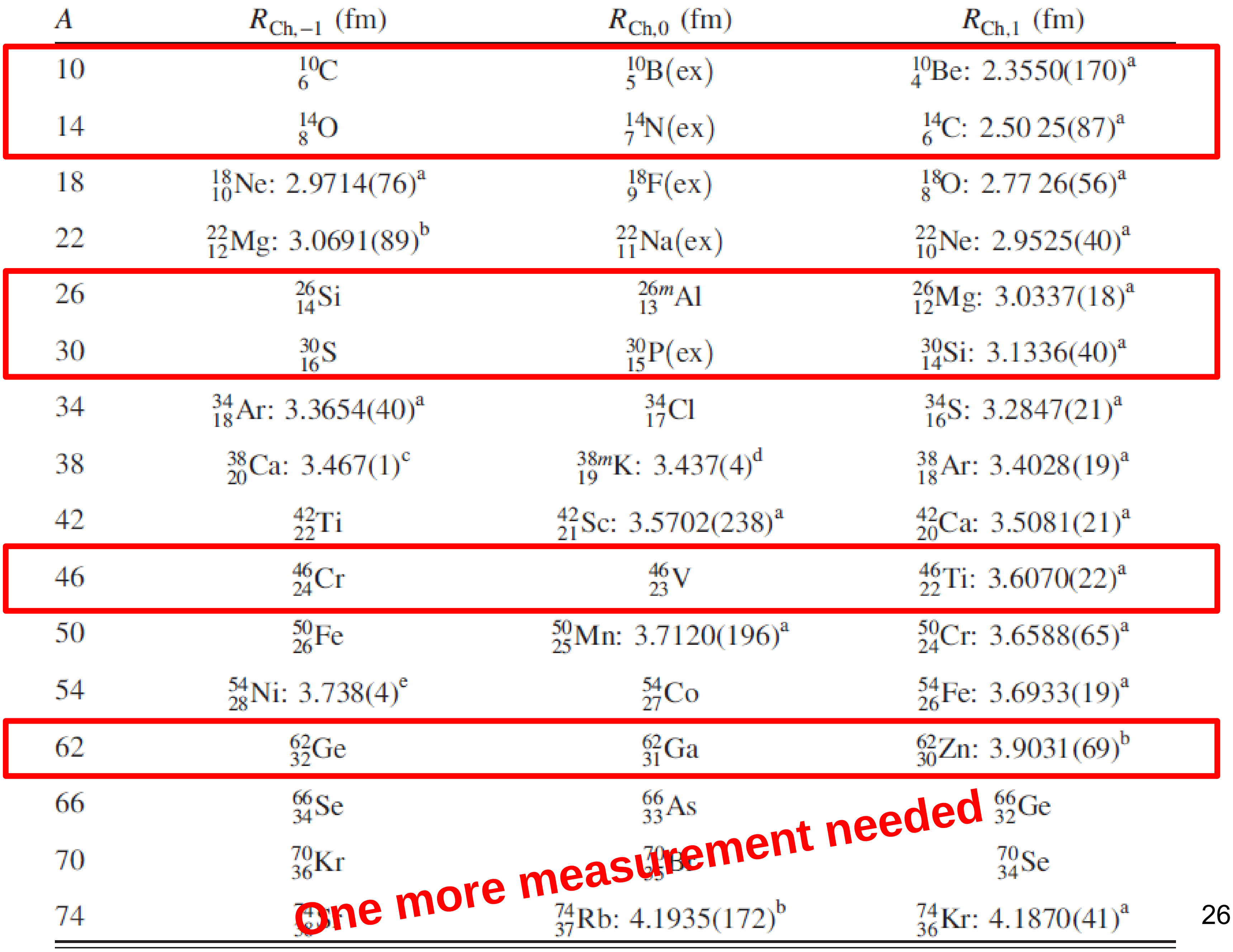
Sufficient data available



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