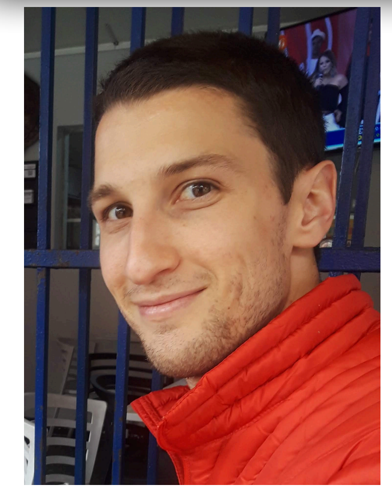
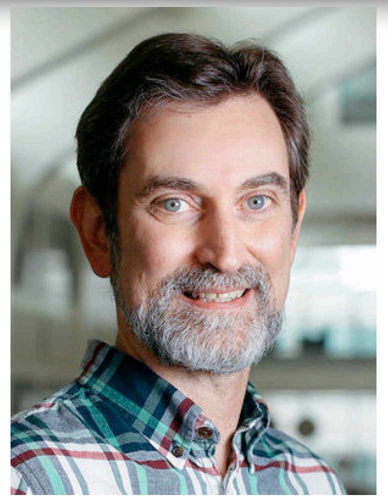
Scale and scheme dependence in structure and reactions

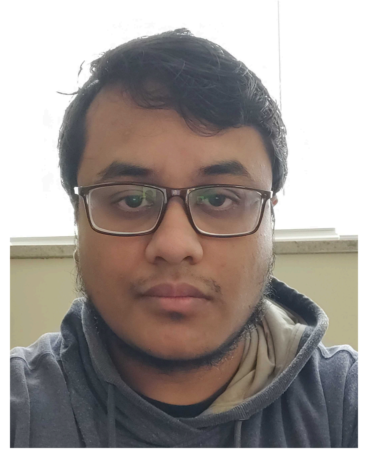
FRIB-TA Program: Theoretical Justifications and Motivations for Early High-Profile FRIB Experiments
May 2023, FRIB, East Lansing, MI



Anthony Tropiano (ANL)



Dick Furnstahl (OSU)



Mostofa Hisham (OSU)

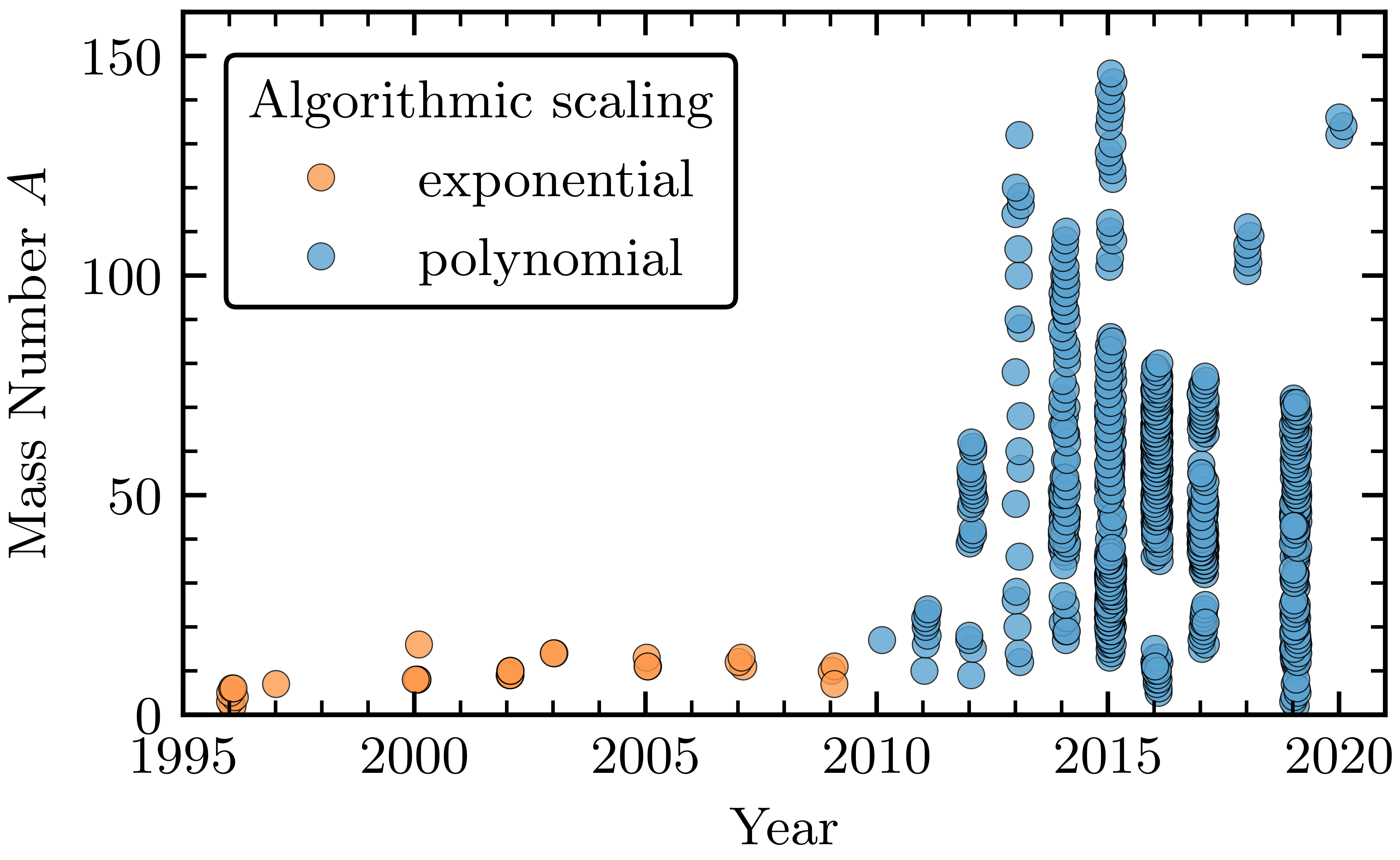
Some references:

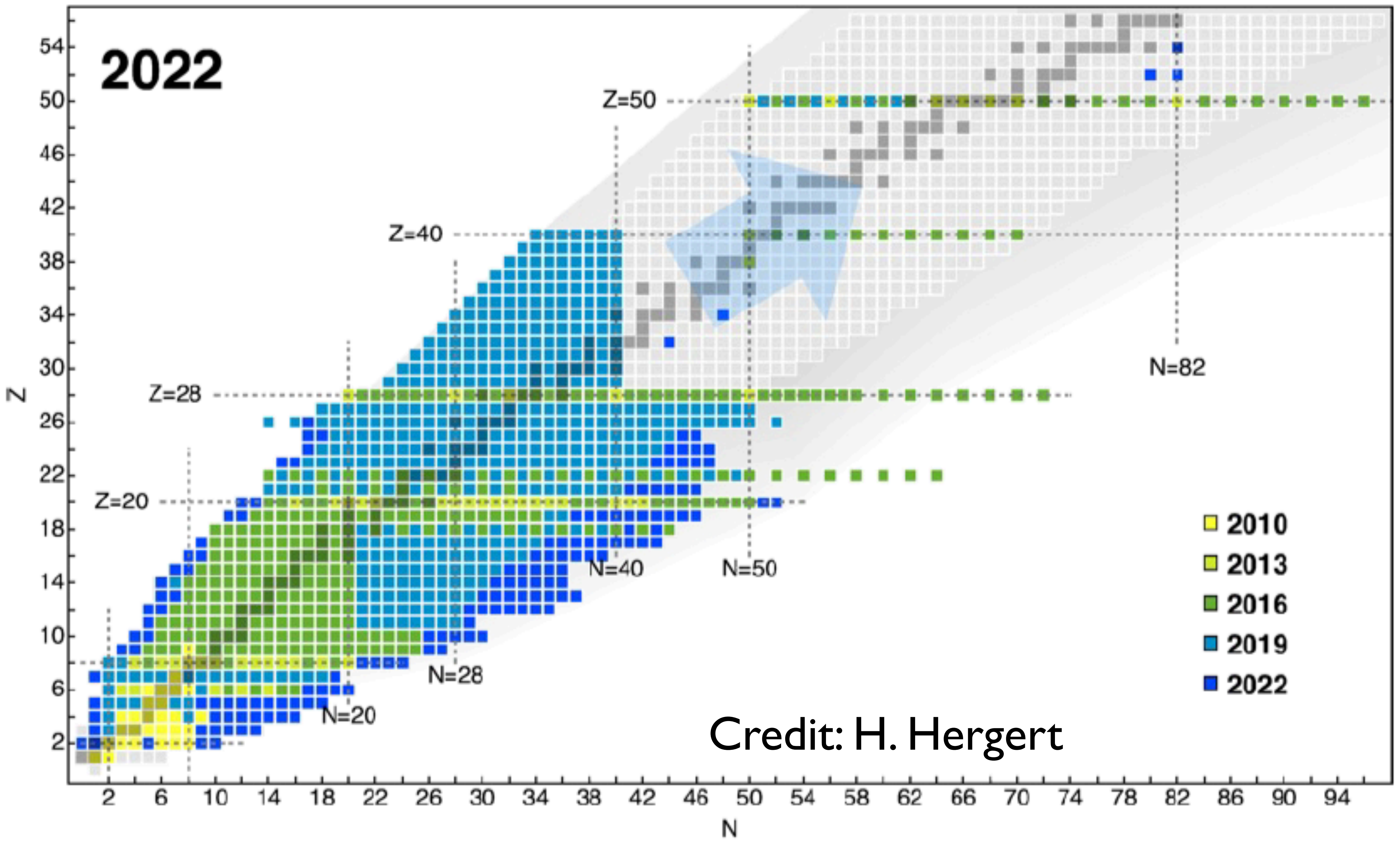
Phys. Rev. C **96**, 054004 (2017)
Phys. Rev. C **104**, 034311 (2021)
Phys. Rev. C **106**, 024324 (2022)

Scott Bogner
Facility for Rare Isotope Beams
Michigan State University

Low and High RG Resolution Scale Pictures

Drischler & Bogner, Few Body Syst. **62**, 109





Explosive progress in ab-initio structure largely due to low resolution Hamiltonians
(See talks of Dean Lee, Jason Holt, Petr Navratil, Heiko Hergert, etc.)

What about operators that couple to external probes (EW currents, optical potentials, etc.) ?

Low and High RG Resolution Scale Pictures

(RG) Resolution Scale $H = H(\Lambda)$ ➡ max. momenta in low-energy wf's ~ Λ

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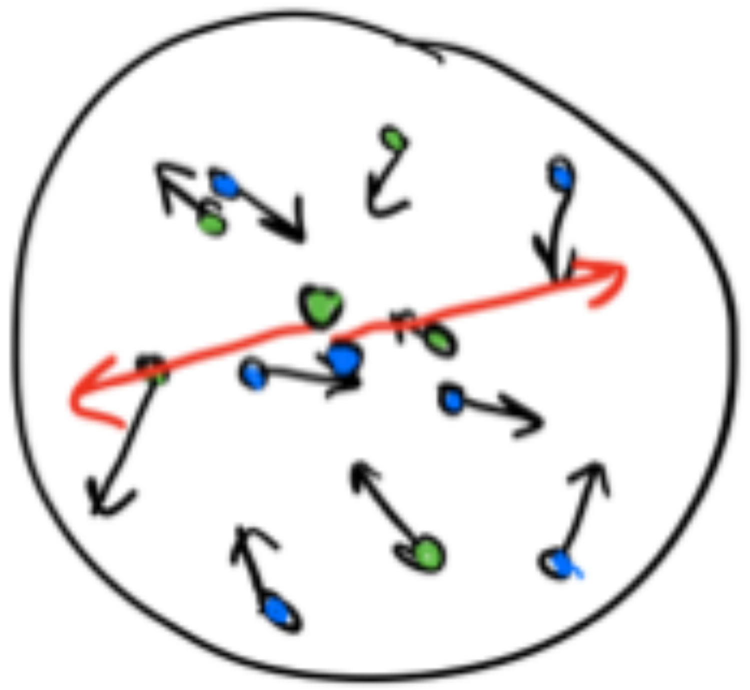
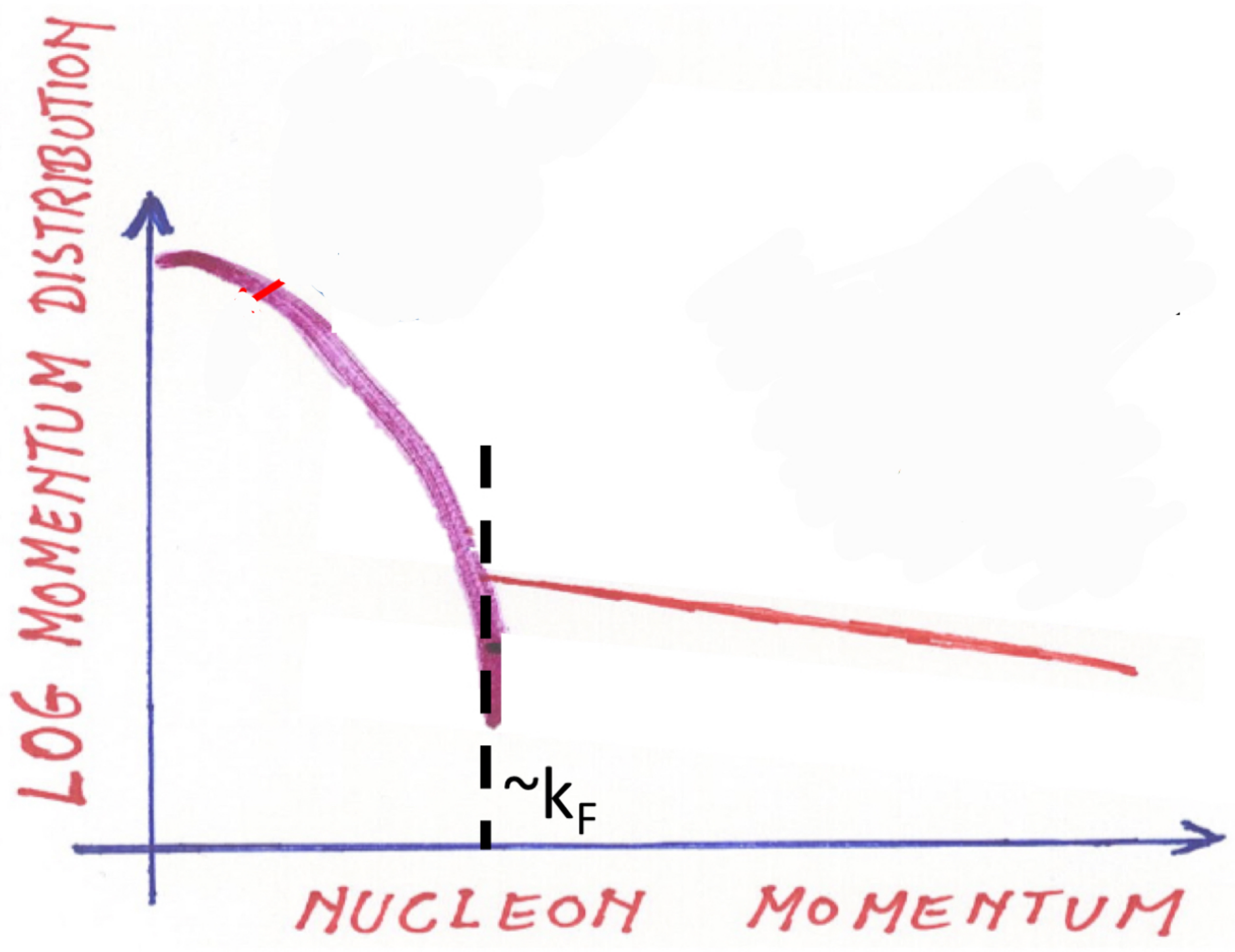
High resolution picture:





correlated SRC pairs

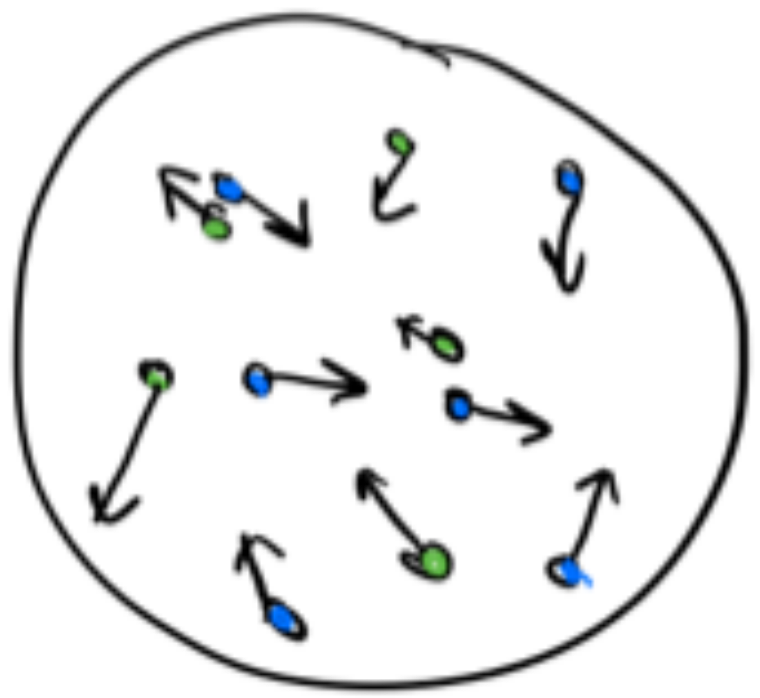
Hard, local interactions
AV18 etc.

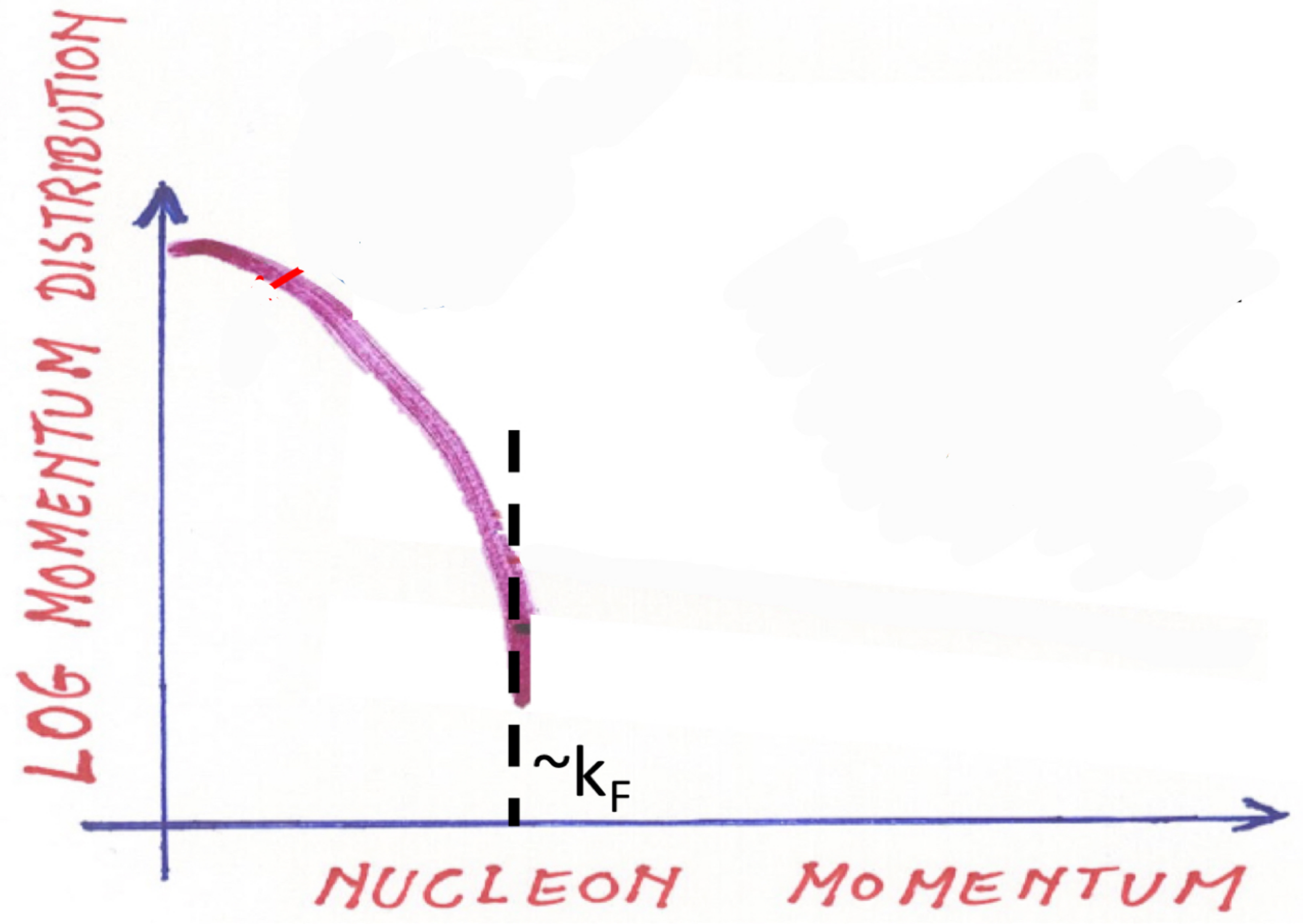
high-k tails ($k \gg k_F$) present

Low and High RG Resolution Scale Pictures

(RG) Resolution Scale $H = H(\Lambda)$ ➡ max. momenta in low-energy wf's ~ Λ

Low resolution picture:





resembles "mean field" picture

no high-k tails ($k >> k_F$)

chiral EFT/soft interactions

shell model

DFT

Low and High RG Resolution Scale Pictures

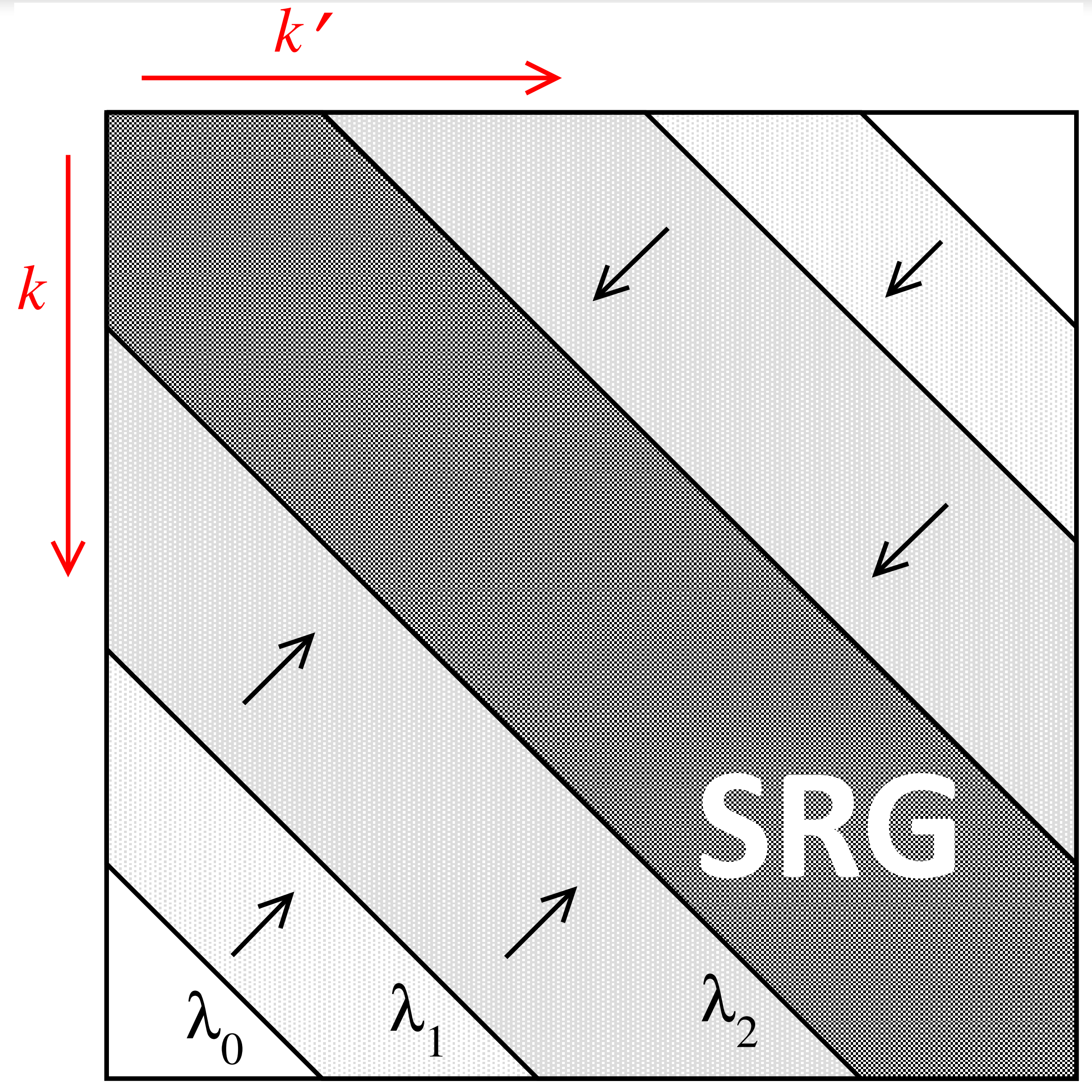
Theories at different resolutions connected by RG evolution

Unitary RG ("**S**imilarity **R**enormalization **G**roup")

$H(\lambda) = U(\lambda) H U^\dagger(\lambda) \qquad O(\lambda) = U(\lambda) O U^\dagger(\lambda)$

preserves all physics (unitary) if no approximations

low E states => $k \gtrsim \lambda$ highly suppressed/decoupled



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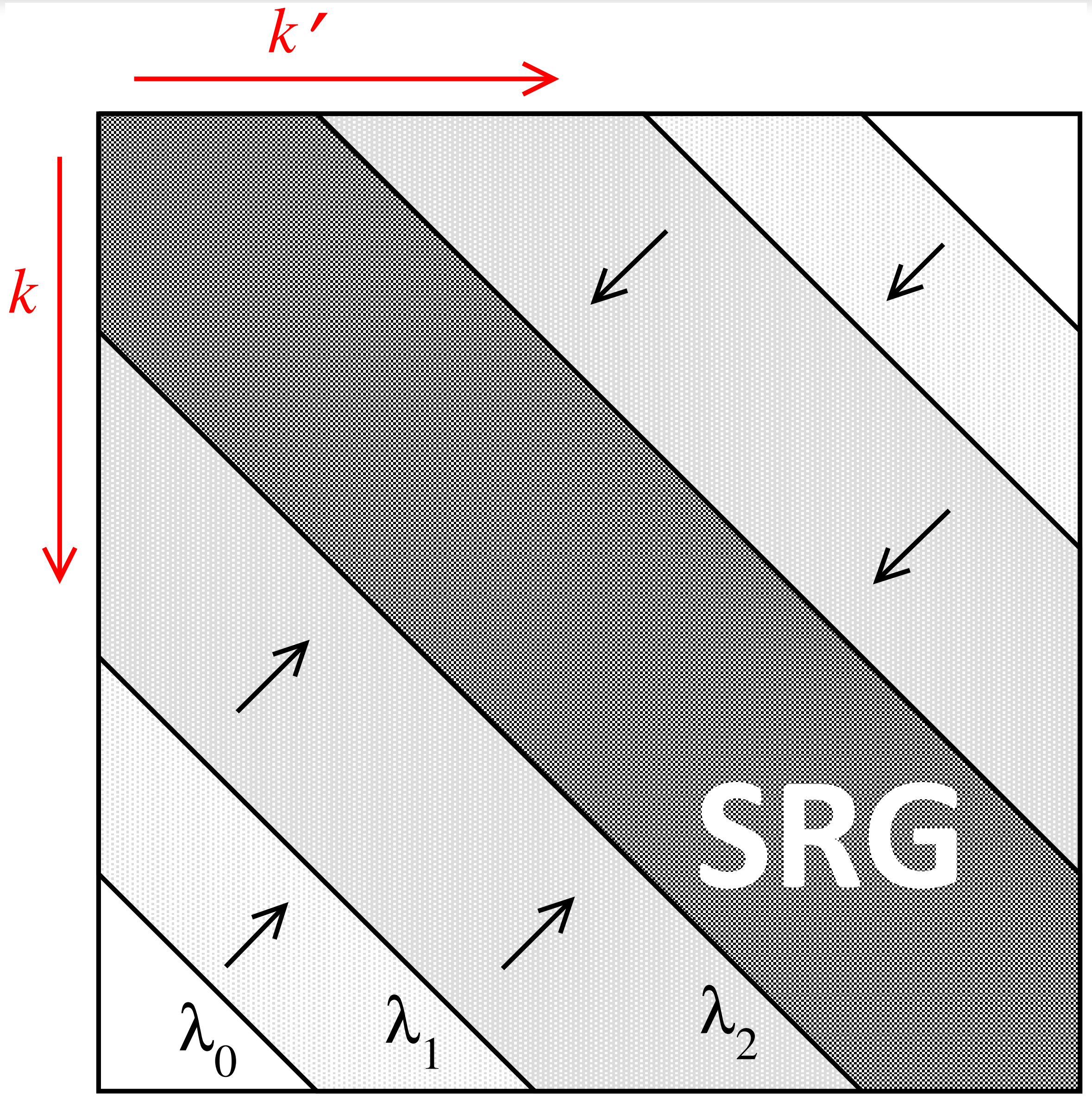
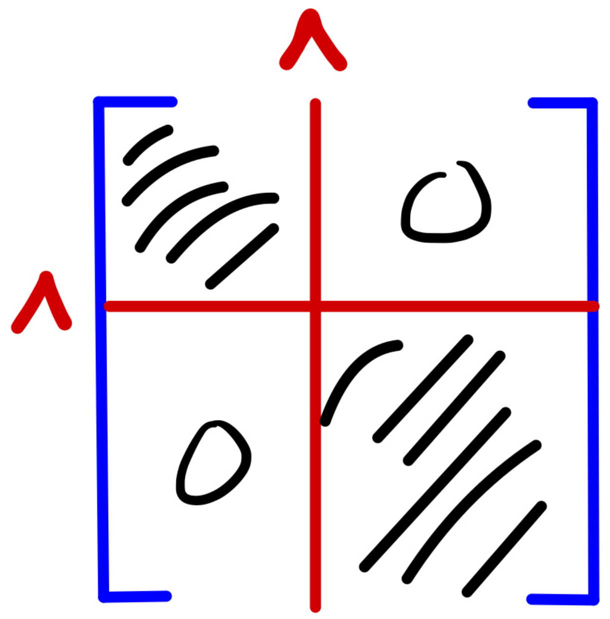


Decoupling pattern of high- and low-k physics not unique (different "schemes")

e.g.



