

Collective phenomena and shell structure far from stability

Frédéric Nowacki¹



Theory Alliance
FACILITY FOR RARE ISOTOPE BEAMS

FRIB-TA Topical Program:
Theoretical Justifications and
Motivations for Early High-Profile
FRIB Experiments

16-26 mai 2023
Facility for Rare Isotope Beams



¹Strasbourg-Madrid Shell-Model collaboration

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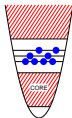
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Fortunately, life has made us more reasonable ... and things more simple !

Shell Model: Physics Goals

Collective excitations:

- Deformation, Superdeformation,
- Dipole/M1 resonances
- Superfluidity
- Symmetries

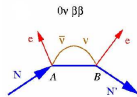


- define Effective Interaction
- $\mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$
- build and diagonalize Energy matrix

Weak processes:

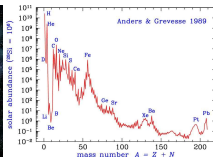
- β decay
- $\beta\beta$ decay

$$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_{\nu} \rangle^2$$



Shell evolution far from stability:

- Vanishing of shell closures
- New magic numbers



Ab Initio calculations:

- Chiral EFT realistic interactions
- 3N forces

Shell Model: Giant Computations

- exponential growth of basis dimensions:

$$D \sim \binom{d_\pi}{p} \cdot \binom{d_\nu}{n}$$

In *pf* shell :

⁴⁸Cr 1,963,461

⁵⁶Ni **1,087,455,228**

In *pf-sdg* space :

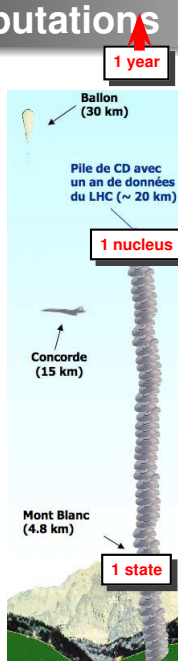
⁷⁸Ni **210,046,691,518**

- Actual limits in limits in giant diagonalizations: **0.2 10¹²** for ¹¹⁴Sn core excitations
- Some of the largest diagonalizations ever are performed in Strasbourg with relatively modest computational resources:

Phys. Rev. C82 (2010) 054301, ibidem 064304

- [m scheme](#) ANTOINE code
- [coupled scheme](#) NATHAN code

E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427;
ANTOINE website



- Largest matrices up to now contain up to $\sim 10^{14}$ non-zero matrix elements.
- This would require more than 1,000,000 CD-ROM's to store the information for a single matrix !
- They cannot be stored on hard disk and are computed on the fly.

Discrete Non-Orthogonal Shell Model

Generator Coordinate Method: $|\Psi_{\text{eff}}\rangle = \sum_i f_i |\Phi_i\rangle$

- 1) Deformed Hartree-Fock (HF) Slater determinants
- 2) Restoration of rotational symmetry
- 3) Mixing of shapes:

$$|\Psi_{\text{eff}}\rangle = \text{[deformed shape]} + \text{[deformed shape]} + \text{[deformed shape]} \dots$$

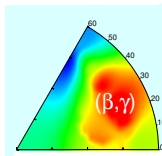
Basis Truncation Method



choice of relevant deformed Hartree-Fock states

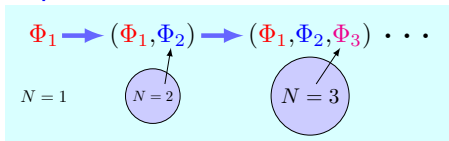
• E. Caurier's Minimization Technique:

(E. Caurier, Proc. on GCM, BLG report **484** (1975))



- ◇ Based on the variational principle
- ◇ Minimization of the energy of given states $\{J^\pi\}$

• Iterative procedure:



Discrete Non-Orthogonal Shell Model

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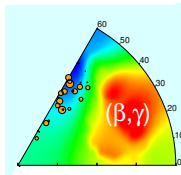
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$$|\Psi_{\text{eff}}\rangle = \text{[deformed sphere]} + \text{[deformed sphere]} + \text{[deformed sphere]} \dots$$

Intrinsic/Laboratory Description

- **Deformation structure of nuclear states:** $\{J_{\alpha}^{\pi}\}$, $q = (\beta, \gamma)$

$$M_{\alpha}^{(J)}(q, K) = \sum_{q', K'} [\hat{N}^{1/2}]_{K'K}^{(J)}(q', q) f_{\alpha}^{(J)}(q', K')$$



- ◇ Probability of a configuration (β, γ) :

$$P_{\alpha}^{(J)}(q) = \sum_K |M_{\alpha}^{(J)}(q, K)|^2$$

- **particle-hole interpretation:**



M-scheme

- ***K*-quantum numbers:**

$$P_{\alpha}^{(J)}(K) = \sum_q |M_{\alpha}^{(J)}(q, K)|^2$$

Discrete Non-Orthogonal Shell Model

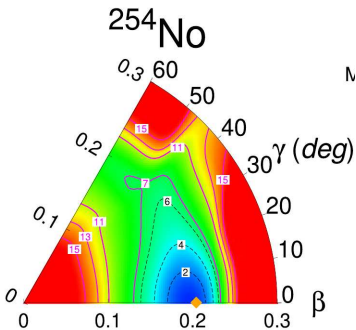
PHYSICAL REVIEW C **105**, 054314 (2022)

Nuclear structure within a discrete nonorthogonal shell model approach: New frontiers

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N

8⁻ ——— 1371
 $K^\pi = 8^-$
 $[\pi h_{\frac{9}{2}} \otimes \nu g_{\frac{9}{2}}]$

10₁⁺ ——— 719

8₁⁺ ——— 472

6₁⁺ ——— 276

4₁⁺ ——— 132

2₁⁺ ——— 40

0₁⁺ $K^\pi = 0^+$

DNO-SM (15 HF states)

254No

8⁻ ——— 1296

7⁺ ——— 1243
 6⁺ ——— 1161
 5⁺ ——— 1091
 4₁⁺ ——— 1033
 3⁺ ——— 987

10₁⁺ ——— 786

7⁺ ——— 658

6⁺ ——— 567

5⁺ ——— 489

4⁺ ——— 424

3⁺ ——— 372

$K^\pi = 3^+$
 $[\pi h_{\frac{9}{2}} \otimes \nu j_{\frac{15}{2}}]$

8₁⁺ ——— 518

6₁⁺ ——— 304

4₁⁺ ——— 145

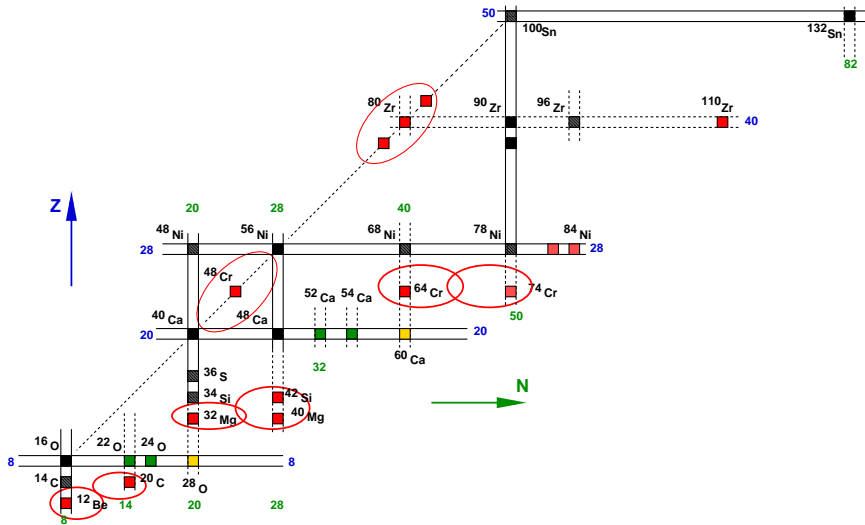
2₁⁺ ——— 44

0₁⁺ ———

EXP

First "SM" calculations for superheavies !!!

Landscape of medium mass nuclei



Landscape of medium mass nuclei

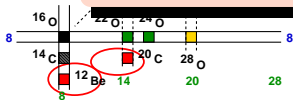
- New gaps: ^{24}O , ^{48}Ni , ^{54}Ca , ^{78}Ni , ^{100}Sn
- Vanishing of shell closure: ^{12}Be , ^{32}Mg , ^{42}Si , ^{64}Cr , ^{80}Zr ...
- Island of deformation around $A \sim 32$, $A \sim 64$
- Low-lying dipole excitations in Ne, Ni isotopes



- Variety of phenomena dictated by shell structure
- Close connection between collective behaviour and underlying shell structure
- Interplay between
 - Monopole field (spherical mean field)
 - Multipole correlations (pairing, Q.Q, ...)