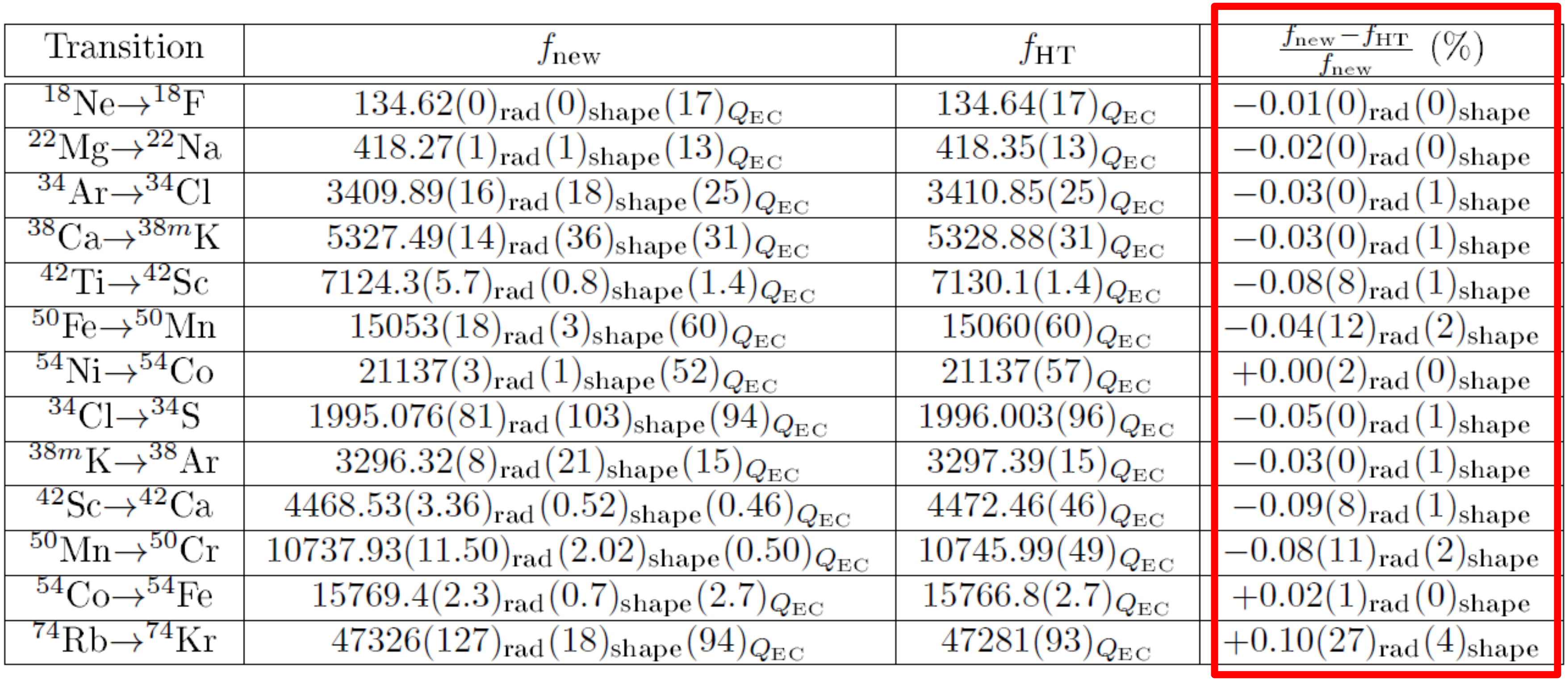
One more measurement needed

Preliminary re-analysis of statistical rate functions:



Transition	$f_{\rm new}$	$f_{\rm HT}$	$\frac{f_{\rm new}-f_{\rm HT}}{f_{\rm new}}$ (%)
^{18}Ne$\to^{18}$F	$134.62(0)_{\rm rad}(0)_{\rm shape}(17)_{Q_{\rm EC}}$	$134.64(17)_{Q_{\rm EC}}$	$-0.01(0)_{\rm rad}(0)_{\rm shape}$
^{22}Mg$\to^{22}$Na	$418.27(1)_{\rm rad}(1)_{\rm shape}(13)_{Q_{\rm EC}}$	$418.35(13)_{Q_{\rm EC}}$	$-0.02(0)_{\rm rad}(0)_{\rm shape}$
^{34}Ar$\to^{34}$Cl	$3409.89(16)_{\rm rad}(18)_{\rm shape}(25)_{Q_{\rm EC}}$	$3410.85(25)_{Q_{\rm EC}}$	$-0.03(0)_{\rm rad}(1)_{\rm shape}$
^{38}Ca$\to^{38m}$K	$5327.49(14)_{\rm rad}(36)_{\rm shape}(31)_{Q_{\rm EC}}$	$5328.88(31)_{Q_{\rm EC}}$	$-0.03(0)_{\rm rad}(1)_{\rm shape}$
^{42}Ti$\to^{42}$Sc	$7124.3(5.7)_{\rm rad}(0.8)_{\rm shape}(1.4)_{Q_{\rm EC}}$	$7130.1(1.4)_{Q_{\rm EC}}$	$-0.08(8)_{\rm rad}(1)_{\rm shape}$
^{50}Fe$\to^{50}$Mn	$15053(18)_{\rm rad}(3)_{\rm shape}(60)_{Q_{\rm EC}}$	$15060(60)_{Q_{\rm EC}}$	$-0.04(12)_{\rm rad}(2)_{\rm shape}$
^{54}Ni$\to^{54}$Co	$21137(3)_{\rm rad}(1)_{\rm shape}(52)_{Q_{\rm EC}}$	$21137(57)_{Q_{\rm EC}}$	$+0.00(2)_{\rm rad}(0)_{\rm shape}$
^{34}Cl$\to^{34}$S	$1995.076(81)_{\rm rad}(103)_{\rm shape}(94)_{Q_{\rm EC}}$	$1996.003(96)_{Q_{\rm EC}}$	$-0.05(0)_{\rm rad}(1)_{\rm shape}$
^{38m}K$\to^{38}$Ar	$3296.32(8)_{\rm rad}(21)_{\rm shape}(15)_{Q_{\rm EC}}$	$3297.39(15)_{Q_{\rm EC}}$	$-0.03(0)_{\rm rad}(1)_{\rm shape}$
^{42}Sc$\to^{42}$Ca	$4468.53(3.36)_{\rm rad}(0.52)_{\rm shape}(0.46)_{Q_{\rm EC}}$	$4472.46(46)_{Q_{\rm EC}}$	$-0.09(8)_{\rm rad}(1)_{\rm shape}$
^{50}Mn$\to^{50}$Cr	$10737.93(11.50)_{\rm rad}(2.02)_{\rm shape}(0.50)_{Q_{\rm EC}}$	$10745.99(49)_{Q_{\rm EC}}$	$-0.08(11)_{\rm rad}(2)_{\rm shape}$
^{54}Co$\to^{54}$Fe	$15769.4(2.3)_{\rm rad}(0.7)_{\rm shape}(2.7)_{Q_{\rm EC}}$	$15766.8(2.7)_{Q_{\rm EC}}$	$+0.02(1)_{\rm rad}(0)_{\rm shape}$
^{74}Rb$\to^{74}$Kr	$47326(127)_{\rm rad}(18)_{\rm shape}(94)_{Q_{\rm EC}}$	$47281(93)_{Q_{\rm EC}}$	$+0.10(27)_{\rm rad}(4)_{\rm shape}$

CYS, **PRELIMINARY RESULTS**

Shifts at 0.01% level are observed!

Upward shift of V_{ud} ?

Role of charge radii measurements:

1. Beta decay form factors
2. Isospin-symmetry-breaking corrections

Isospin symmetry breaking **(ISB)** correction alters the Fermi matrix element:

$$|M_F|^2 = |\langle f|\hat{\tau}_+|i\rangle|^2 = |M_F^0|^2(1-\delta_C)$$

Caused by **isospin mixing** of nuclear states, predominantly due to Coulomb repulsion between protons