Bogner, Furnstahl, Schwenk Prog. Part. Nucl. Phys. 2010

Scale and scheme dependence of structure/reactions

Can we use low-RG scale pictures to directly compute cross sections, etc?

$$\underbrace{\langle\psi_f|}_{\text{structure}}\overbrace{\widehat{O}(q)}^{\text{reaction}}\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger\widehat{O}(q)U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

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General questions:

scale/scheme dependence of extracted properties? (e.g., SFs)

extract at one scale, evolve to another? (a-la Parton distribution functions)

how do FSIs, physical interpretations, etc. depend on RG scale?

Scale and scheme dependence of structure/reactions

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Disclaimer

The examples I'll show concern SRC physics from high energy electron scattering and photoabsorption where one is probing the short-distance or high-momentum structure of nuclear wfs.

Extensions to nucleonic probes are in their infancy (e.g., behavior of optical potentials under RG evolution)

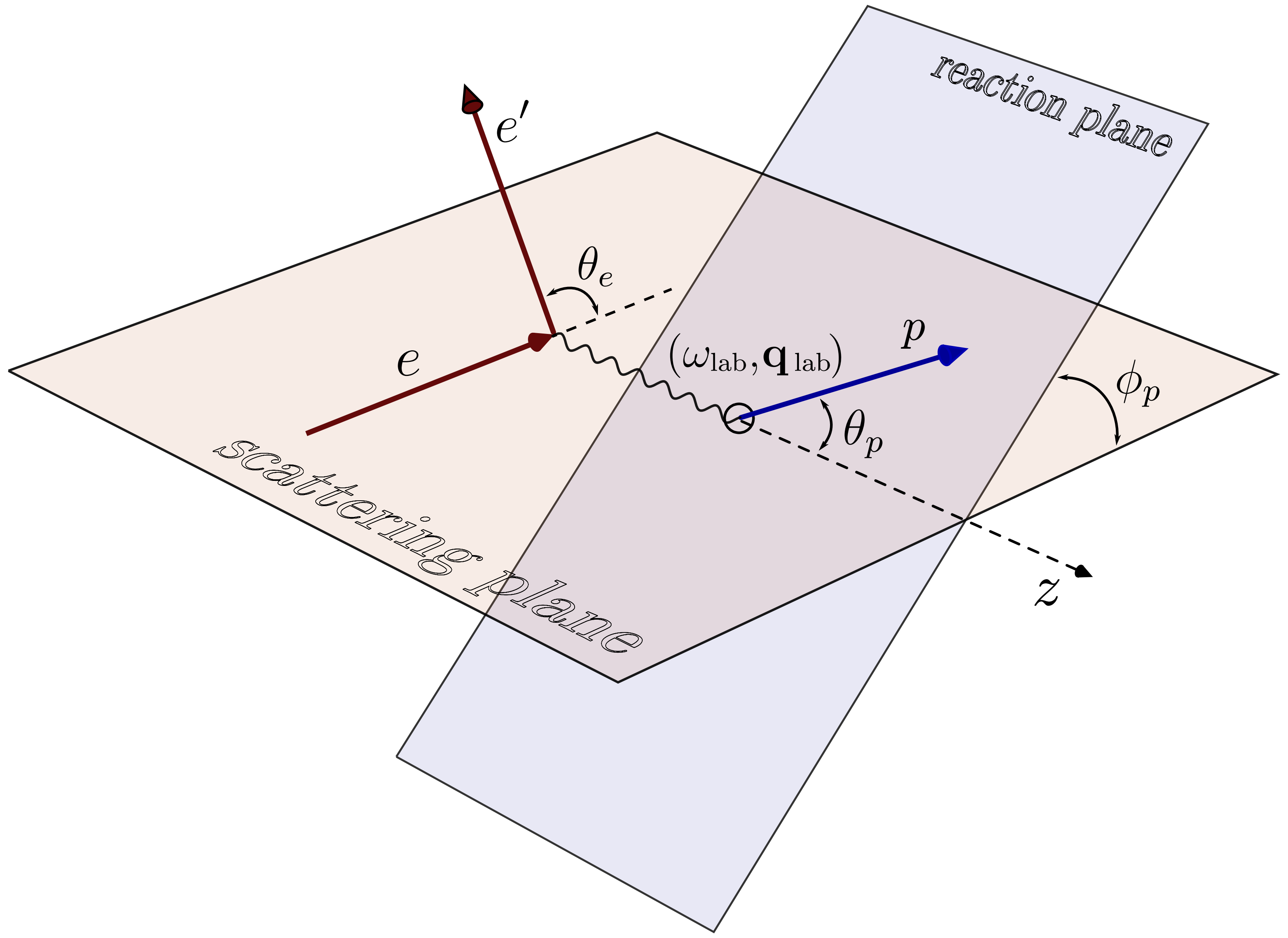
Still, matching structure and reactions to the same scale and scheme should be a desiderata for any consistent treatment of structure and reactions

Example 1: ^{2}H(e,e′p)n

- Simplest knockout process (no induced 3N forces/currents)
- Focus on longitudinal structure function f_L in SRC kinematics

$$f_L \sim \sum_{m_s, m_J} \left|\langle \psi_f | J_0 | \psi_i \rangle\right|^2$$

- $$f_L^\lambda \sim \left|\underbrace{\langle \psi_f | U_\lambda^\dagger}_{\psi_f^\lambda} \underbrace{U_\lambda J_0 U_\lambda^\dagger}_{J_0^\lambda} \underbrace{U_\lambda | \psi_i \rangle}_{\psi_i^\lambda}\right|^2; \quad U_\lambda^\dagger U_\lambda = I; \quad f_L^\lambda = f_L$$



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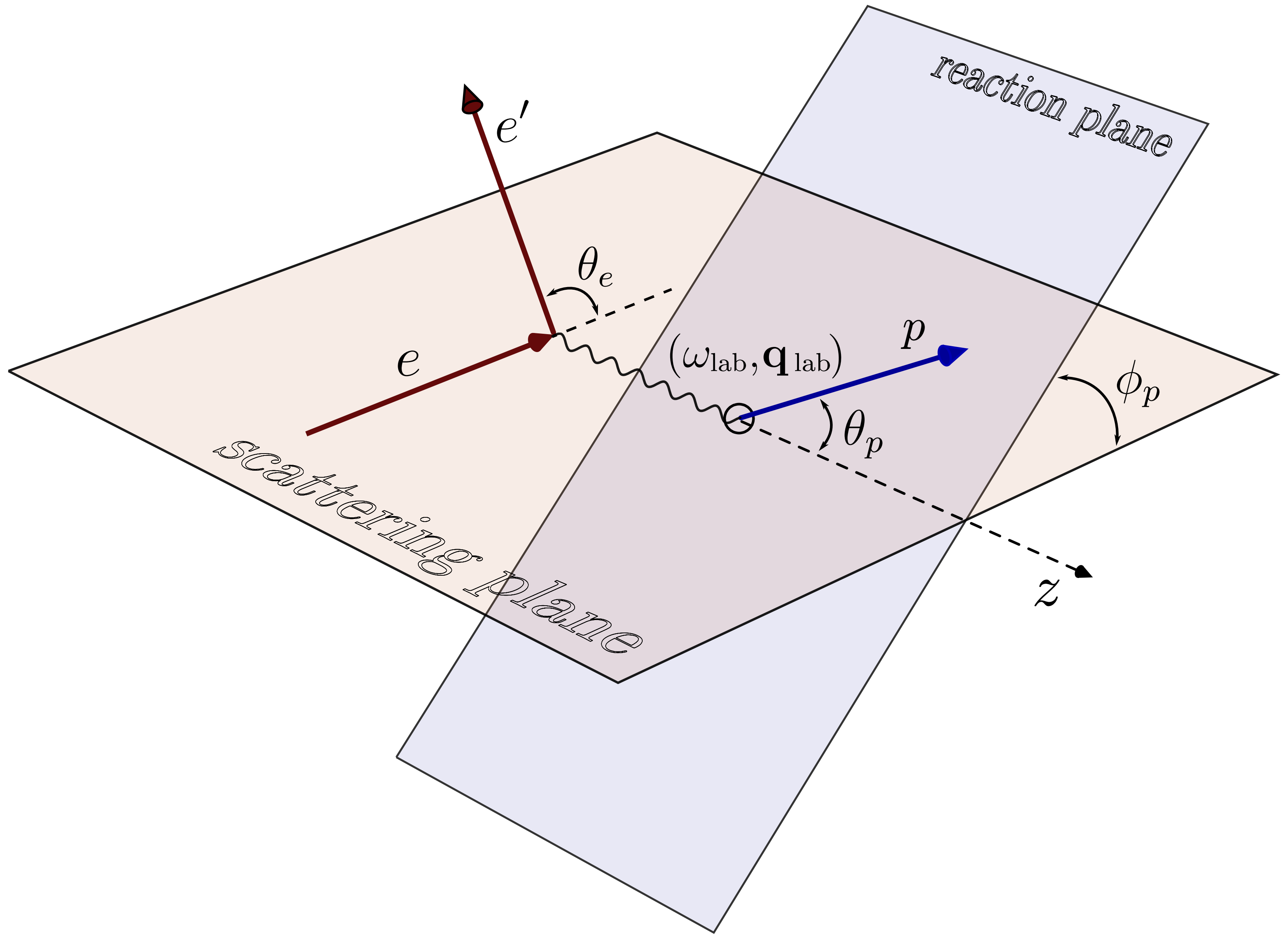

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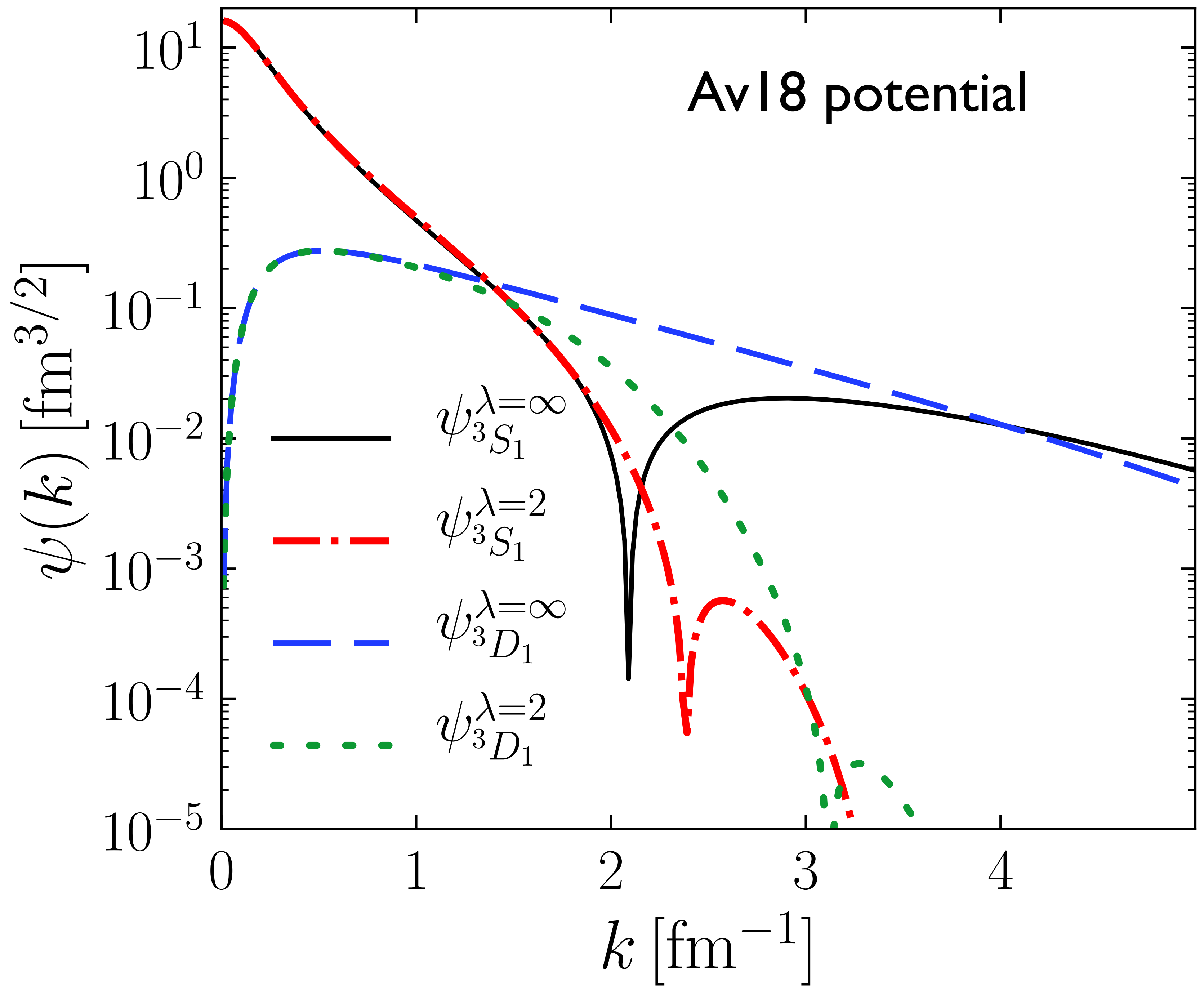
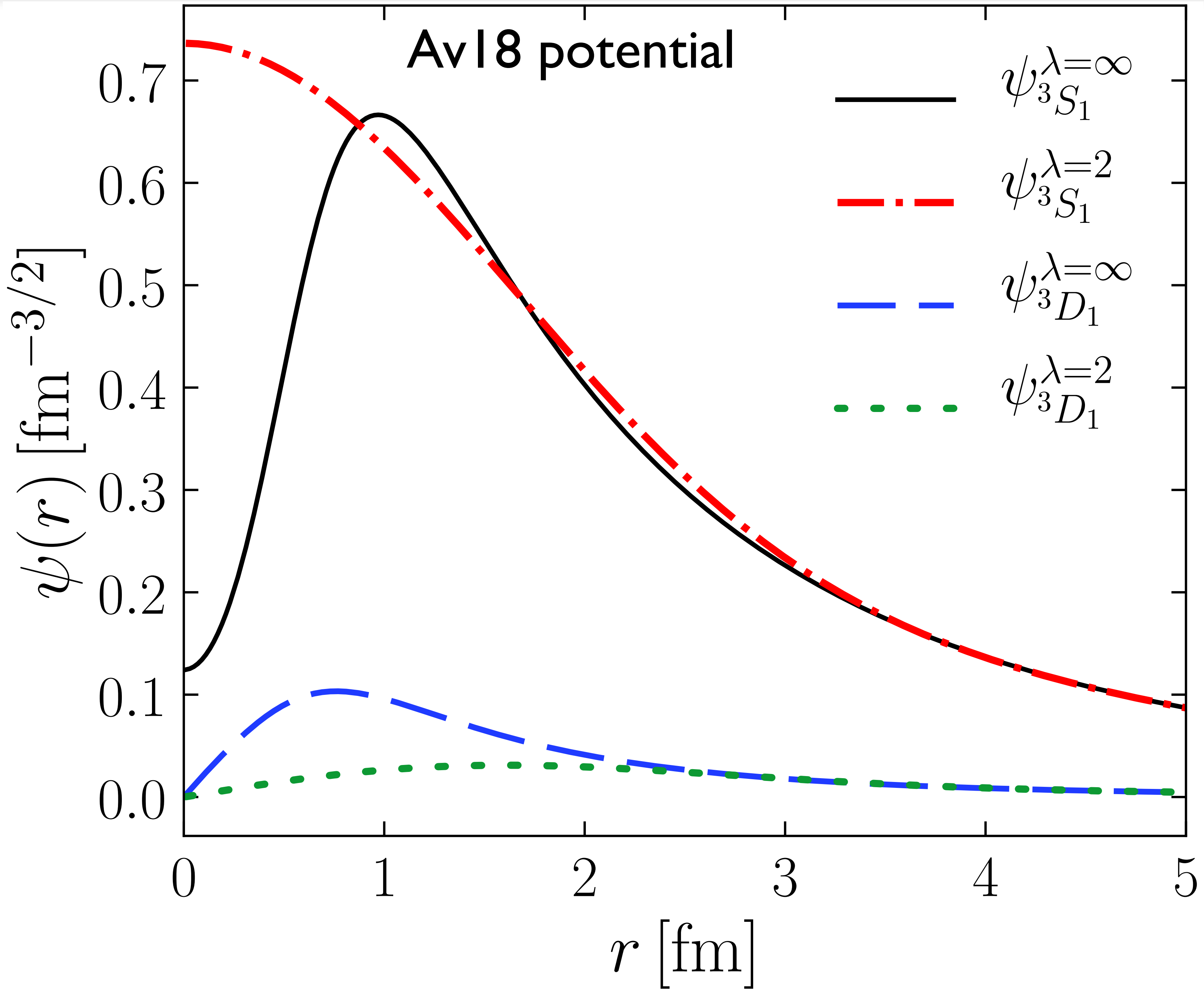
- Components (deuteron wf, transition operator, FSI) scale-dependent, total is not.
- How do the components evolve? How do physical interpretations change? Are some resolution scales "easier" to calculate with?