"Pairing plus Quadrupole propose, Monopole disposes"

A. Zuker, Coherent and Random Hamiltonians, CRN Preprint 1994



132Sn
82

Development of deformation at N=8,20,40,70

F. Nowacki, A. Obertelli and A. Poves

Progress in Particle and Nuclear Physics 120 (2021) 103866

- Magic numbers are associated to energy gaps in the spherical mean field. Therefore, to promote particles above the Fermi levels costs energy
- However some intruders configurations can overwhelm their loss of monopole energy with their huge gain in correlation energy
- Several examples of this phenomenon exist in stable magic nuclei (as in ^{40}Ca nucleus) in the form of coexisting spherical, deformed and superdeformed states in a very narrow energy range
- At the very neutron rich or very proton rich edges, the T=0 and T=1 channels of the effective nuclear interaction weight very differently than they do at the stability line. Therefore the effective single particle structure may suffer important changes, leading in some cases to the vanishing of established shell closures or to the appearance of new ones

Fig. 40. Schematic view of the valence spaces at N = 8, 20, 40 and 70. The intruder configurations that develop quadrupole collectivity are highlighted.

Development of deformation at N=8,20,40,70

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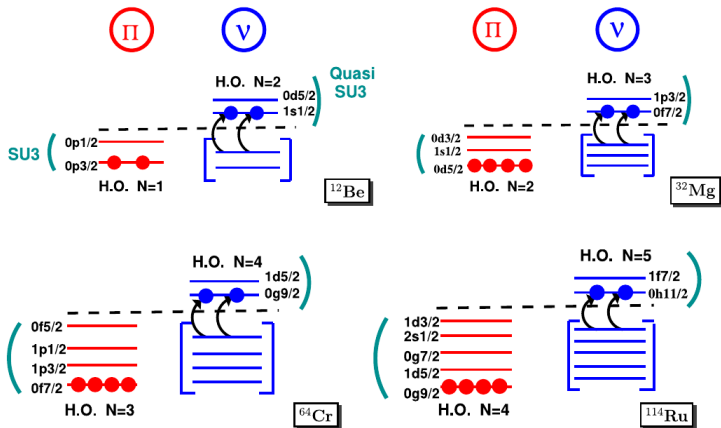


Fig. 40. Schematic view of the valence spaces at N = 8, 20, 40 and 70. The intruder configurations that develop quadrupole collectivity are highlighted.

Development of deformation at N=14,28,50,82

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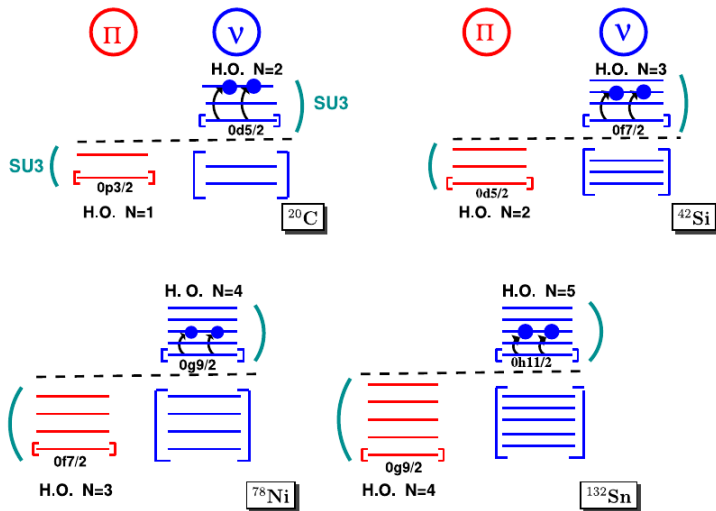
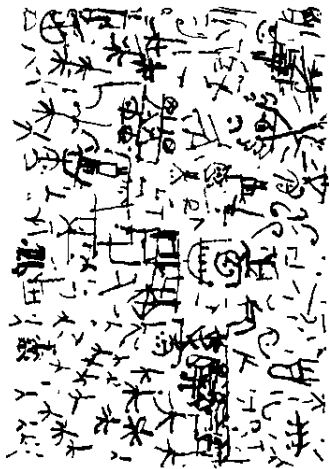
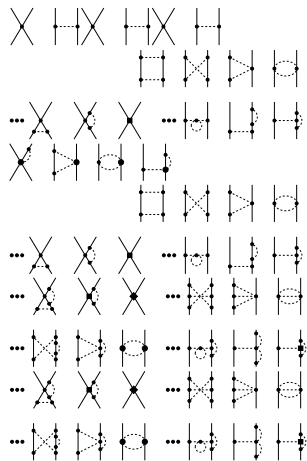


Fig. 41. Schematic view of the valence spaces at N = 14, 28, 50 and 82. The intruder configurations that develop quadrupole collectivity are highlighted.

The nuclear interaction: the complex view

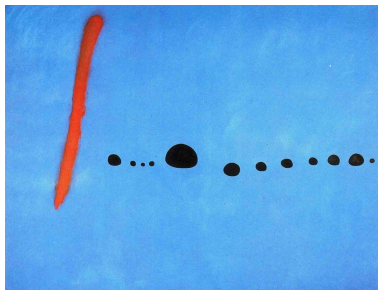


P. Klee, art

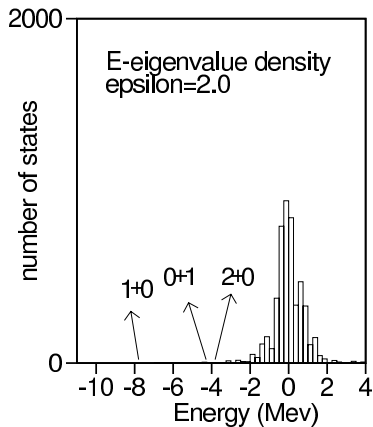


E. Epelbaum, physics

The nuclear interaction: the simple view



J. Miro, art



A. Zuker, physics

Separation of the effective Hamiltonian

Monopole and multipole

From the work of M. Dufour and A. Zuker (PRC 54 1996 1641)
Separation theorem:

Any effective interaction can be split in two parts:

$$H = H_{monopole} + H_{multipole}$$

$H_{monopole}$: *spherical mean-field*

☞ responsible for the global saturation properties and for the evolution of the spherical single particle levels.

$H_{multipole}$: *correlator*

☞ pairing, quadrupole, octupole...

Important property:

$$\langle CS \pm 1 | H | CS \pm 1 \rangle = \langle CS \pm 1 | H_{monopole} | CS \pm 1 \rangle$$

Multipole Hamiltonian

$H_{\text{multipole}}$ can be written in two representations, particle-particle and particle-hole. Both can be brought into a diagonal form. When this is done, it comes out that only a few terms are coherent, and those are the simplest ones:

- $L = 0$ isovector and isoscalar pairing
- Elliott's quadrupole
- $\vec{\sigma}\vec{\tau} \cdot \vec{\sigma}\vec{\tau}$
- Octupole and hexadecapole terms of the type $r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$

Besides, they are universal (all the realistic interactions give similar values) and scale simply with the mass number

	pp(JT)				ph($\lambda\tau$)		
	10	01	21	20	40	10	11
KB	-5.83	-4.96	-3.21	-3.53	-1.38	+1.61	+3.00
USD-A	-5.62	-5.50	-3.17	-3.24	-1.60	+1.56	+2.99
CCEI	-6.79	-4.68	-2.93	-3.40	-1.39	+1.21	+2.83
NN+NNN-MBPT	-6.40	-4.36	-2.91	-3.28	-1.23	+1.10	+2.43
NN-MBPT	-6.06	-4.38	-2.92	-3.35	-1.31	+1.03	+2.49

Multipole Hamiltonian

$H_{multipole}$ can be written in two representations, particle-particle

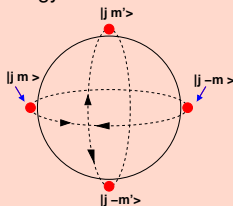
and pair
When
cohere

- $L =$
- EI
- $\vec{\sigma}\vec{\tau}$
- Oc

- **Pairing regime: spherical nuclei**

ground state = pairs of like-particles coupled at $J=0$ (seniority $\nu=0$)
 2^+ state (break of pair; $\nu=2$) at high energy

superfluid nucleus:



Typical example: **Tin isotopes**

- **Quadrupole regime: deformed nuclei**

prolate nucleus:



Typical example: **open shell $N=Z$ nuclei**

Beside
similar

KB
USD
CCE
NN+
NN-

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Multipole Hamiltonian

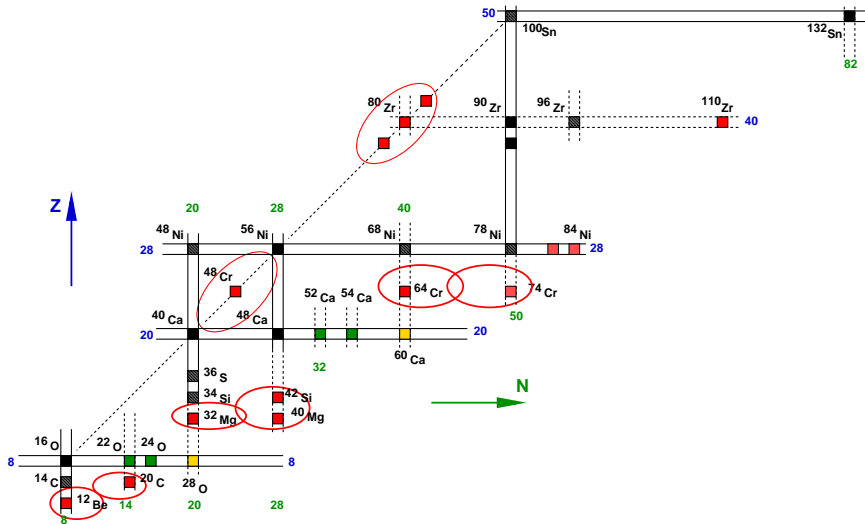
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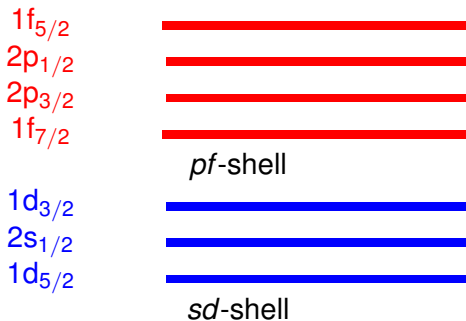
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Landscape of medium mass nuclei



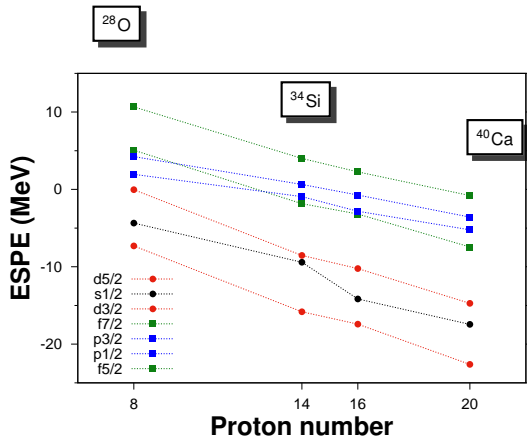
Playground

In the valence space of two major shells



EFFECTIVE INTERACTION: SDPF-U-MIX (update 2020)

Island of Inversion: Trends



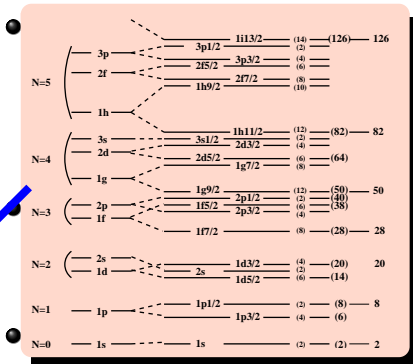
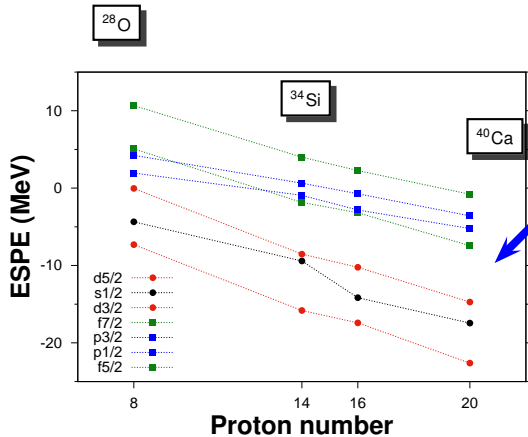
- At the neutron drip line, the ESPE's of ^{28}O are completely at variance with those of ^{40}Ca at the stability valley. The change from the standard ESPE's of ^{16}O to the anomalous ones in ^{28}O is totally due to the interactions of *sd* shell neutrons among themselves

- Notice that the *sd* shell orbits remain always below the *pf* shell with the $\nu 0f_{7/2}$ and $\nu 0p_{3/2} - 0p_{1/2}$ orbitals DO get inverted

- The monopole part of the neutron-proton interaction restores the N=20 shell gap when the valley of stability is approached

- Spin-Tensor decomposition shows it is mainly a **Central** and **Tensor** effect

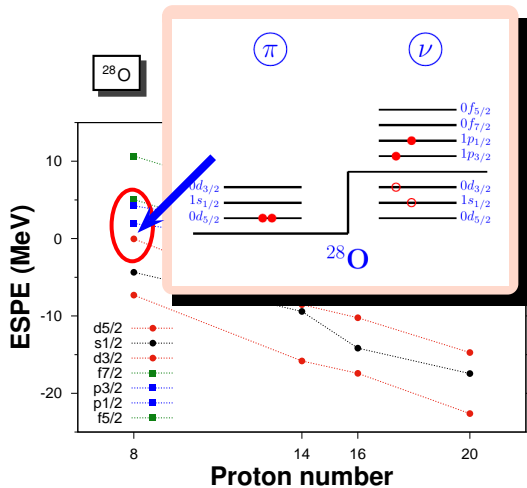
Islands Of Inversion: Trends



N=20 shell gap when the valley of stability is approached

Spin-Tensor decomposition shows it is mainly a **Central** and **Tensor** effect

Further away from Stability



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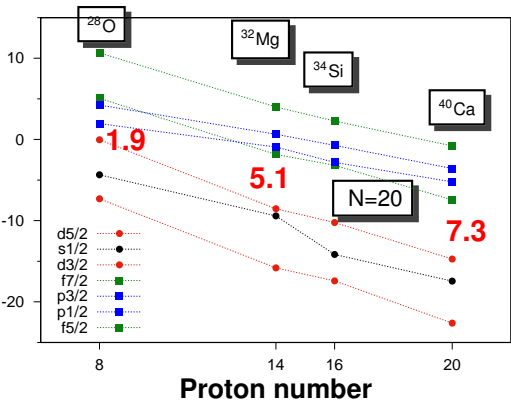
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- Shell Evolution favors natural geometry for **low-lying M1** excitations

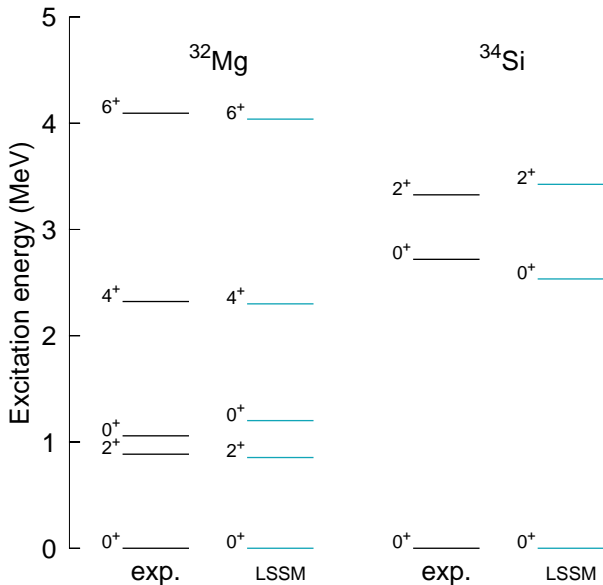
$$\begin{array}{cc} \nu 1s_{1/2} & \nu 1p_{3/2} \\ \nu 0d_{3/2} & \nu 1p_{1/2} \end{array} \otimes$$