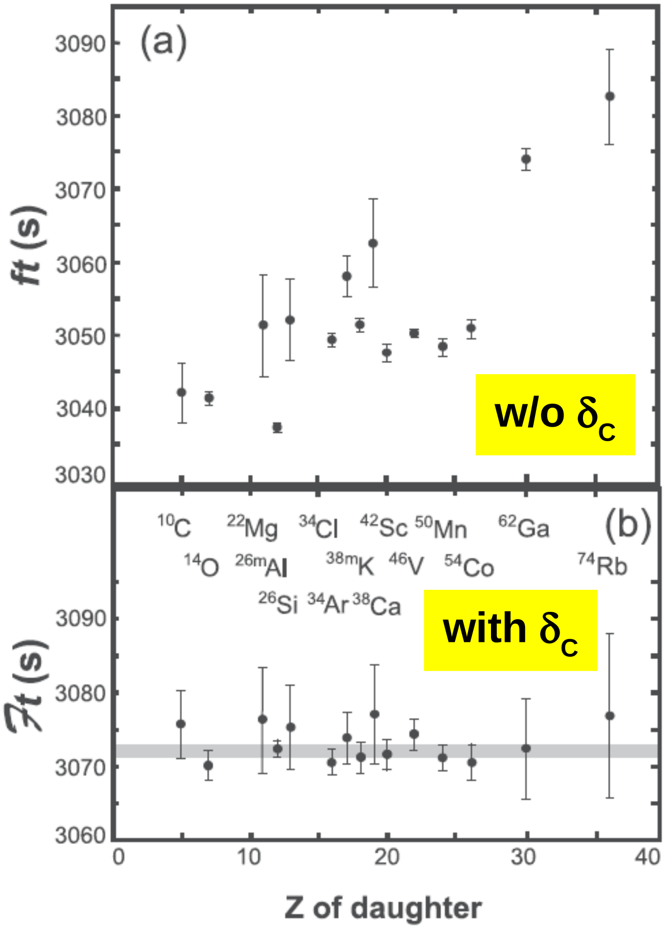
$$H = H_0 + \boxed{V} \quad \textbf{ISB}$$

$$\delta_C = \mathcal{O}(V^2) \quad \textbf{0.1\%\textasciitilde1\%}$$

Crucial in obtaining a nucleus-independent Ft-value from the nucleus-dependent ft-values:

$$|V_{ud}|^2 \boxed{\mathcal{F}t}(1+\Delta_R^V) = 2984.43\,\mathrm{s}$$

$$\mathcal{F}t = ft\,(1+\delta_R')\,(1+\delta_{NS}-\delta_C)$$

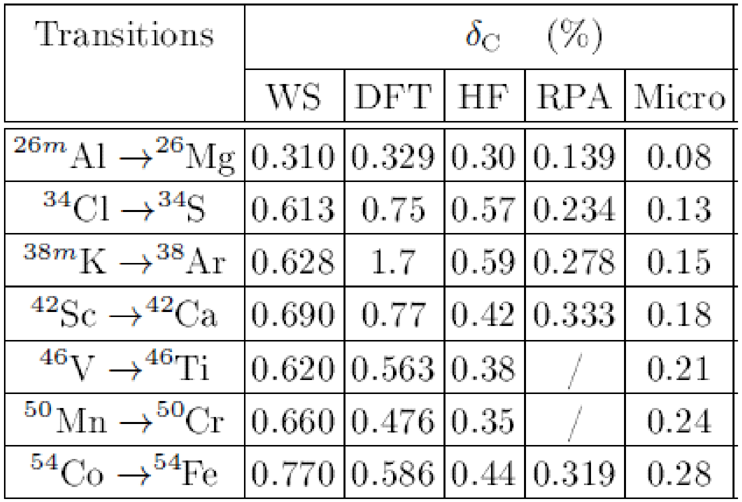


Hardy and Towner, 2020 PRC

- Computing δ_c: Classic problem over ***6 decades***! *MacDonald, 1958 Phys.Rev*

- Current input adopted in global analysis: Shell model + Woods-Saxon (WS) potential
- Successful in aligning Ft values of different superallowed transitions