Deuteron wave function evolution

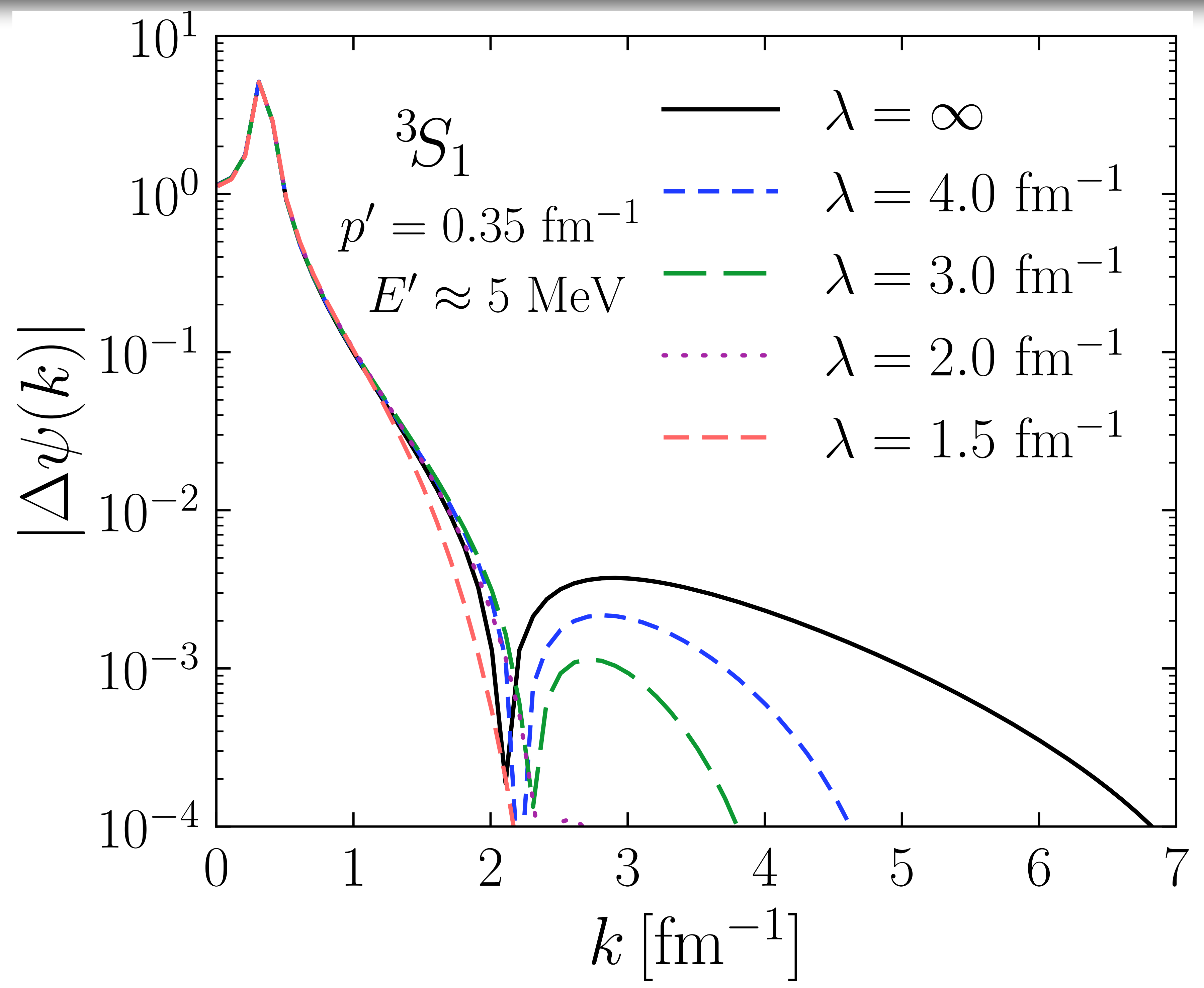


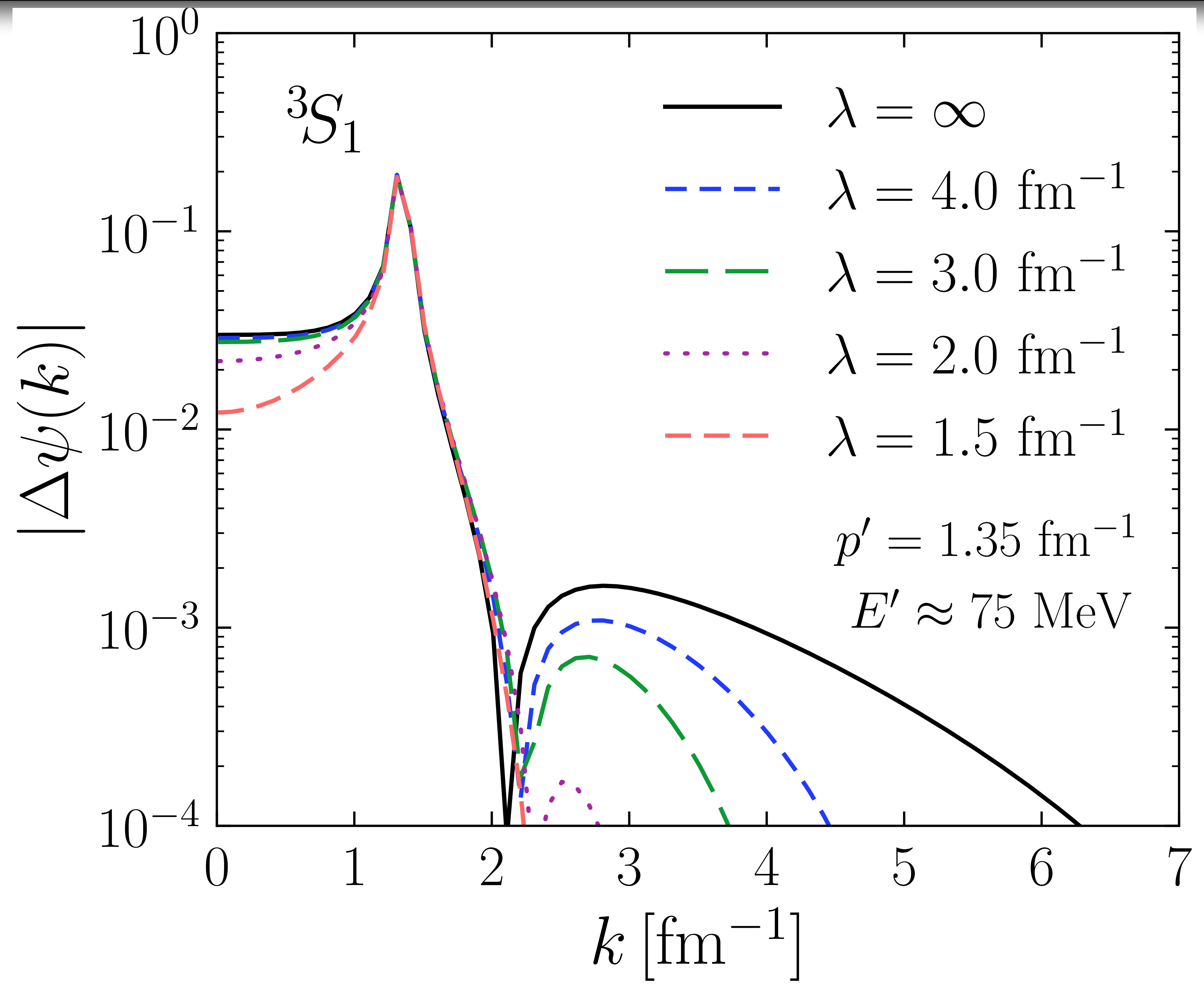


k < λ components invariant <==> RG preserves long-distance physics

k > λ components suppressed <==> short-range correlations blurred out

Final-state wave function evolution





$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

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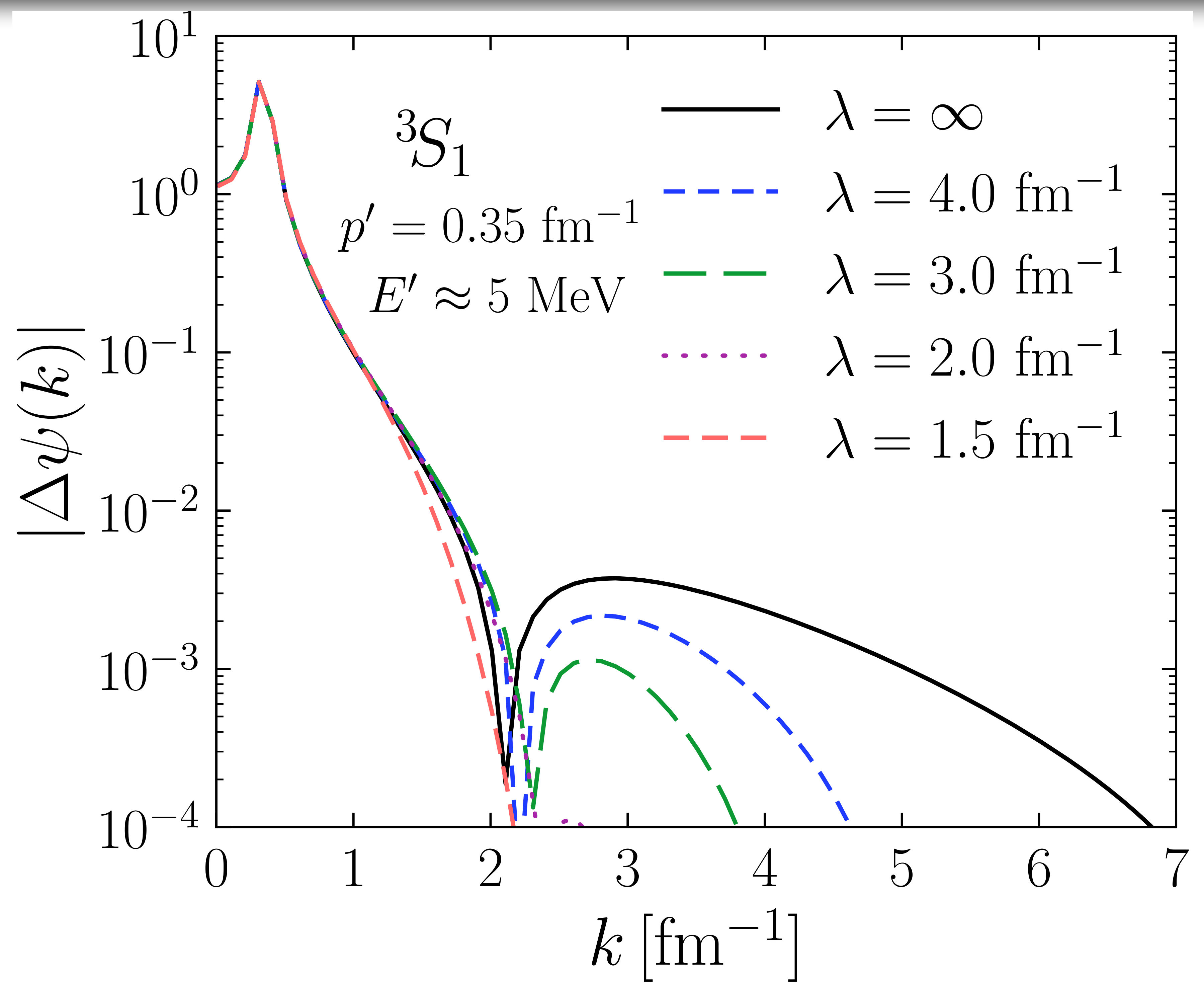
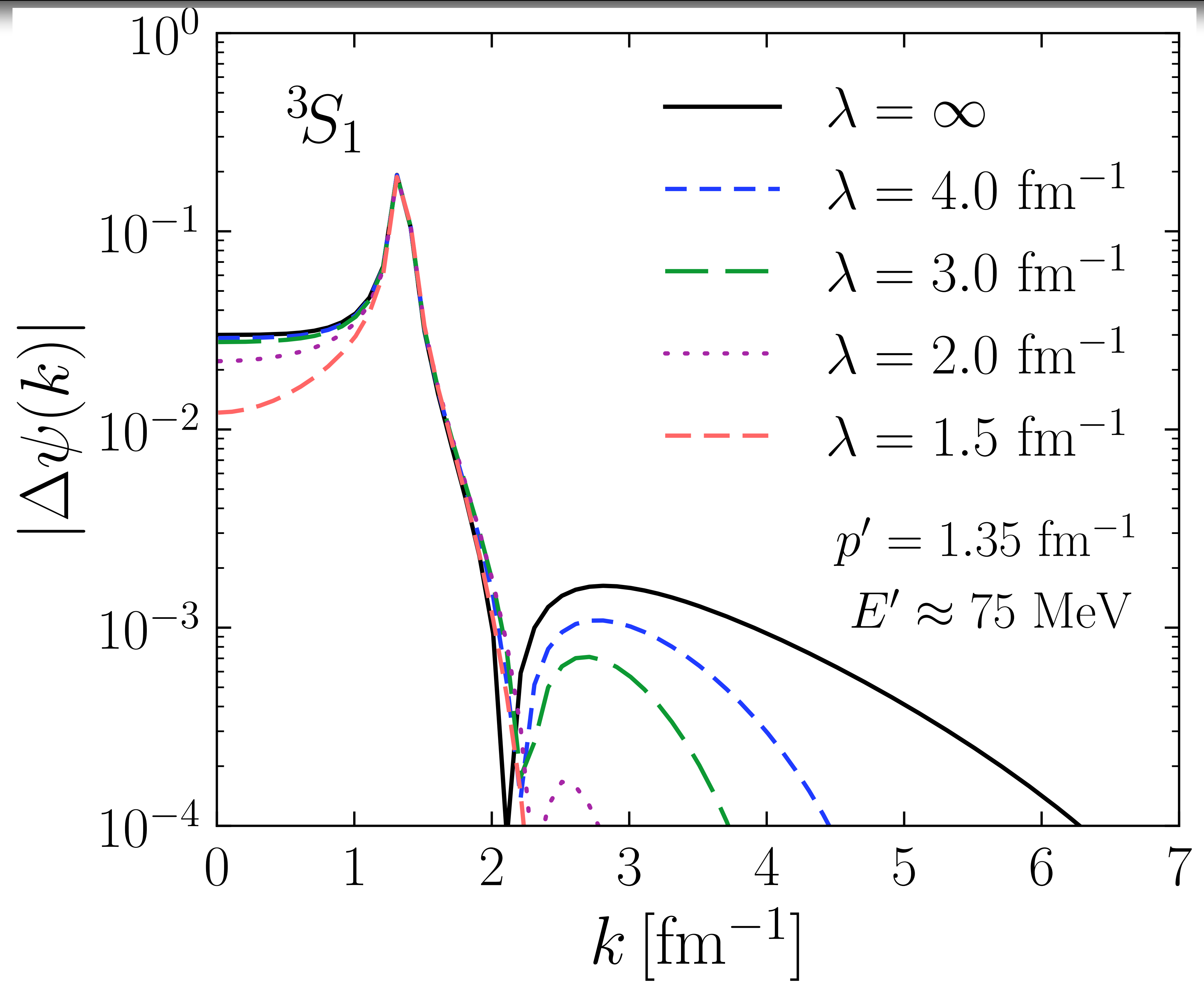