Island of Inversion: Trends

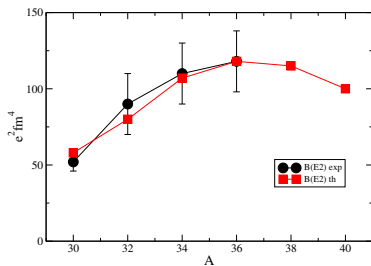
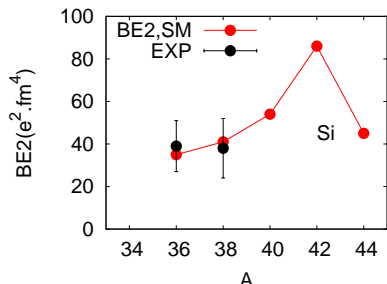
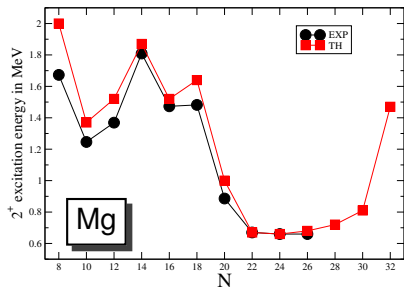
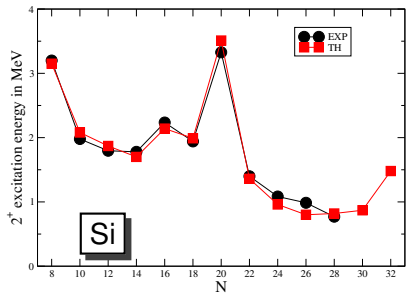


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- Notice that the *sd* shell orbits remain always below the *pf* shell with the $\nu 0f_{7/2}$ and $\nu 0p_{3/2} - 0p_{1/2}$ orbitals DO get inverted
- The monopole part of the neutron-proton interaction restores the N=20 shell gap when the valley of stability is approached
- Spin-Tensor decomposition shows it is mainly a **Central** and **Tensor** effect

Inverse shape coexistence Shell closure in ^{32}Mg



Merging of IOIs at N=20 and N=28



Low-lying neutron fp -shell intruder states in ^{27}Ne

S. M. Brown,¹ W. N. Catford,¹ J. S. Thomas,¹ B. Fernández-Domínguez,^{2,3} N. A. Orr,² M. Labiche,⁴ M. Rejmund,⁵ N. L. Achouri,⁶ H. Al Falou,⁶ N. I. Ashwood,⁶ D. Beaumel,⁷ Y. Blumenfeld,⁷ B. A. Brown,⁸ R. Chapman,⁹ M. Chartier,⁵ N. Curtis,⁹ G. de France,⁵ N. de Sereville,⁷ F. Delaunay,² A. Drouart,¹⁰ C. Force,⁵ S. Franchoo,⁷ J. Guillot,⁷ P. Haigh,⁹ F. Hammache,⁷ V. Lapoux,¹⁰ R. C. Lemmon,⁴ A. Leprince,² F. Maréchal,⁷ X. Mougeot,¹⁰ B. Mougionot,⁷ L. Nalpas,¹⁰ A. Navin,⁵ N. P. Patterson,¹ B. Pietras,³ E. C. Pollacco,¹⁰ A. Ramus,⁷ J. A. Scarpaci,⁷ I. Stefan,⁷ and G. L. Wilson¹

LOW-LYING NEUTRON fp -SHELL INTRUDER STATES . . .

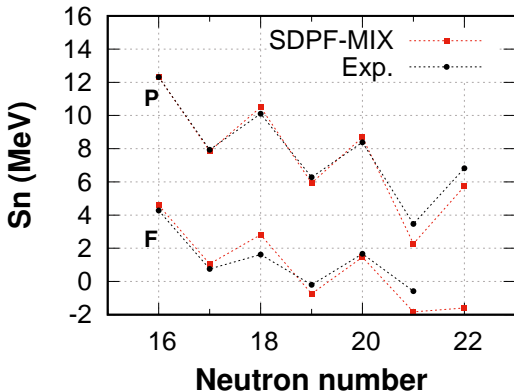
TABLE I. Comparison between experimental and calculated (see text) excitation energies and spectroscopic factors for states in ^{27}Ne . Experimental excitation energies are from [10] except for the 1.74-MeV state (present work). For C^2S , the errors include uncertainties from the reaction model.

J^π	E_{exp}^* (MeV)	$E_{\text{WBP-M}}^*$ (MeV)	C^2S		
			Ref. [10]	Present	WBP-M
$3/2^+$	0	0	0.2(2)	0.42(22)	0.63
$3/2^-$	0.765	0.809	0.6(2)	0.64(33)	0.67
$1/2^+$	0.885	0.869	0.3(1)	0.17(14)	0.17
$7/2^-$	1.74	1.686	–	0.35(10)	0.40

At the neutron drip line, the ESPE's of ^{28}O are completely at variance with those of ^{40}Ca at the stability valley. The change from the standard ESPE's of ^{16}O to the anomalous ones in ^{28}O is totally due to the interactions of sd shell neutrons among themselves

- Notice that the sd shell orbits remain always below the pf shell with the $\nu 0f_{7/2}$ and $\nu 0p_{3/2} - 0p_{1/2}$ orbitals DO get inverted
- The monopole part of the neutron-proton interaction restores the $N=20$ shell gap when the valley of stability is approached
- Evidence for shell inversion towards ^{28}O

At the drip line



Nowacki/Poves 2014

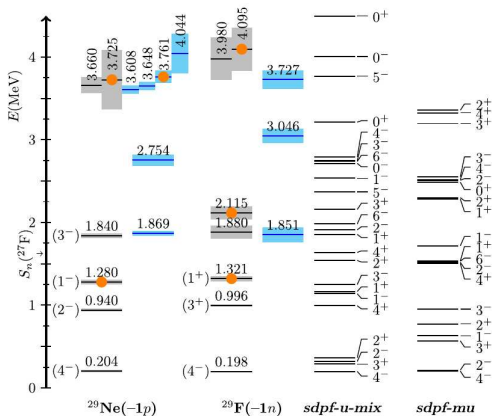
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- The monopole part of the neutron-proton interaction restores the N=20 shell gap when the valley of stability is approached
- "ill" behaviour mainly due to ^{38}P separation energy

At the drip line

PHYSICAL REVIEW LETTERS **124**, 152502 (2020)

Extending the Southern Shore of the Island of Inversion to ^{28}F

A. Revel,^{1,2} O. Sorlin,¹ F. M. Marqués,² Y. Kondo,³ J. Kahlbow,^{4,5} T. Nakamura,³ N. A. Orr,² F. Nowacki,^{6,7} ...



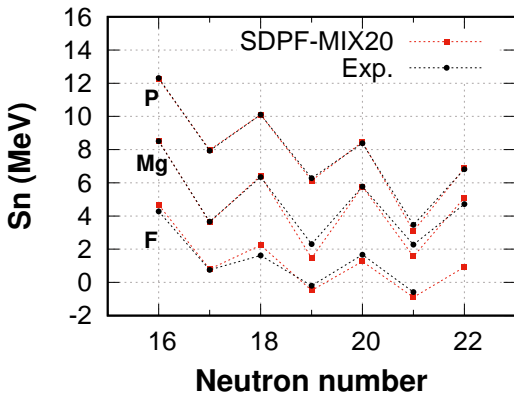
- At the neutron drip line, the ESPE's of ^{28}O are completely at variance with those of ^{40}Ca at the stability valley.

- Notice that the sd shell orbits remain always below the pf shell with the $\nu 0f_{7/2}$ and $\nu 0p_{3/2} - 0p_{1/2}$ orbitals DO get inverted

- Recent evidence for intruder states in ^{28}F low-lying spectrum

- In addition, extraction of 80% of "l=1" content in the GS

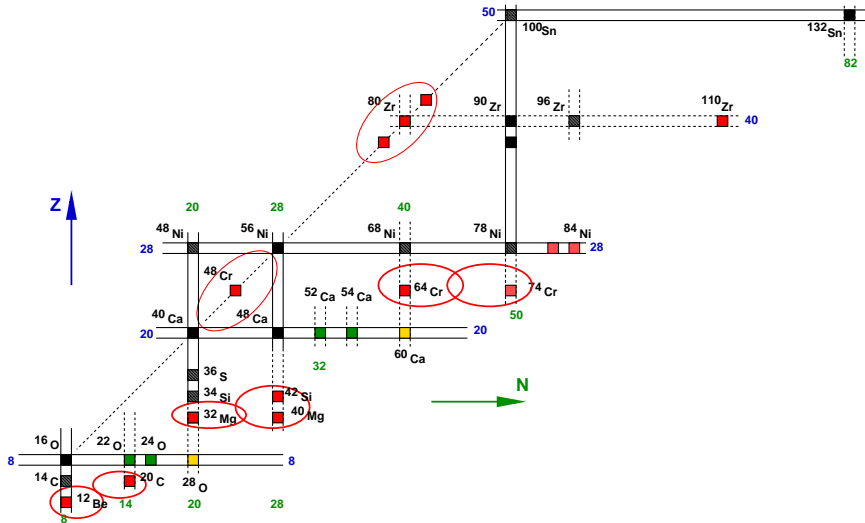
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Nowacki/Poves 2020