Hardy and Towner, 2020 PRC

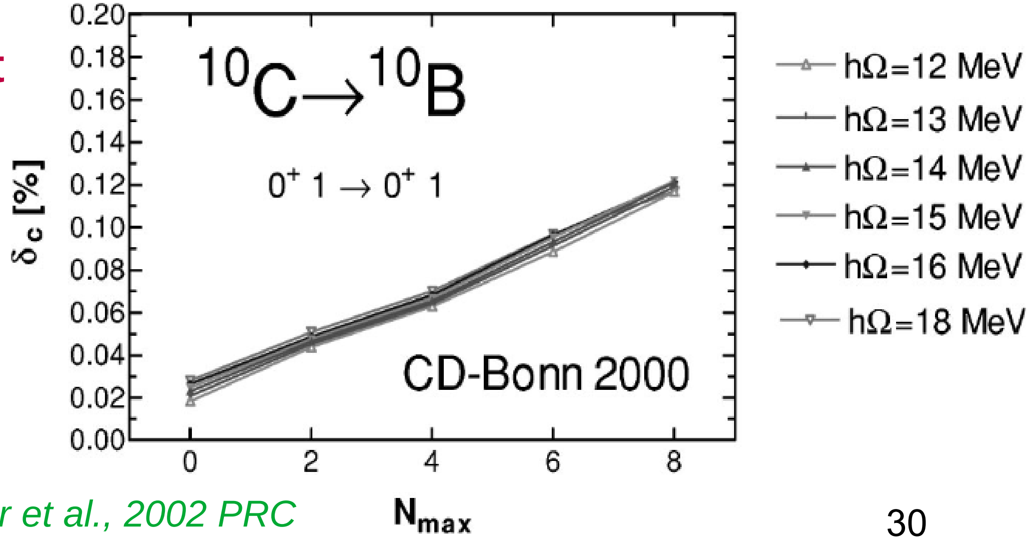


Transitions	δ_C (%)				
	WS	DFT	HF	RPA	Micro
$^{26m}\mathrm{Al} \to {}^{26}\mathrm{Mg}$	0.310	0.329	0.30	0.139	0.08
$^{34}\mathrm{Cl} \to {}^{34}\mathrm{S}$	0.613	0.75	0.57	0.234	0.13
$^{38m}\mathrm{K} \to {}^{38}\mathrm{Ar}$	0.628	1.7	0.59	0.278	0.15
$^{42}\mathrm{Sc} \to {}^{42}\mathrm{Ca}$	0.690	0.77	0.42	0.333	0.18
$^{46}\mathrm{V} \to {}^{46}\mathrm{Ti}$	0.620	0.563	0.38	/	0.21
$^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}$	0.660	0.476	0.35	/	0.24
$^{54}\mathrm{Co} \to {}^{54}\mathrm{Fe}$	0.770	0.586	0.44	0.319	0.28

(Selected results)

Caveats:

- Significant model dependence.
- **No direct experimental constraint**
- Ab-initio calculations still in preliminary stages



Caurier et al., 2002 PRC

Probing isospin mixing in experiment?

Nuclear mass splitting: $\delta E_n = {}_0\langle n|V|n\rangle_0$ **X**

$$|n\rangle = |n\rangle_0 + \underbrace{\sum_{k\neq n} |k\rangle_0 \frac{{}_0\langle k|V|n\rangle_0}{E_n^{(0)} - E_k^{(0)}}}_{\textbf{We need this}} + \dots$$

Second-class current: doubly-suppressed (kinematics*ISB), null results so far

Minamisono et al., 2011 PRC

$$V_\mu = \bar{u}(p_2)\left(g_V\gamma_\mu - i\frac{g_M - g_V}{2M}\sigma_{\mu\nu}q^\nu + \frac{\boxed{g_S}}{2M}q_\mu\right)u(p_1),$$

$$A_\mu = -\bar{u}(p_2)\gamma_5\left(g_A\gamma_\mu - i\frac{\boxed{g_T}}{2M}\sigma_{\mu\nu}q^\nu + \frac{g_P}{2M}q_\mu\right)u(p_1).$$

Isospin mixing can be probed through the **variation of charge radii within the same nuclear isomultiplet**