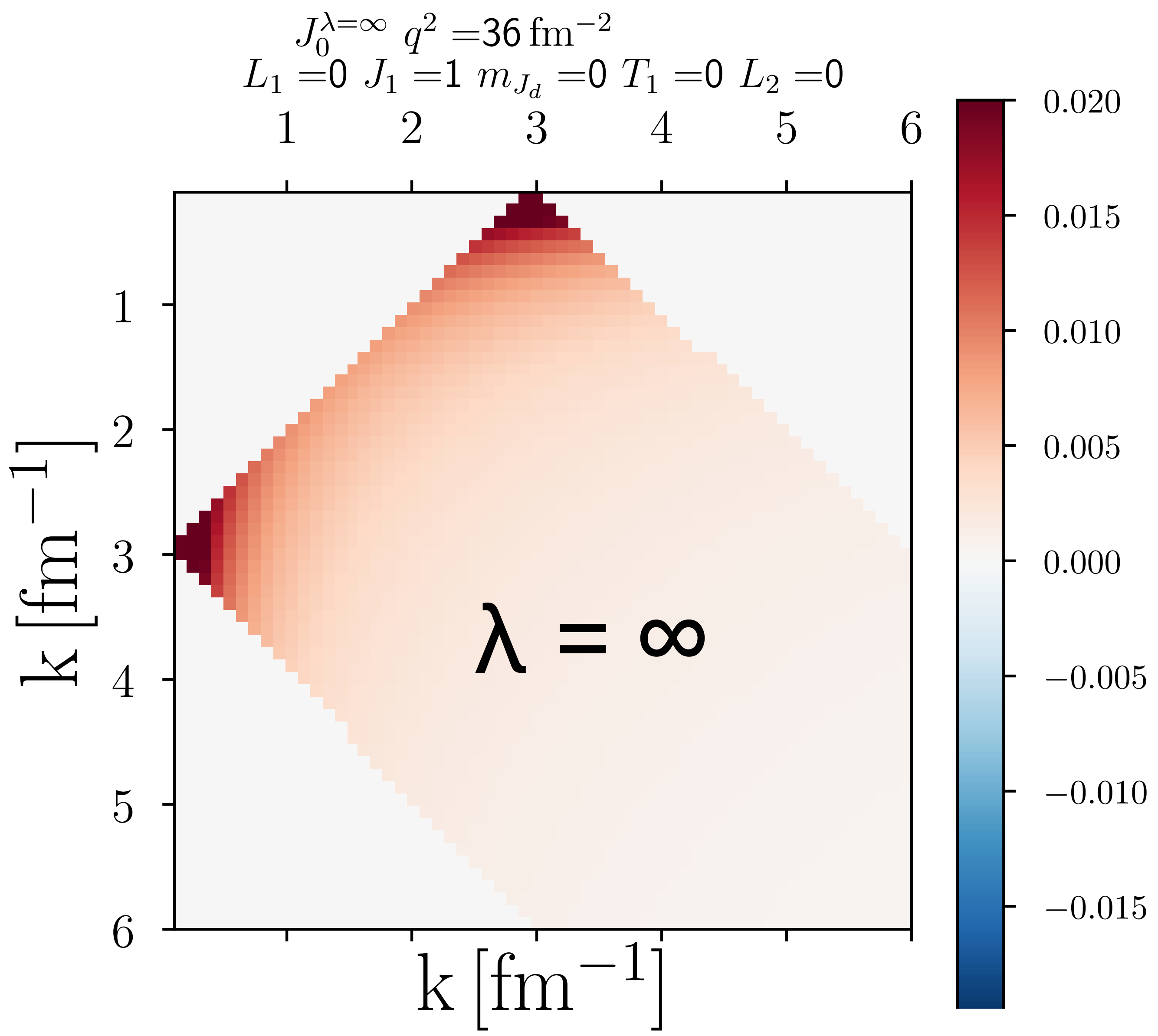


$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

- High-k tail suppressed with evolution
- For $p' \gtrsim \lambda,\ \Delta\psi_f^\lambda(p';k)$ localized around outgoing p′

Scheme-dependent "local decoupling" Dainton et al. PRC 89 (2014)

Current operator evolution



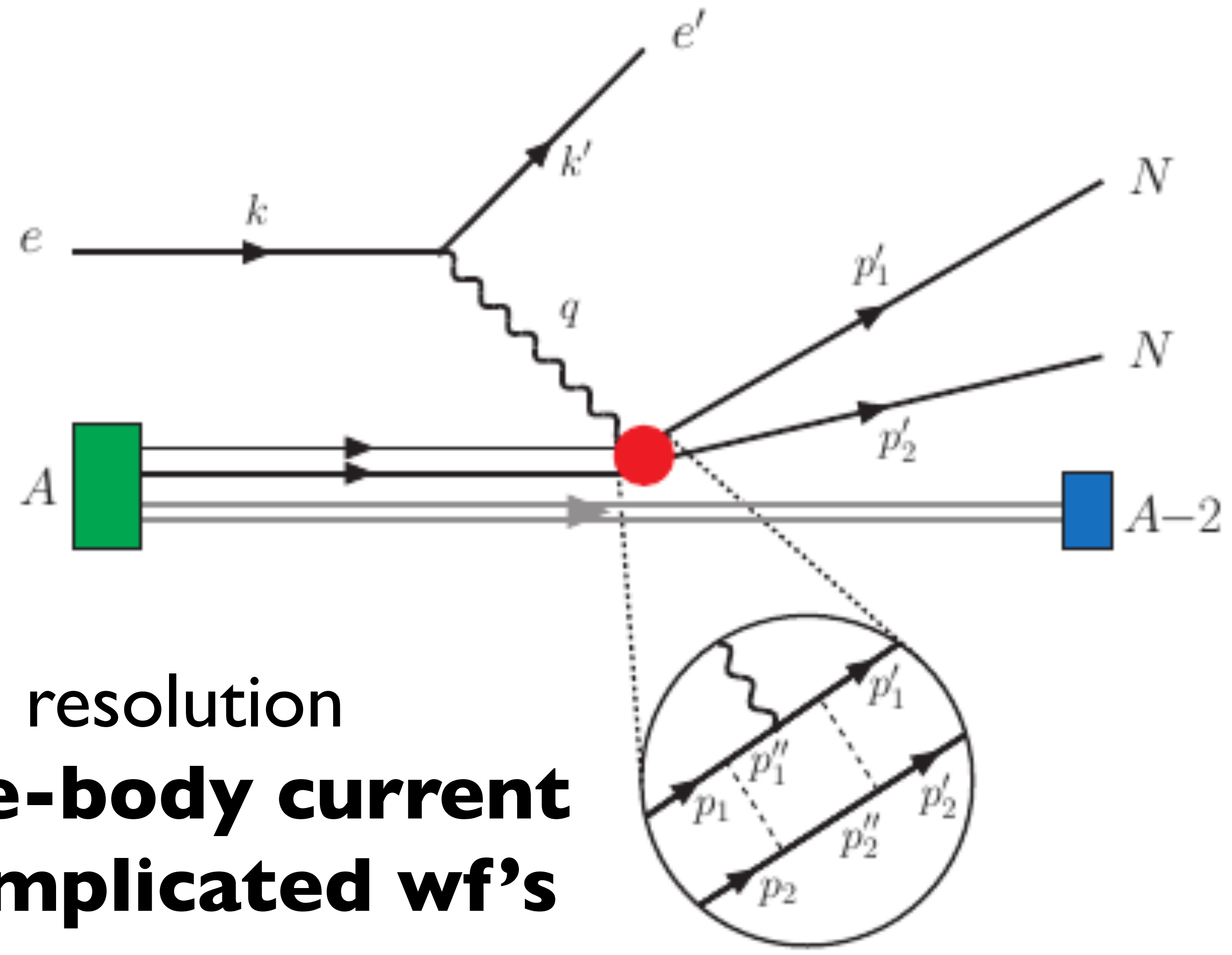
$^3S_1$ channel

$q^2 = 36\ \mathrm{fm}^{-2}$

high resolution

one-body current

Complicated wf's



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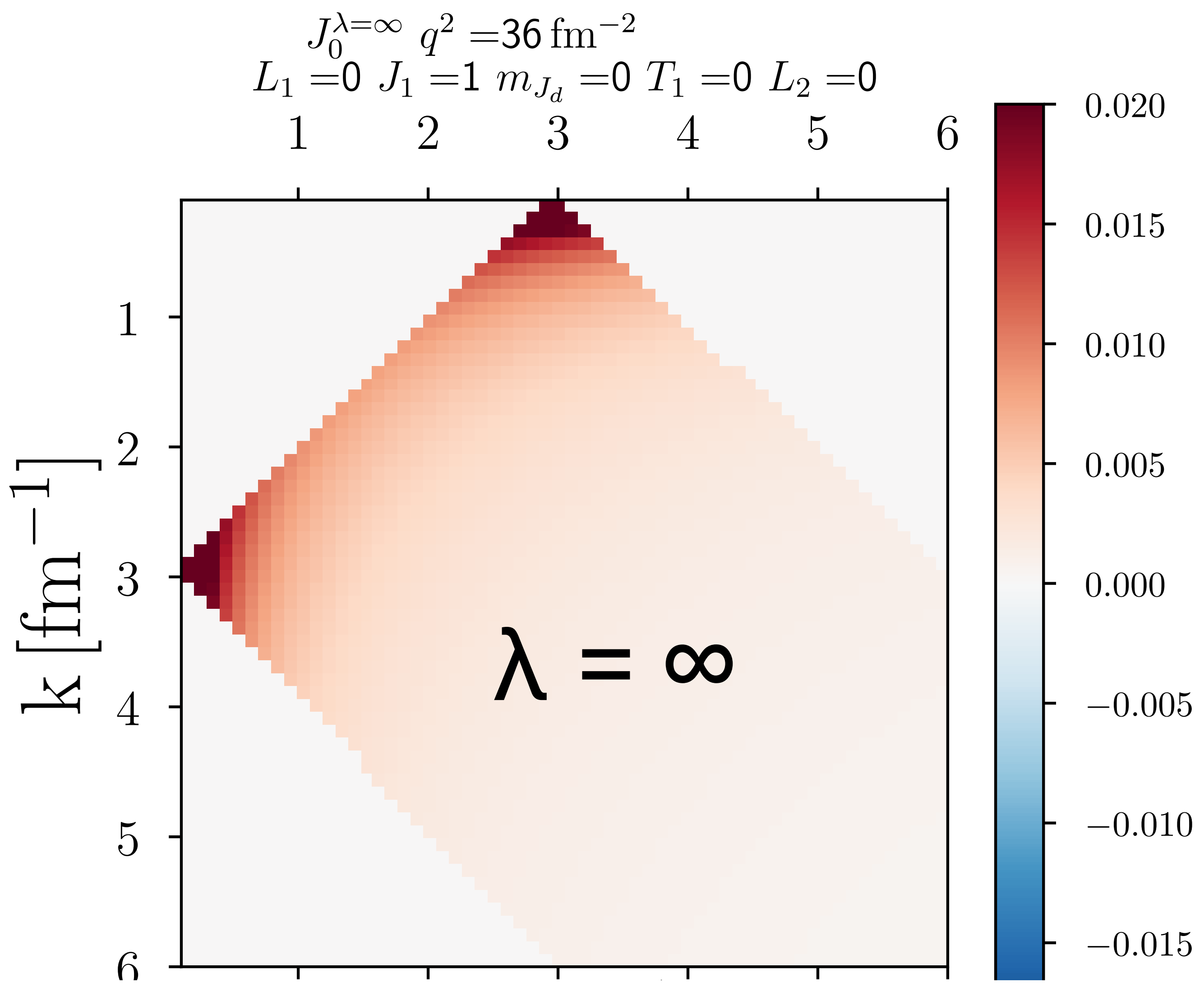