- At the neutron drip line, the ESPE's of ^{28}O are completely at variance with those of ^{40}Ca at the stability valley. The change from the standard ESPE's of ^{16}O to the anomalous ones in ^{28}O is totally due to the interactions of *sd* shell neutrons among themselves
- Notice that the *sd* shell orbits remain always below the *pf* shell with the $\nu 0f_{7/2}$ and $\nu 0p_{3/2} - 0p_{1/2}$ orbitals DO get inverted
- The monopole part of the neutron-proton interaction restores the N=20 shell gap when the valley of stability is approached
- New ^{30}F data from NeuLAND-SAMOURAI collaboration (J. Kahlbow PhD work, submitted)
- ^{38}P separation energy + $p_{3/2} - f_{7/2}$ splitting matches Fluorine chain S_n trend

Landscape of medium mass nuclei



Island of inversion at $N=40$, an old story: 1996

The Physics around the doubly-magic ^{78}Ni Nucleus

Leuven, Belgium
November 4th, 1996

A. Poves

^{64}Cr

$$g(\text{Oph} - 2\text{ph}) = 5.70$$

$$g(\text{Oph} - \text{Yph}) = 8.30$$

$$Q = -9.0 \text{ b}^2$$

$$CS < 1\%$$

$$B\text{E}2 = 19.8 \text{ b}^4$$

$$u(d_{5/2}) = 1.1$$

$$\frac{E(4^+)}{E(2^+)} = 2.7$$

$$\left[\frac{E(4^+)}{E(2^+)} = (3.2)(3.4) \right]$$

in the intruder configurations.

A SITUATION THAT REMINDS WHAT IS KNOWN AT $N=20$ FFS.

PHYSICAL REVIEW C **81**, 051304(R) (2010)

Collectivity at $N = 40$ in neutron-rich ^{64}Cr

A. Gade,^{1,2} R. V. F. Janssens,³ T. Baugher,^{1,2} D. Bazin,¹ B. A. Brown,^{1,2} M. P. Carpenter,³ C. J. Chiara,^{3,4} A. N. Deacon,⁵ S. J. Freeman,⁵ G. F. Grinyer,¹ C. R. Hoffman,³ B. P. Kay,³ F. G. Kondev,⁶ T. Lauritsen,³ S. McDaniel,^{1,2} K. Meierbachtol,^{1,7} A. Ratkiewicz,^{1,2} S. R. Stroberg,^{1,2} K. A. Walsh,^{1,2} D. Weisshaar,¹ R. Winkler,¹ and S. Zhu³

¹National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA

²Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

³Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

⁴Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

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⁷Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

PHYSICAL REVIEW C **81**, 061301(R) (2010)

Onset of collectivity in neutron-rich Fe isotopes: Toward a new island of inversion?

J. Ljungvall,^{1,2,3} A. Görgen,¹ A. Obertelli,¹ W. Korten,¹ E. Clément,² G. de France,² A. Bürger,⁴ J.-P. Delaroche,⁵ A. Dewald,⁶ A. Gadea,⁷ L. Gaudefroy,⁵ M. Girod,⁵ M. Hackstein,⁶ J. Libert,⁸ D. Mengoni,⁹ F. Nowacki,¹⁰ T. Pissulla,⁶ A. Poves,¹¹ F. Recchia,¹² M. Rejmund,² W. Rother,⁶ E. Sahin,¹² C. Schmitt,² A. Shrivastava,² K. Sieja,¹⁰ J. J. Valiente-Dobón,¹² K. O. Zell,⁶ and M. Zielińska¹³

¹CEA Saclay, IRFU, Service de Physique Nucléaire, F-91191 Gif-sur-Yvette, France

²GANIL, CEA/DSM-CNRS/IN2P3, Bd Henri Becquerel, BP 55027, F-14076 Caen, France

³GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

⁴GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

⁵GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

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⁷GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

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¹⁰GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

¹¹GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

¹²GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

¹³GANIL, CEA/DSM-CNRS/IN2P3, F-14076 Caen, France

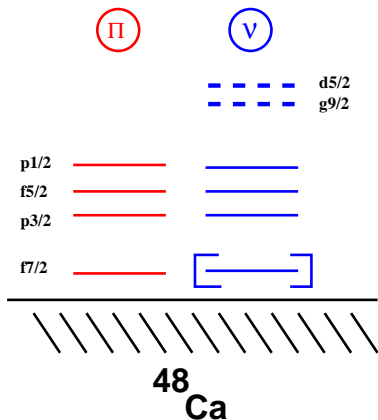
SM framework



Island of inversion around ^{64}Cr

S. Lenzi, F. Nowacki, A. Poves and K. Sieja

Phys. Rev. C82, 054301, 2010



LNPS interaction:

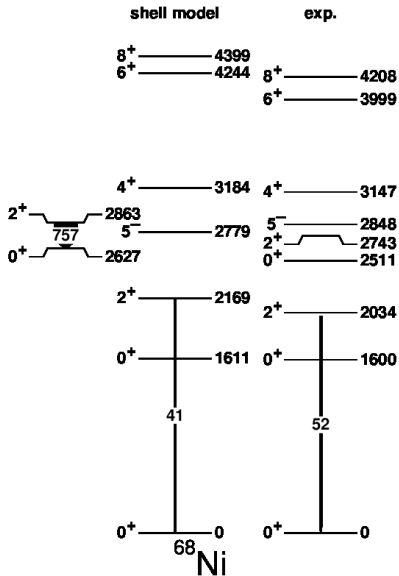
- based on realistic TBME
- new fit of the pf shell (KB3GR, E. Caurier)
- monopole corrections
- $g_{9/2}-d_{5/2}$ gap now constrained to 2.5 MeV in ^{68}Ni

Calculations:

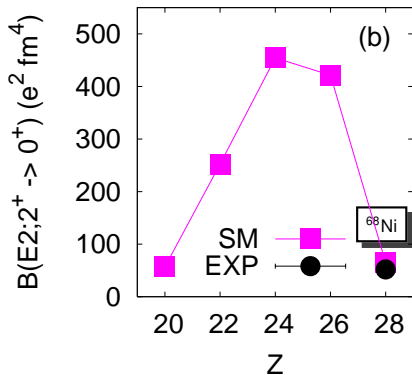
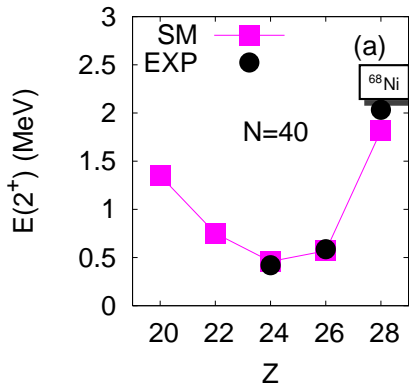
- Up to $14\hbar\omega$ excitations across $Z=28$ and $N=40$ gaps
- Matrix diagonalizations up to $2 \cdot 10^{10}$
- m-scheme code ANTOINE (non public parallel version)

Triple coexistence in ^{68}Ni

- at first approximation, ^{68}Ni has a double closed shell structure for GS
- But low lying structure much more complex
- three coexisting 0^+ states appear between 0 and ~ 2.5 MeV
- new location of 0_2^+ state !
Configuration mixing and relative transition rates between low-spin states in ^{68}Ni :
F. Recchia et al.
Phys. Rev. **C88**, 041302(R) (2013)
- prediction of very low-lying **superdeformed band** ($\beta_2 \sim 0.4$) of $6p6h$ nature!
• S. Lenzi et al.
Phys. Rev. **C82**, 054301 (2010)
• A. Dijon et al.
Phys. Rev. **C85**, 0311301(R) (2012)

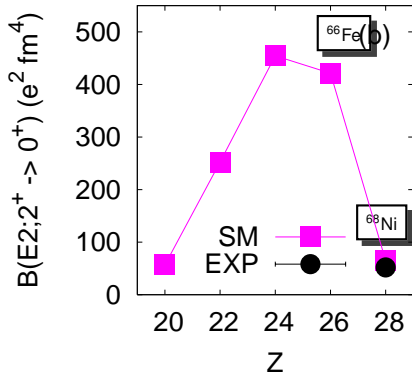
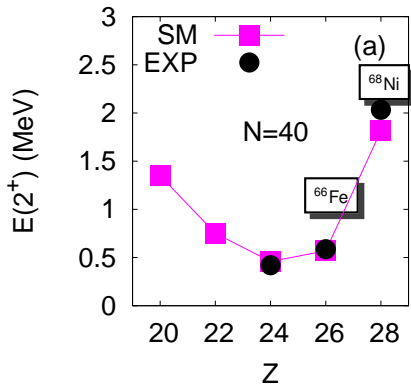