Consider T=1 system. The non-zeroness of

$$\boxed{\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_{+1} r^2_{\mathrm{ch},+1} + Z_{-1} r^2_{\mathrm{ch},\text{-}1}\right) - Z_0 r^2_{\mathrm{ch},0}}$$

signifies ISB. No double suppression! *Seng and Gorchtein, 2023 PLB*

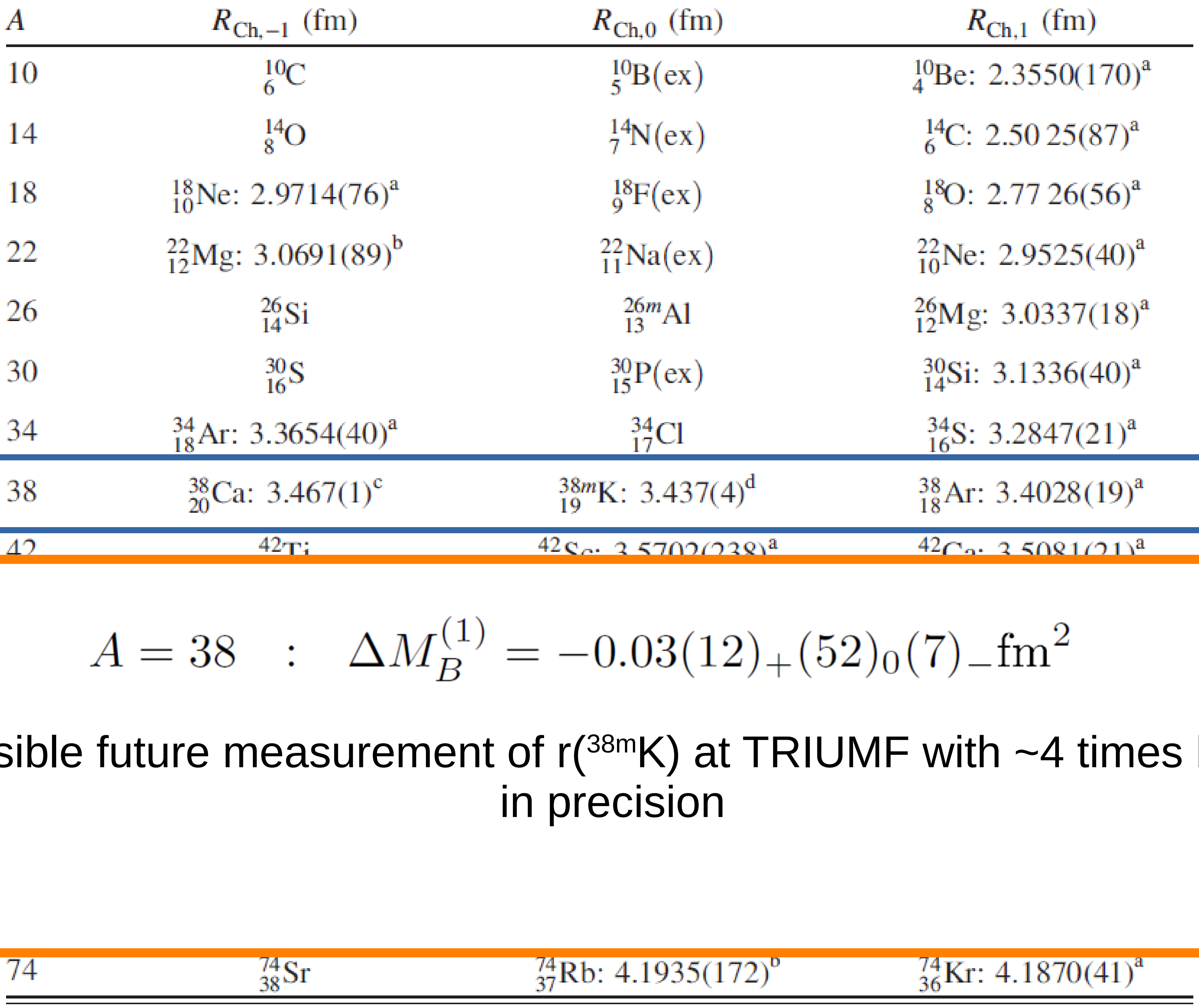
Simple isovector-monopole-dominance picture: $\delta_{\mathrm{C}} \propto \Delta M_B^{(1)}$

Auerbach, 2009 PRC

Therefore, the ability to reproduce $\Delta M_B^{(1)}$ serves as a test of the theory accuracy of δ_C

Challenge: ***THREE*** charge radii need to be measured!