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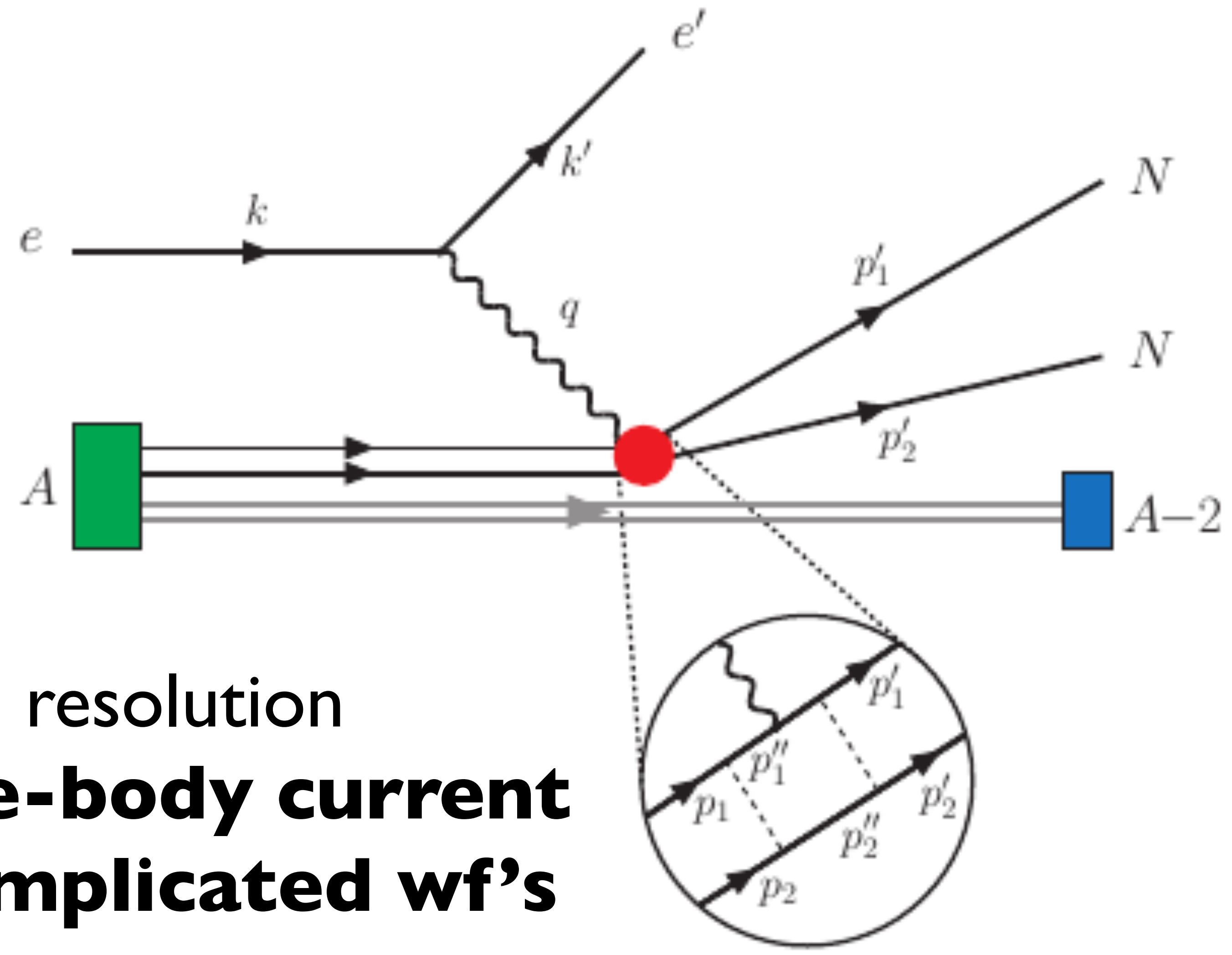
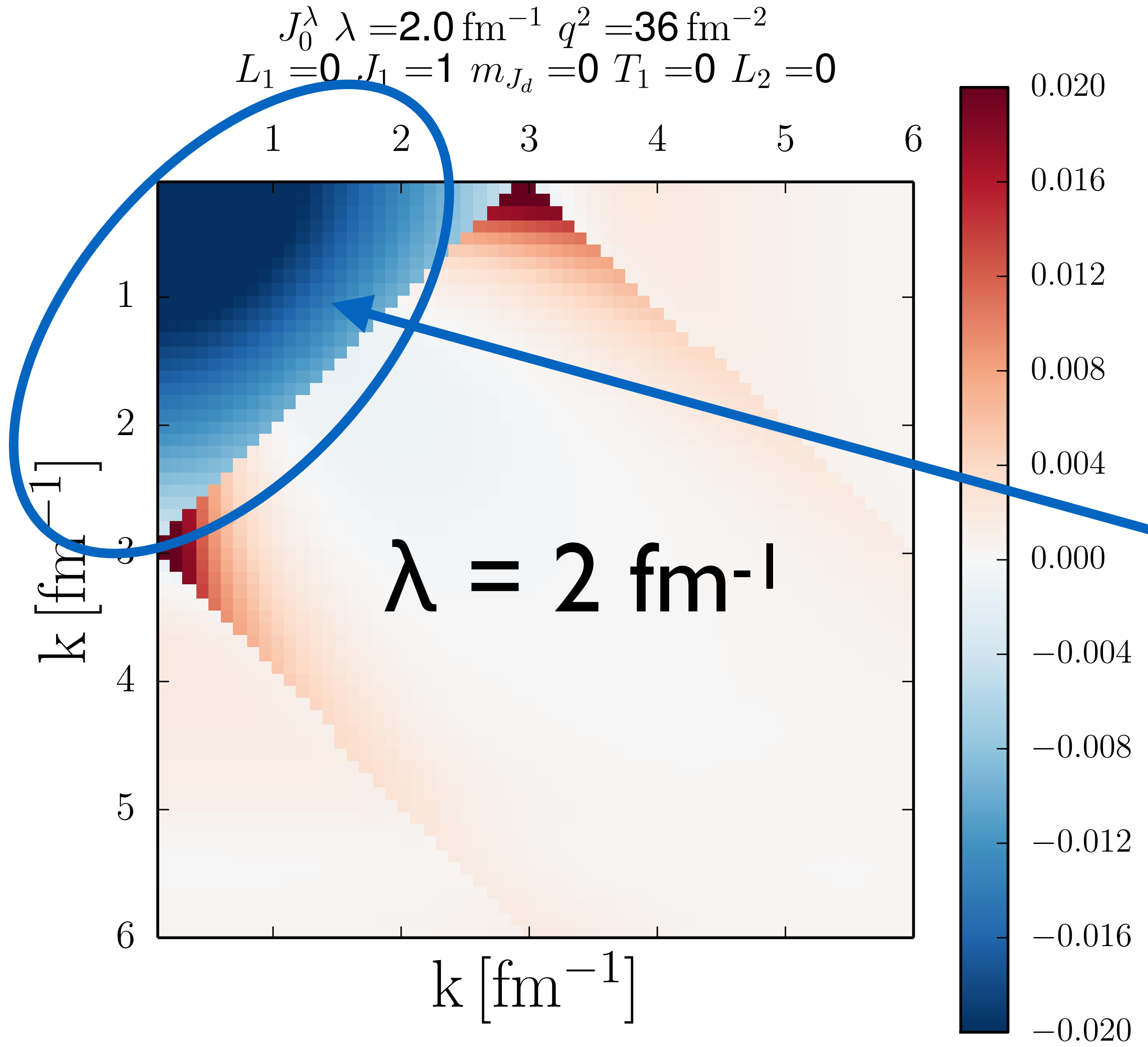
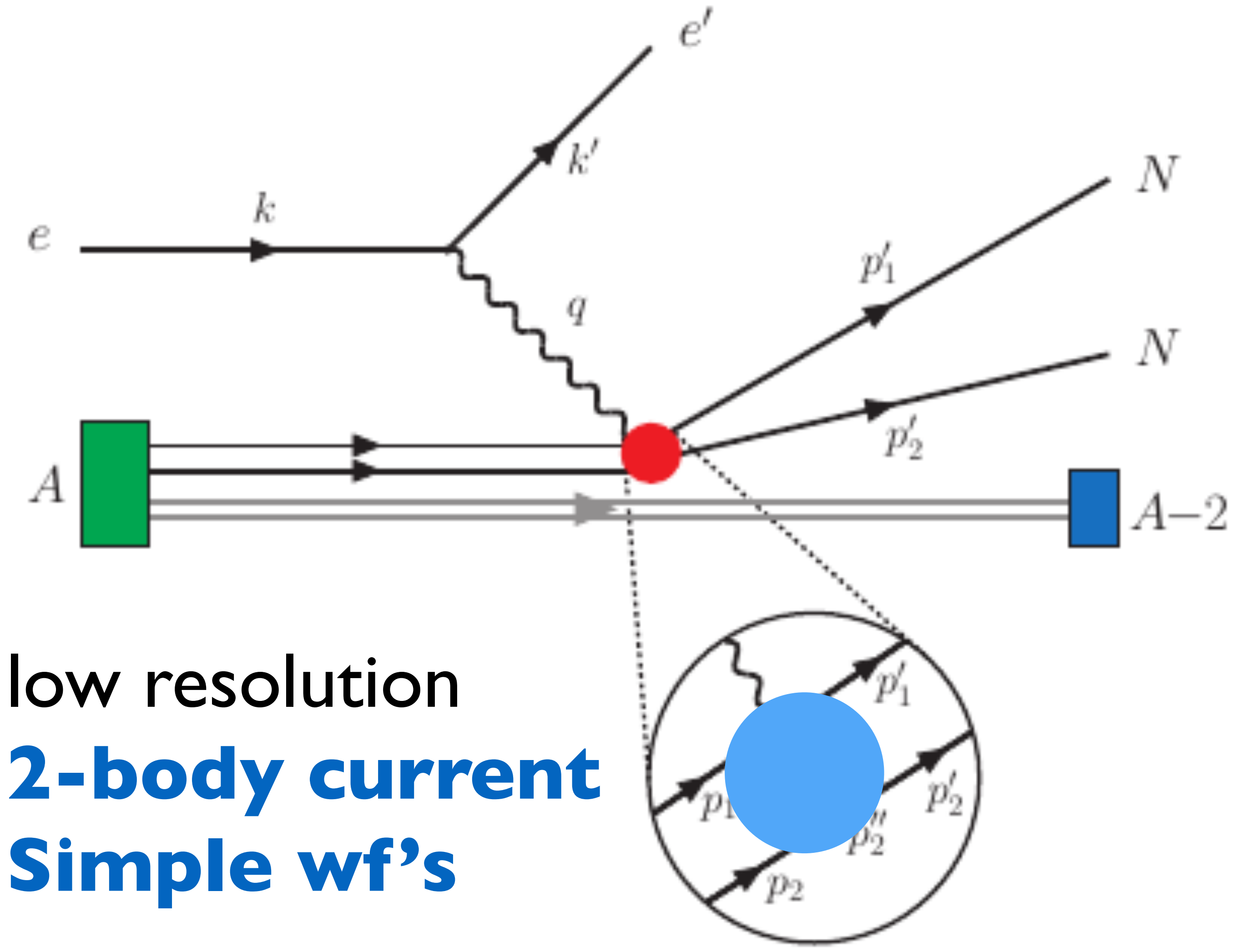


high resolution
one-body current
Complicated wf's



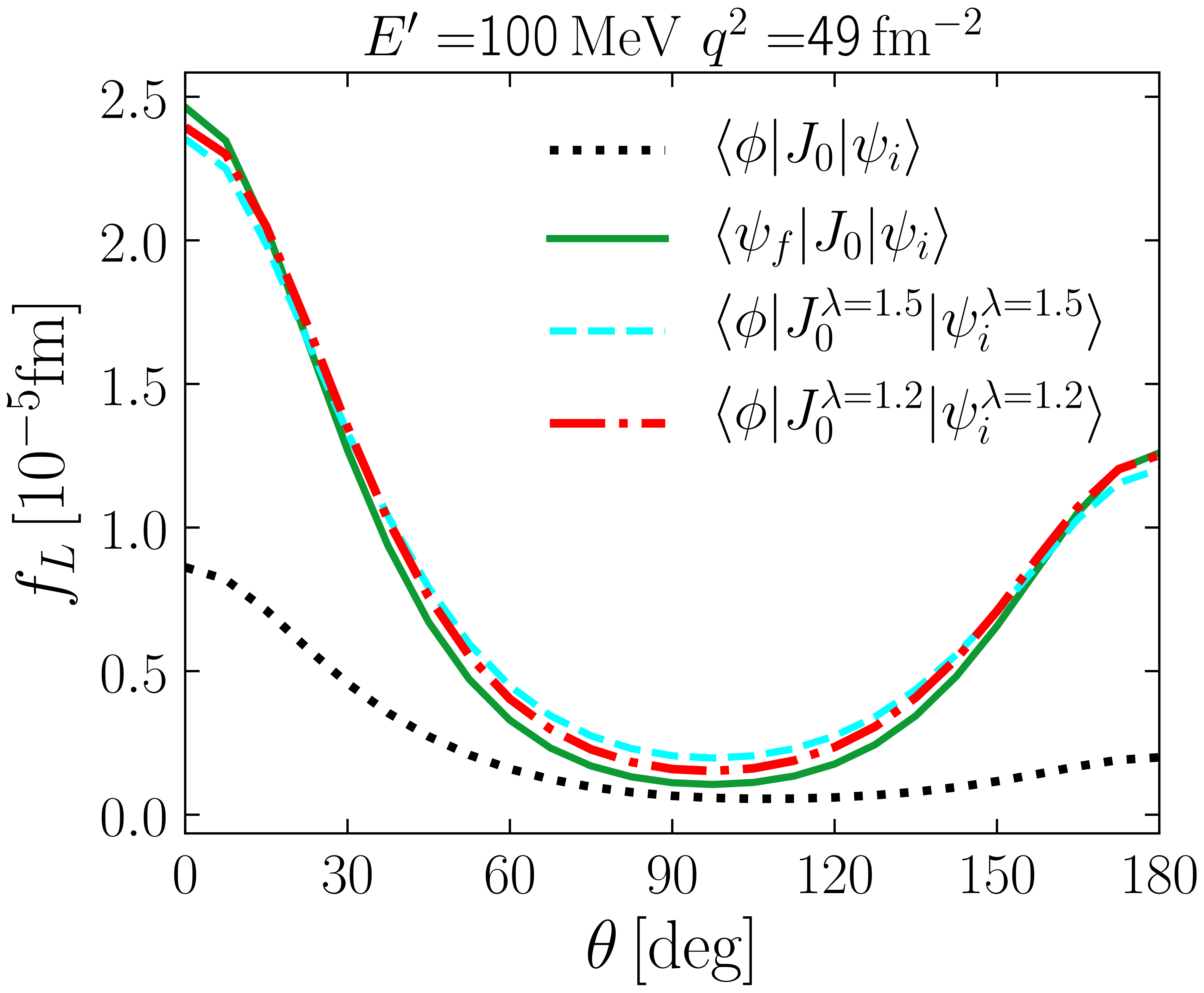
Smeared contact operator (smooth)