Shape transition at N=40



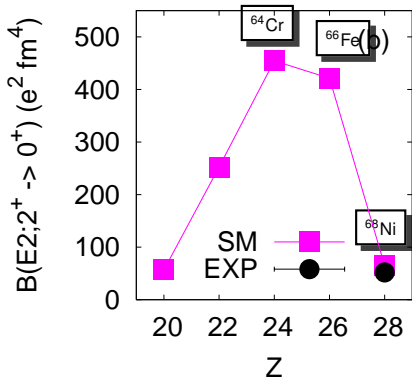
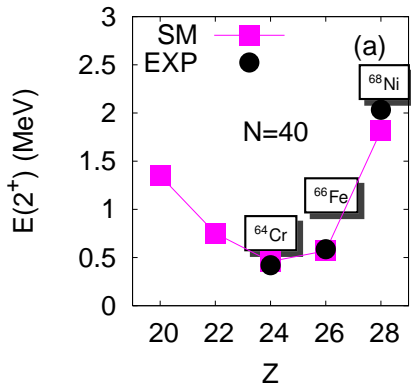
Nucleus	$\nu g_{9/2}$	$\nu d_{5/2}$	configuration
^{68}Ni	0.98	0.10	0p0h(51%)
^{66}Fe	3.17	0.46	4p4h(26%)
^{64}Cr	3.41	0.76	6p6h(23%)
^{62}Ti	3.17	1.09	4p4h(48%)

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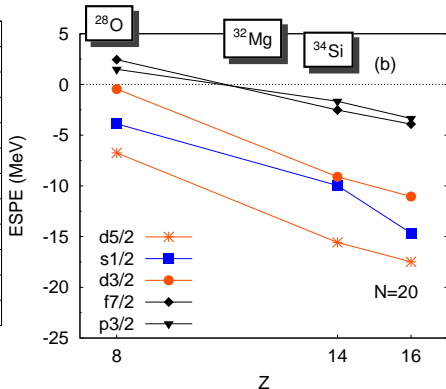
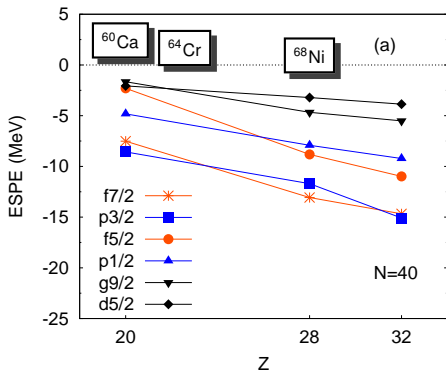
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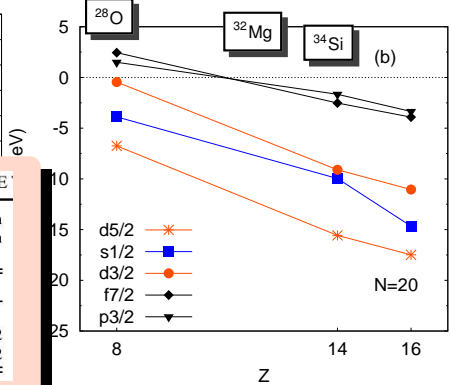
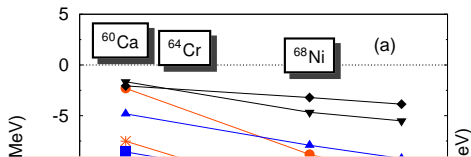
Neutron effective single particle energies



- reduction of the $\nu f_{5/2}-g_{9/2}$ gap with removing $f_{7/2}$ protons
- proximity of the quasi-SU3 partner $d_{5/2}$
- inversion of $d_{5/2}$ and $g_{9/2}$ orbitals
same ordering as CC calculations

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Neutron effective single particle energies



PRL **109**, 032502 (2012) PHYSICAL REVIEW LETTERS

TABLE II. Energies of the $5/2^+$ and $9/2^+$ resonances in $^{53,55,61}\text{Ca}$. $\text{Re}[E]$ is the energy relative to the one-neutron emission threshold, and the width is $\Gamma = -2\text{Im}[E]$ (in MeV).

	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	$\text{Re}[E]$	Γ	$\text{Re}[E]$	Γ	$\text{Re}[E]$	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

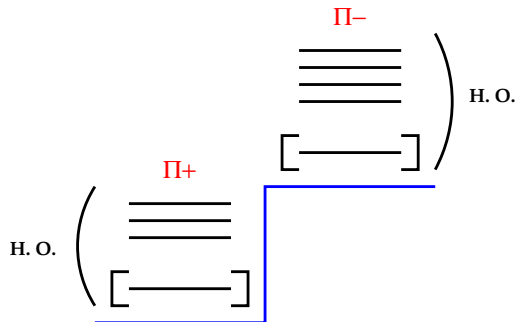
G. Hagen et al.

Phys. Rev. Lett. **109**, 032502 (2012)

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Spin-orbit shell closure far from stability



- sd-pf: ^{42}Si deformed

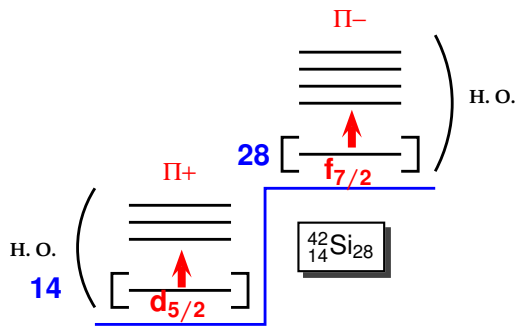
- pf-sdg: ^{78}Ni ???

- sdg-phf: ^{132}Sn
doubly magic

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Spin-orbit shell closure far from stability



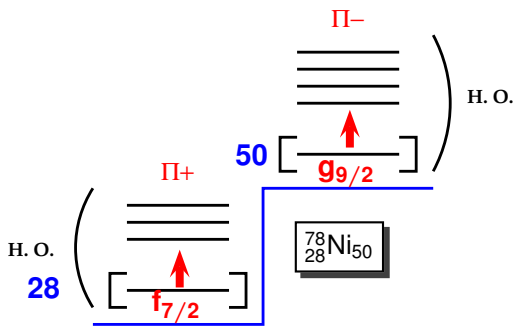
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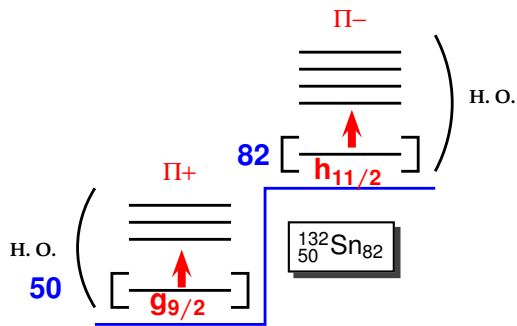
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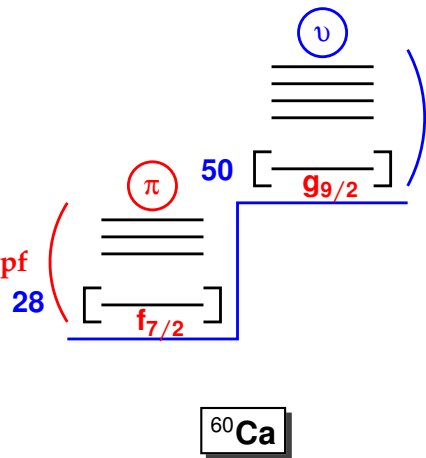
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Physics around ^{78}Ni



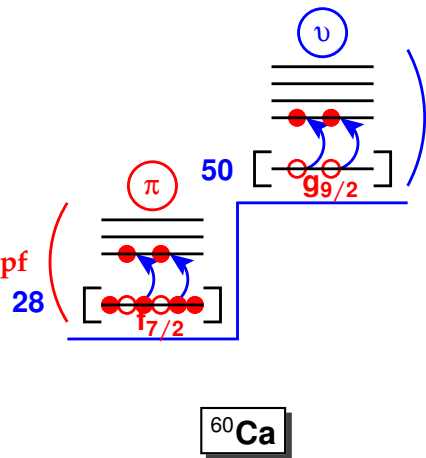
PFSDG-U interaction:

- realistic TBME
- pf shell for protons and gds shell for neutrons
- monopole corrections (3N forces)
- proton and neutrons gap ^{78}Ni fixed to phenomenological derived values

Calculations:

- excitations across Z=28 and N=50 gaps
- up to $5 \cdot 10^{10}$ Slater Determinant basis states
- up to $3 \cdot 10^{13}$ non-zero terms in the matrix!
- m-scheme code ANTOINE (non public version)
- J-scheme code NATHAN (parallelized version): $0.5 \cdot 10^9$ J basis states

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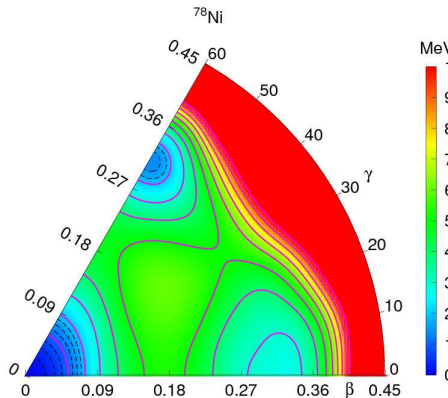
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Shape coexistence in ^{78}Ni

- At first approximation, ^{78}Ni has a double closed shell structure for GS
- But very low-lying competing structures
- From the diagonalization, the first excited states in ^{78}Ni are :
 - $0_2^+ - 2_1^+$ predicted at 2.6-2.9 MeV and to be deformed intruders of a **rotationnal band** !!!
- “ $1p1h$ ” 2_2^+ predicted at ~ 3.1 MeV
- Necessity to go **beyond** ($fp_{g_{\frac{9}{2}}} d_{\frac{5}{2}}$) **LNPS space** and **beyond ab-initio description**
- Portal to a new **Island of Inversion**

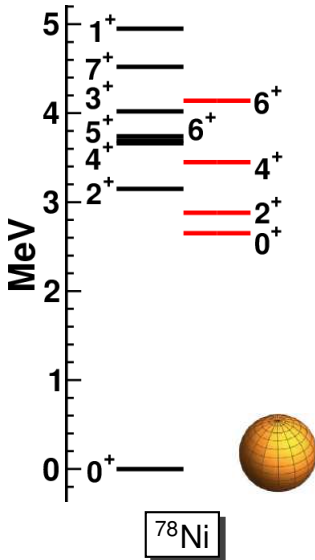


Constrained deformed HF in the SM basis

(Duy Duc Dao, DNO-SM calc., Strasbourg)

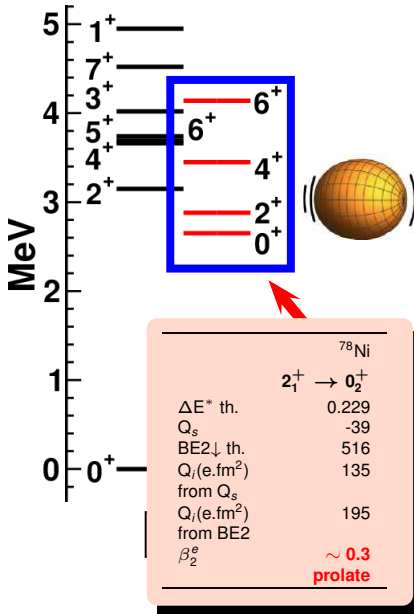
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F. Nowacki et al., PRL **177**, 272501 (2016)



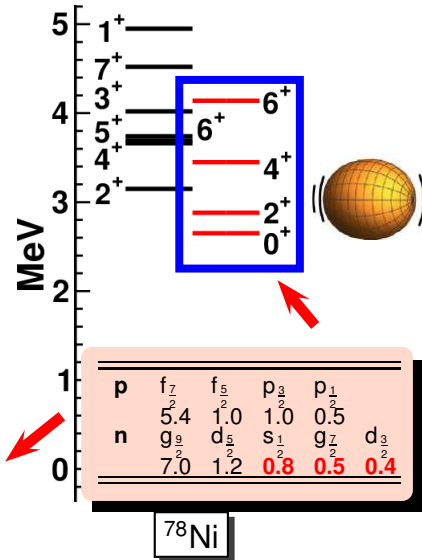
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F. Nowacki et al., PRL **177**, 272501 (2016)



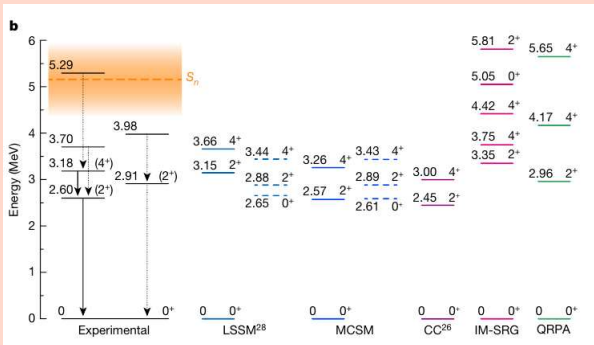
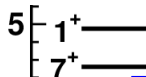
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F. Nowacki et al., PRL **177**, 272501 (2016)



Shape coexistence in ^{78}Ni

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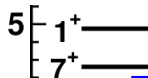
R. Taniuchi et al., NATURE 569, 53-58 (2019)

Portal to a new island of inversion
F. Nowacki et al., PRL 177, 272501 (2016)

^{78}Ni



$d_{3/2}$
0.4



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ARTICLE

<https://doi.org/10.1038/s41586-019-1155-x>

^{78}Ni revealed as a doubly magic stronghold against nuclear deformation

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R. Taniuchi et al., NATURE 569, 53-58 (2019)

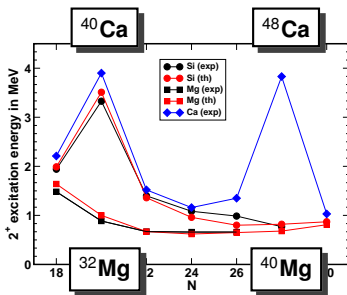
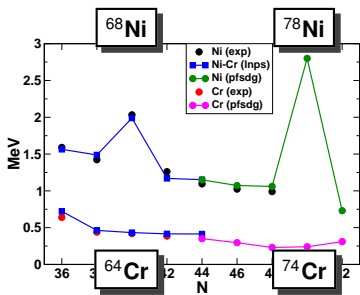
● Portal to a new island of inversion
F. Nowacki et al., PRL 177, 272501 (2016)

^{78}Ni

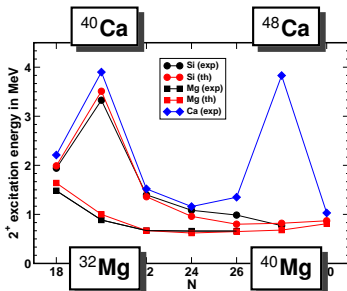
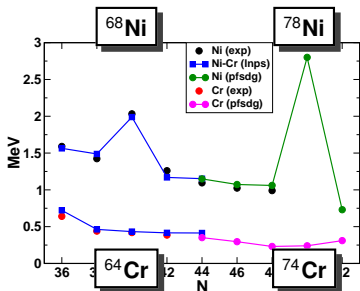


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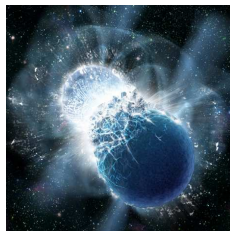
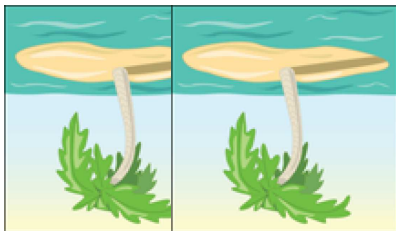
Island of Inversion Mergers