A	$R_{\mathrm{Ch},-1}$ (fm)	$R_{\mathrm{Ch},0}$ (fm)	$R_{\mathrm{Ch},1}$ (fm)
10	${}^{10}_{6}\mathrm{C}$	${}^{10}_{5}\mathrm{B}$(ex)	${}^{10}_{4}\mathrm{Be}$: 2.3550(170)[a]
14	${}^{14}_{8}\mathrm{O}$	${}^{14}_{7}\mathrm{N}$(ex)	${}^{14}_{6}\mathrm{C}$: 2.50 25(87)[a]
18	${}^{18}_{10}\mathrm{Ne}$: 2.9714(76)[a]	${}^{18}_{9}\mathrm{F}$(ex)	${}^{18}_{8}\mathrm{O}$: 2.77 26(56)[a]
22	${}^{22}_{12}\mathrm{Mg}$: 3.0691(89)[b]	${}^{22}_{11}\mathrm{Na}$(ex)	${}^{22}_{10}\mathrm{Ne}$: 2.9525(40)[a]
26	${}^{26}_{14}\mathrm{Si}$	${}^{26m}_{13}\mathrm{Al}$	${}^{26}_{12}\mathrm{Mg}$: 3.0337(18)[a]
30	${}^{30}_{16}\mathrm{S}$	${}^{30}_{15}\mathrm{P}$(ex)	${}^{30}_{14}\mathrm{Si}$: 3.1336(40)[a]
34	${}^{34}_{18}\mathrm{Ar}$: 3.3654(40)[a]	${}^{34}_{17}\mathrm{Cl}$	${}^{34}_{16}\mathrm{S}$: 3.2847(21)[a]
38	${}^{38}_{20}\mathrm{Ca}$: 3.467(1)[c]	${}^{38m}_{19}\mathrm{K}$: 3.437(4)[d]	${}^{38}_{18}\mathrm{Ar}$: 3.4028(19)[a]
42	${}^{42}_{22}\mathrm{Ti}$	${}^{42}_{21}\mathrm{Sc}$: 3.5702(238)[a]	${}^{42}_{20}\mathrm{Ca}$: 3.5081(21)[a]
46	${}^{46}_{24}\mathrm{Cr}$	${}^{46}_{23}\mathrm{V}$	${}^{46}_{22}\mathrm{Ti}$: 3.6070(22)[a]
50	${}^{50}_{26}\mathrm{Fe}$	${}^{50}_{25}\mathrm{Mn}$: 3.7120(196)[a]	${}^{50}_{24}\mathrm{Cr}$: 3.6588(65)[a]
54	${}^{54}_{28}\mathrm{Ni}$: 3.738(4)[e]	${}^{54}_{27}\mathrm{Co}$	${}^{54}_{26}\mathrm{Fe}$: 3.6933(19)[a]
62	${}^{62}_{32}\mathrm{Ge}$	${}^{62}_{31}\mathrm{Ga}$	${}^{62}_{30}\mathrm{Zn}$: 3.9031(69)[b]
66	${}^{66}_{34}\mathrm{Se}$	${}^{66}_{33}\mathrm{As}$	${}^{66}_{32}\mathrm{Ge}$
70	${}^{70}_{36}\mathrm{Kr}$	${}^{70}_{35}\mathrm{Br}$	${}^{70}_{34}\mathrm{Se}$
74	${}^{74}_{38}\mathrm{Sr}$	${}^{74}_{37}\mathrm{Rb}$: 4.1935(172)[b]	${}^{74}_{36}\mathrm{Kr}$: 4.1870(41)[a]