low resolution
2-body current
Simple wf's

Scale Dependence of Final State Interactions

Look at kinematics relevant to SRC studies



x_B=1.64, Q^2= 1.78 GeV^2

Scale Dependence of Final State Interactions

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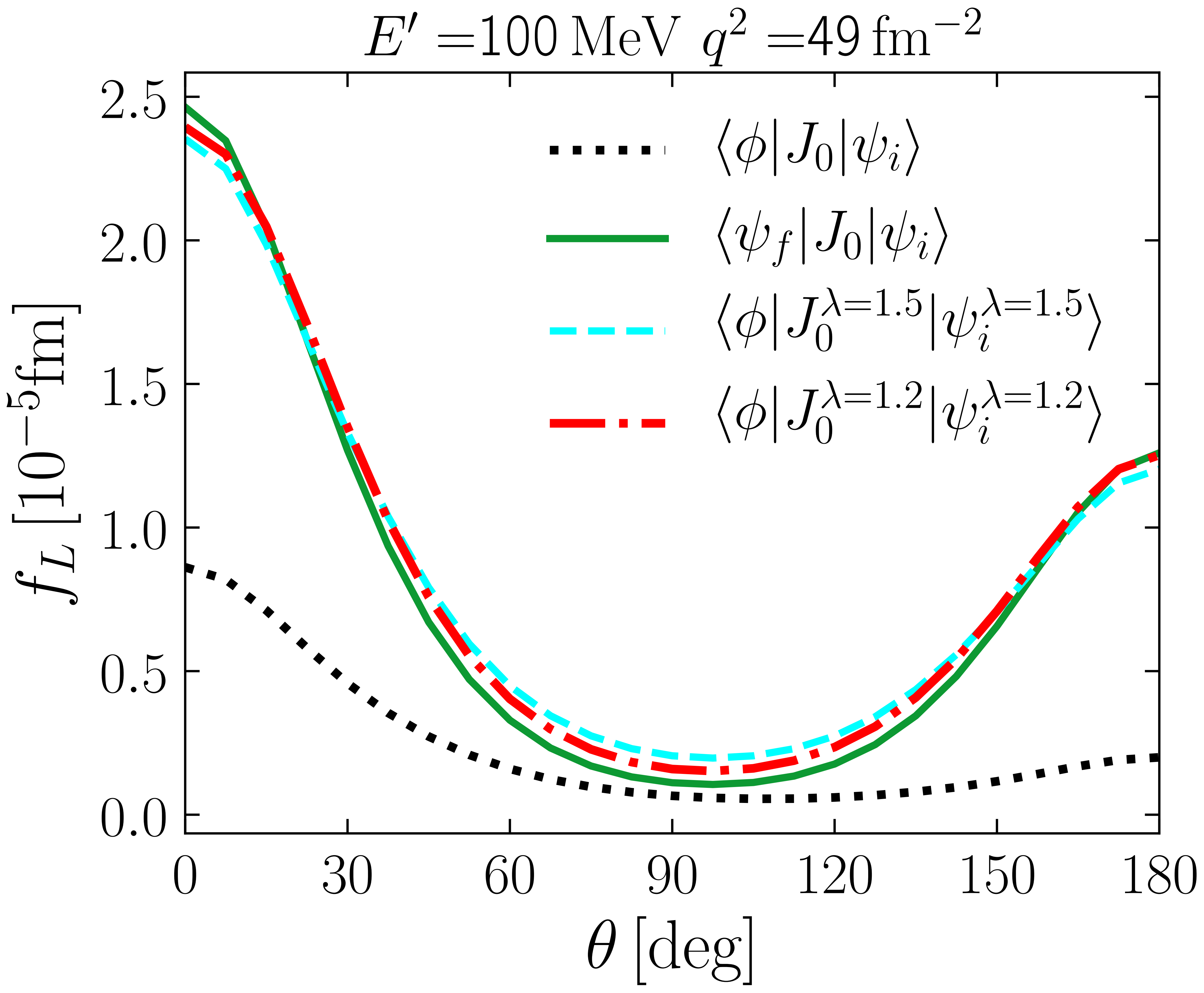
Look at kinematics relevant to SRC studies



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FSI sizable at large λ
but negligible at low-resolution!

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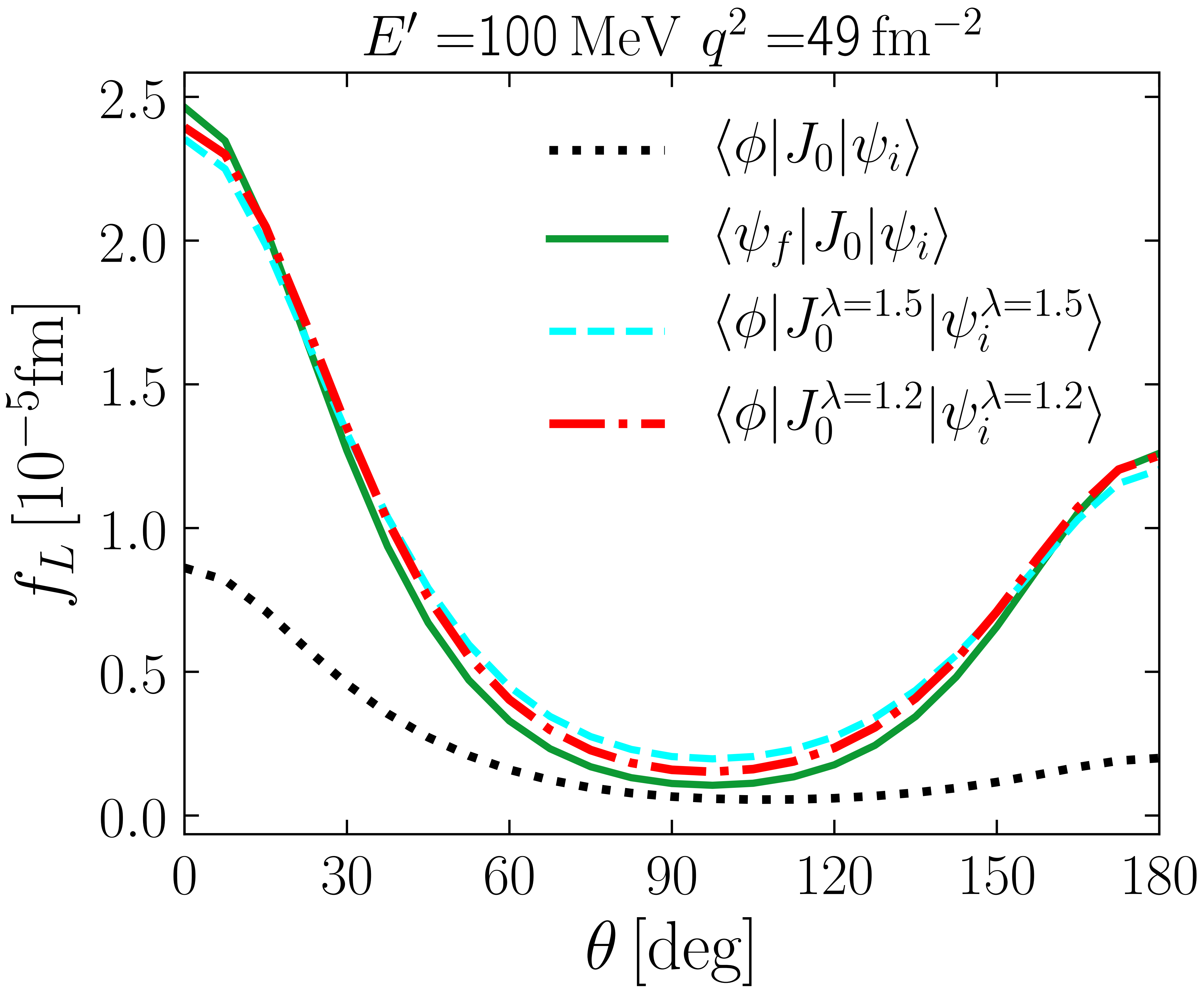


x_B=1.64, Q^2= 1.78 GeV^2

FSI sizable at large λ
but negligible at low-resolution!

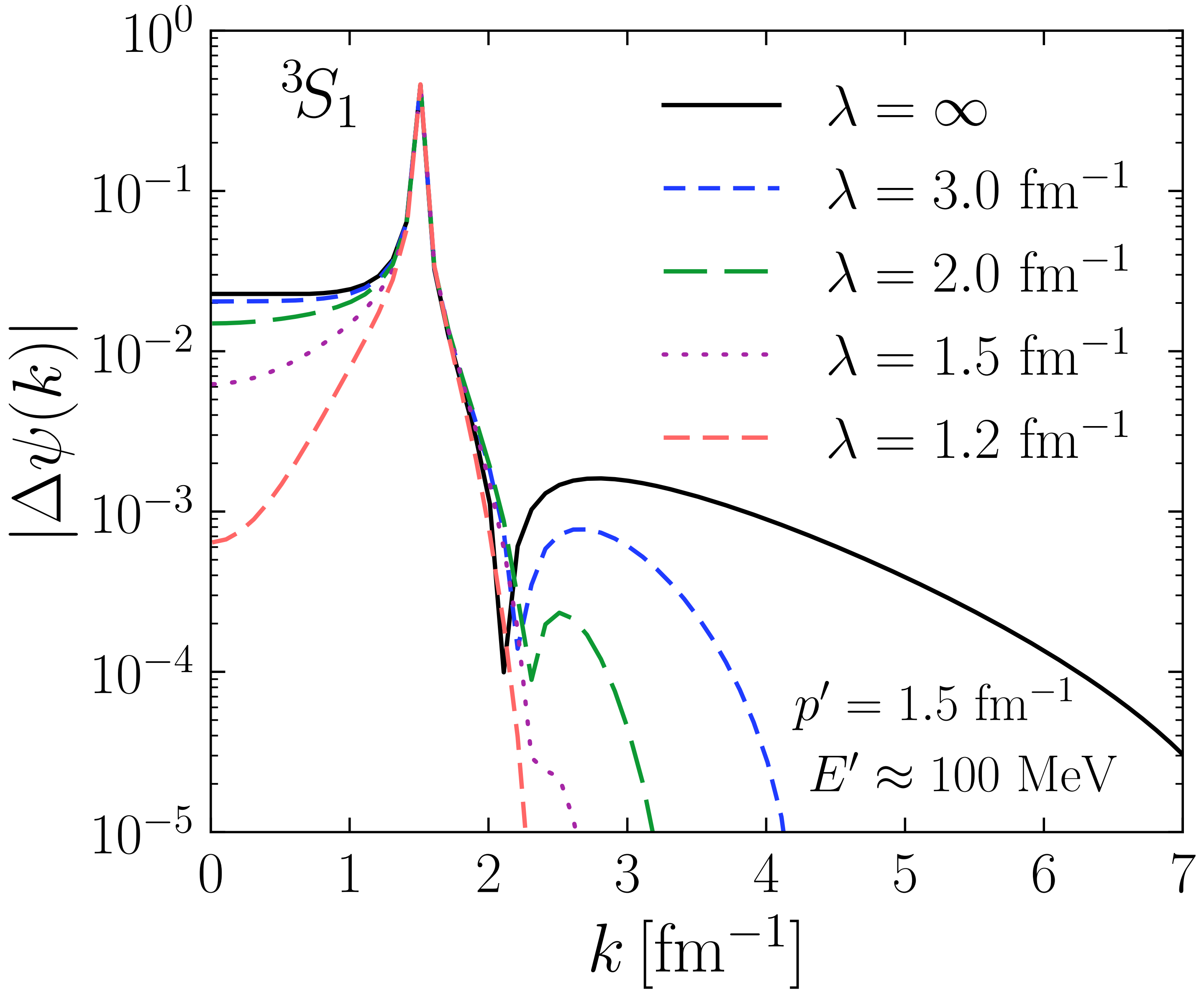
Folklore:

hard scattering processes complicated in low resolution (λ << q) pictures

Why are FSI so small at low λ in these kinematics ?

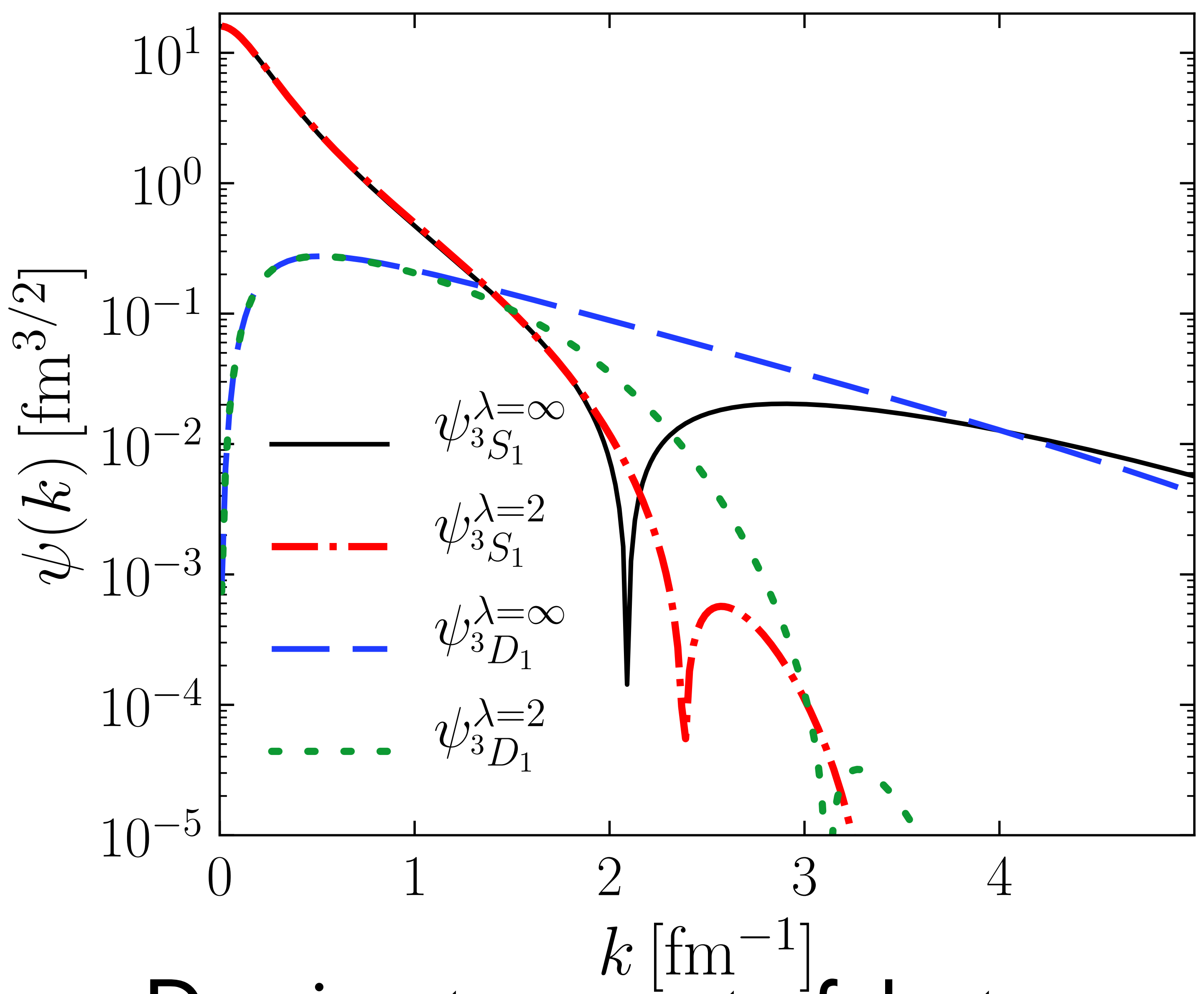
Scale Dependence of Final State Interactions

final state wf (FSI piece)