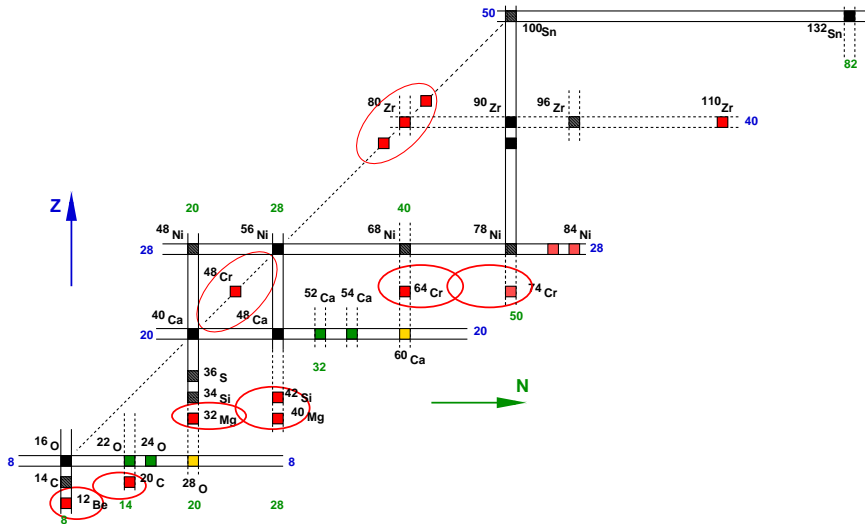
Island of Inversion Mergers



The N=40 and N=50 lol's merge like the N=20 and N=28 lol's did



Landscape of medium mass nuclei

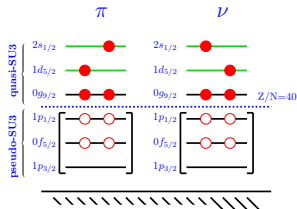


Island of Inversion at the $N=Z$ line

Strongly deformed states at $N = Z$:

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr , ^{80}Zr
- Shape transition between ^{84}Mo and ^{86}Mo

NSCL/GRETINA Experiment



R.D.O. Llewellyn *et al.*, Phys. Rev. Lett. **124**, 152501 (2020)

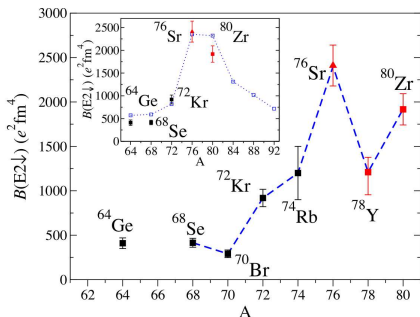
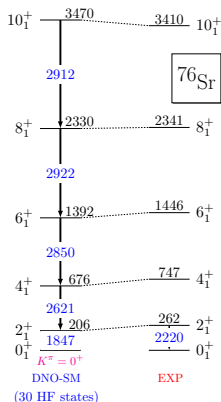


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei

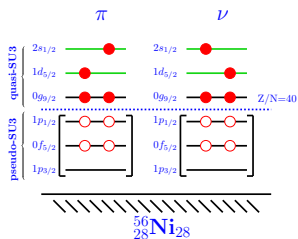
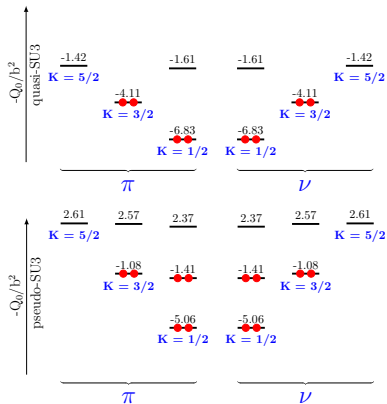


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NCSL/GRETINA Experiment



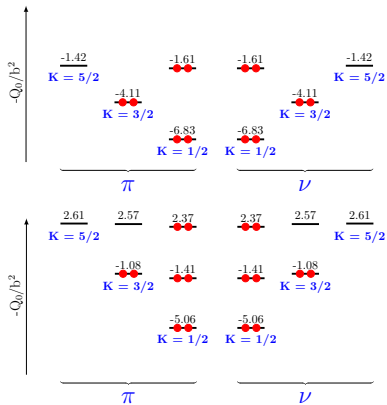
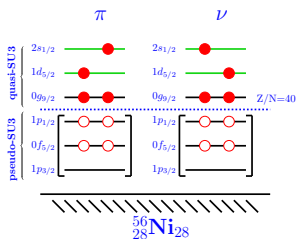
nucleus	NpNh*	B(E2)(e ² .fm ⁴)			Exp.
		ZRP	PHF	DNO-SM	
^{76}Se	4p-4h	924	806		
	8p-8h	2189	2101	1847	2220
	12p-12h	2316	-		
^{80}Zr	4p-4h	587	637		
	8p-8h	1713	1509	2325	1910
	12p-12h	2663	2396		

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NSCL/GRETINA Experiment



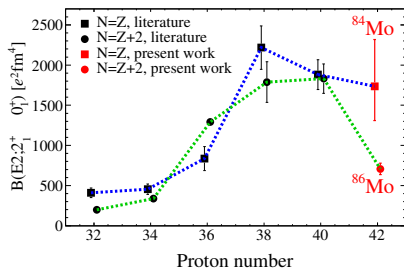
nucleus	Np-Nh*	ZRP	PHF	B(E2)(e ² .fm ⁴)		Exp.
				DNO-SM*	SM	
^{84}Mo	4p-4h	1104	1193	1765	-	1740 ⁺⁵⁸⁰ ₋₇₃₀
	8p-8h	1891	1732			
^{86}Mo	0p-0h	542	196	1184	731	707(71)
	2p-2h	1030	871			
	4p-4h	1416	1179			
	6p-6h	1858	1655			

Island of Inversion at the N=Z line

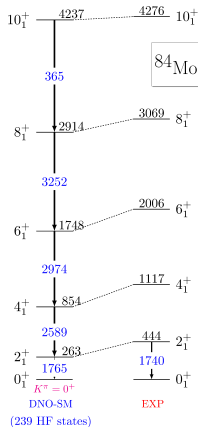
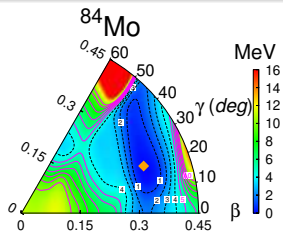
◇ Strongly deformed states at $N = Z$

- Configuration mixing in ^{72}Kr
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NSCL/GRETINA Experiment



J. Ha, F. Recchia *et al.*, submitted

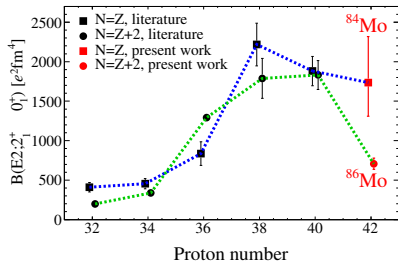
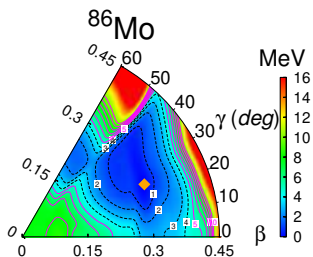


Island of Inversion at the N=Z line

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J. Ha, F. Recchia *et al.*, submitted

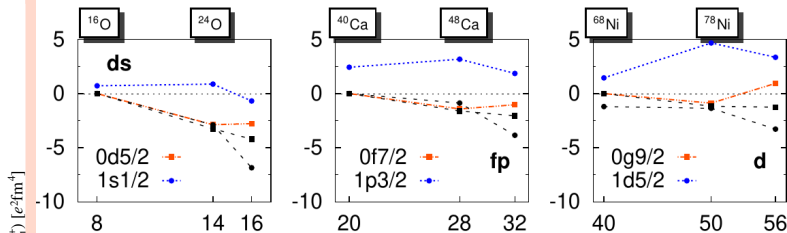
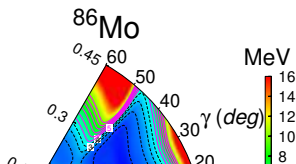
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Island of Inversion at the $N=Z$ line

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NSCL/CRETINA Experiment



K. Sieja, F. Nowacki

Phys. Rev. **C85**, 051301(R) (2012)

SM Exp.

- 1740^{+580}_{-730}

31 707(71)

J. Ha, F. Recchia *et al.*, submitted

Summary

- Monopole drift develops in all regions but the Interplay between **correlations (pairing + quadrupole)** and **spherical mean-field (monopole field)** determines the physics. It can vary from :
 - island of deformation at $N=20$ and $N=40$
 - deformation at $Z=14$, $N=28$ for ^{42}Si and shell weakening at $Z=28$, $N=50$ for ^{78}Ni
- The “islands of inversion” appear due to the effect of the correlations, hence they could also be called “islands of enhanced collectivity”. As quadrupole correlations are dominant in this region, most of their inhabitants are deformed rotors. Shape transitions and coexistence show up everywhere
- Quadrupole energies can be huge and understood in terms of symmetries

Summary

- even at the drip in fluorine isotopes, bound approximation holds
- strong superfluid regime with pair scattering from sd to pf shells
- odd-even S_n energies staggering does not seem to originate from continuum coupling

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- D. D. Dao, G. Martinez-Pinedo, A. Poves, S. Lenzi, K. Sieja
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- J. Herzfeld-Nowacki