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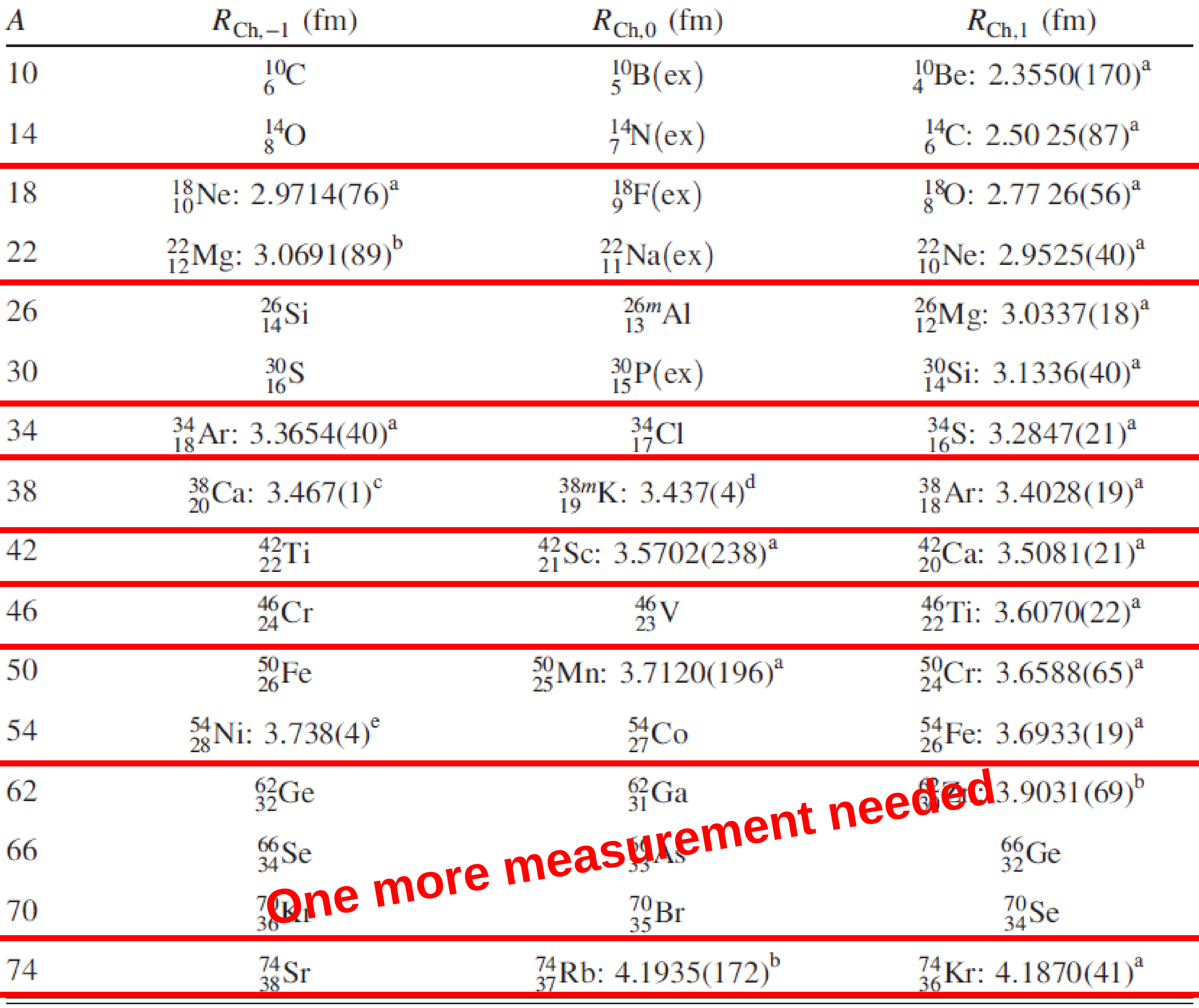
$$A = 38 \quad : \quad \Delta M_B^{(1)} = -0.03(12)_{+}(52)_{0}(7)_{-}\mathrm{fm}^2$$

Possible future measurement of r(^{38m}K) at TRIUMF with ~4 times better in precision



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One more measurement needed

Summary

- **Nuclear beta decays** offer precision tests of SM; **various anomalies** suggest the existence of new physics
- Measurements of **nuclear charge radii** provide important nuclear structure information relevant to beta decays:

  (1) The (vector) decay form factor, determined by **TWO measured radii** through CVC;

  (2) ISB correction in Fermi transitions, probed by **THREE measured radii**
- New data will critically impact the measurement of V_{ud} and the **search of new physics** (BSM couplings)