For p′ ≳ λ, interacting part of final state wf localized at k ≈ p′

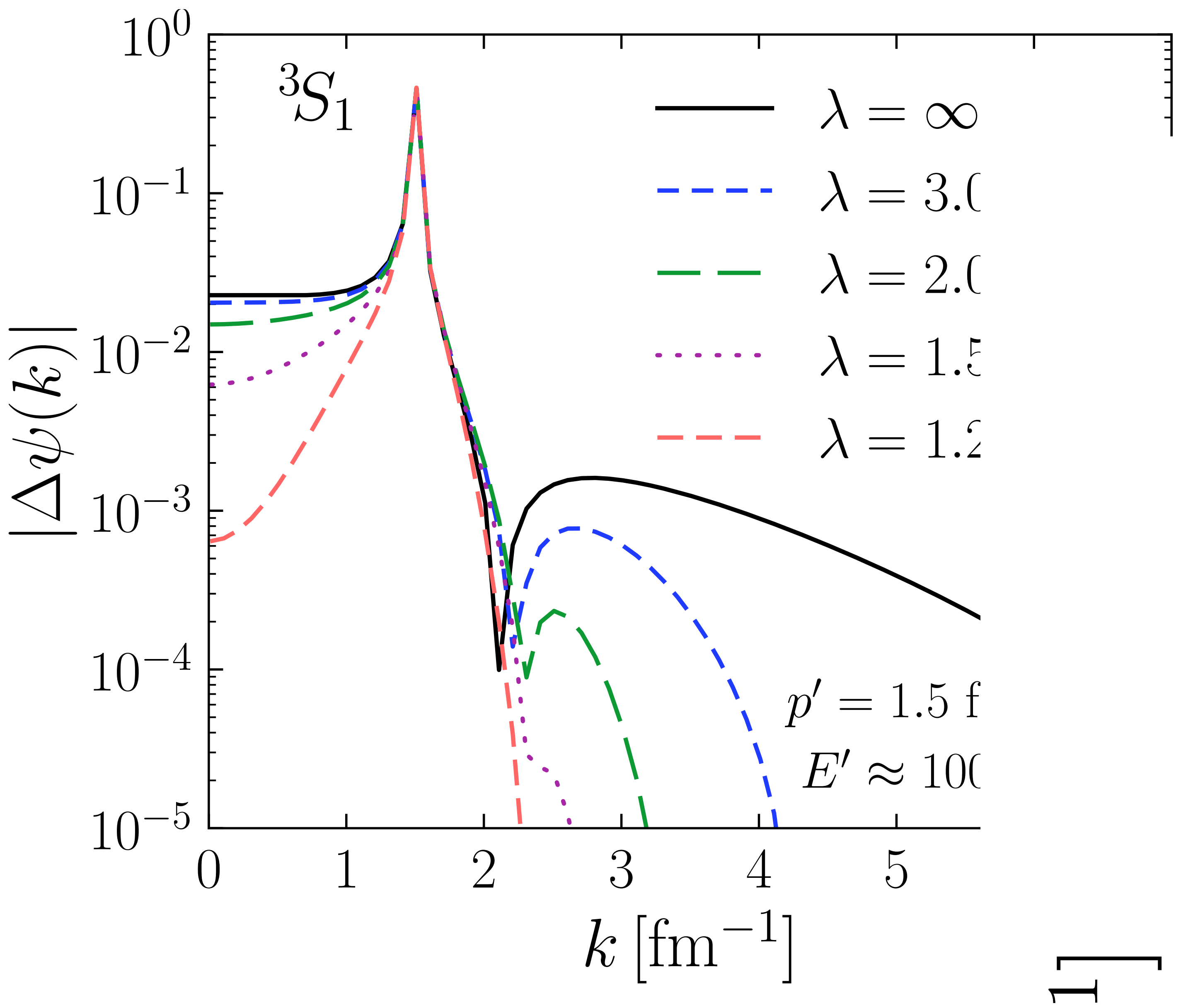
initial state (deuteron) wf

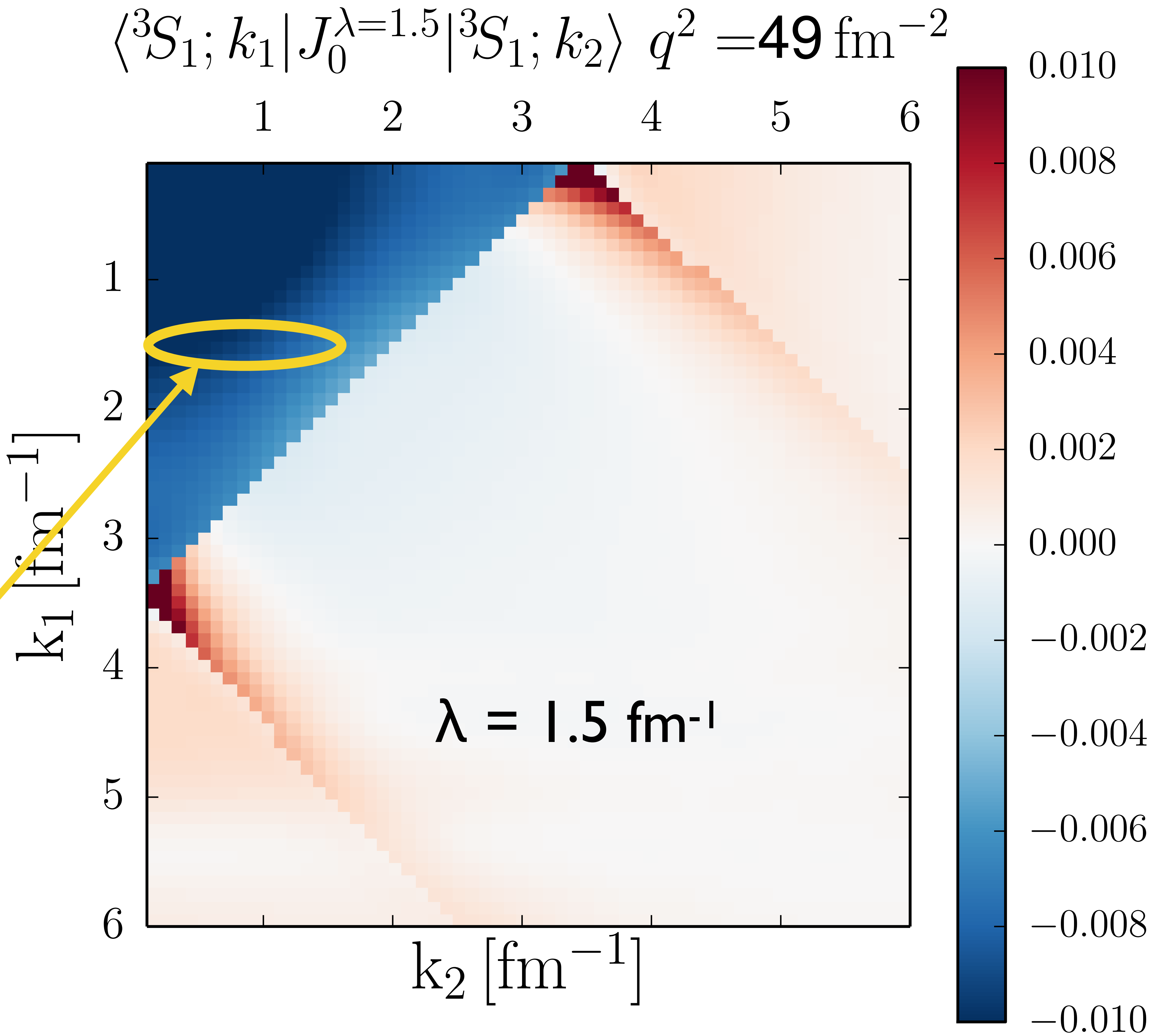


Dominant support of deuteron wf at k ≲ λ

Scale Dependence of Final State Interactions

final state wf (FSI piece)





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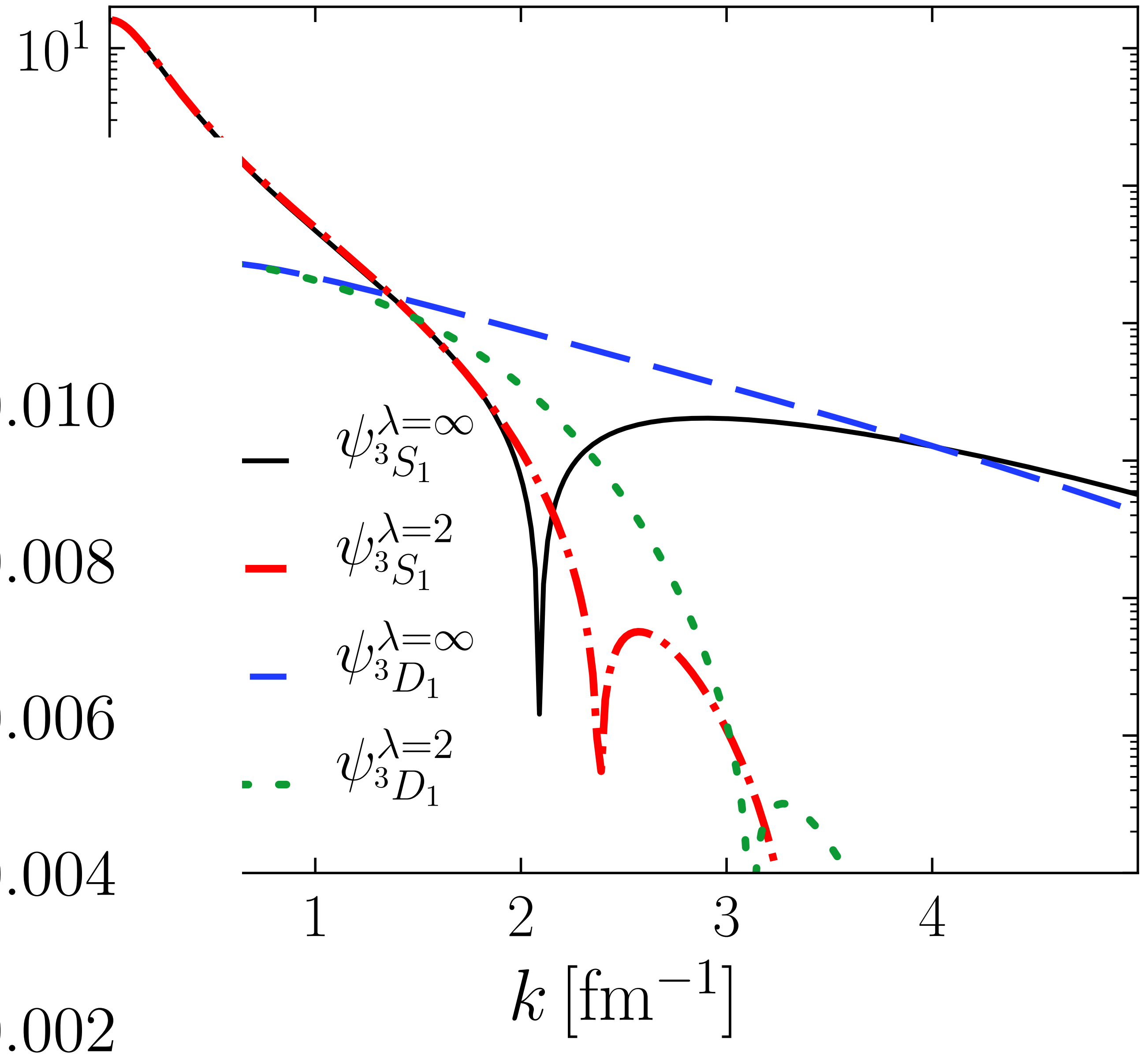

$J^{\lambda}_{\mathbf{q}}(k', k)$ probed by transition

(smooth and non-singular)

∴ FSI ~ T(p',p')

(small!)

Scale Dependence of Interpretations

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- Analysis/interpretation of a reaction involves understanding which part of wave functions probed (highly scale dependent!)
- E.g., claimed sensitivity to D-state w.f. in large **q**^{2} processes

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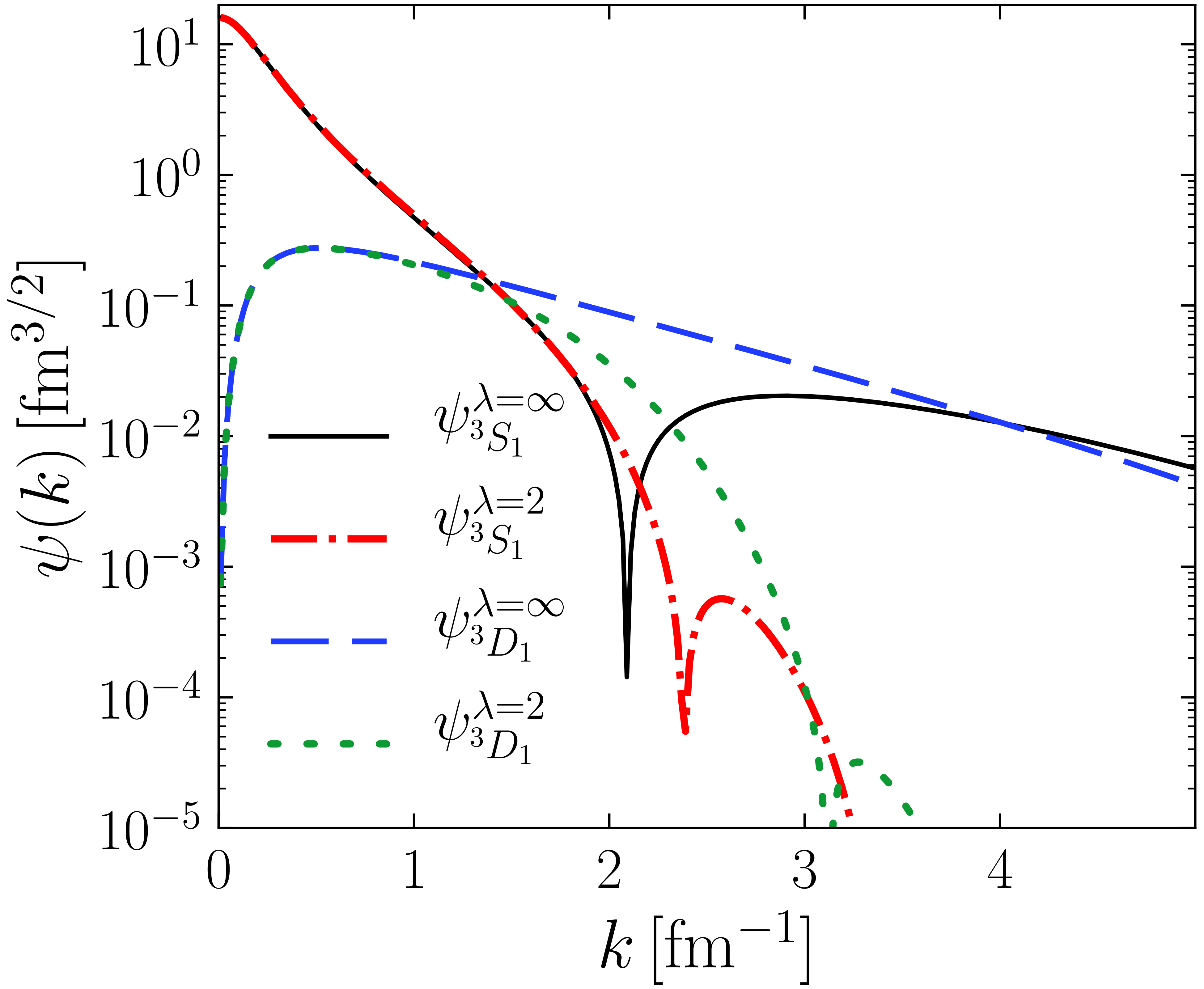
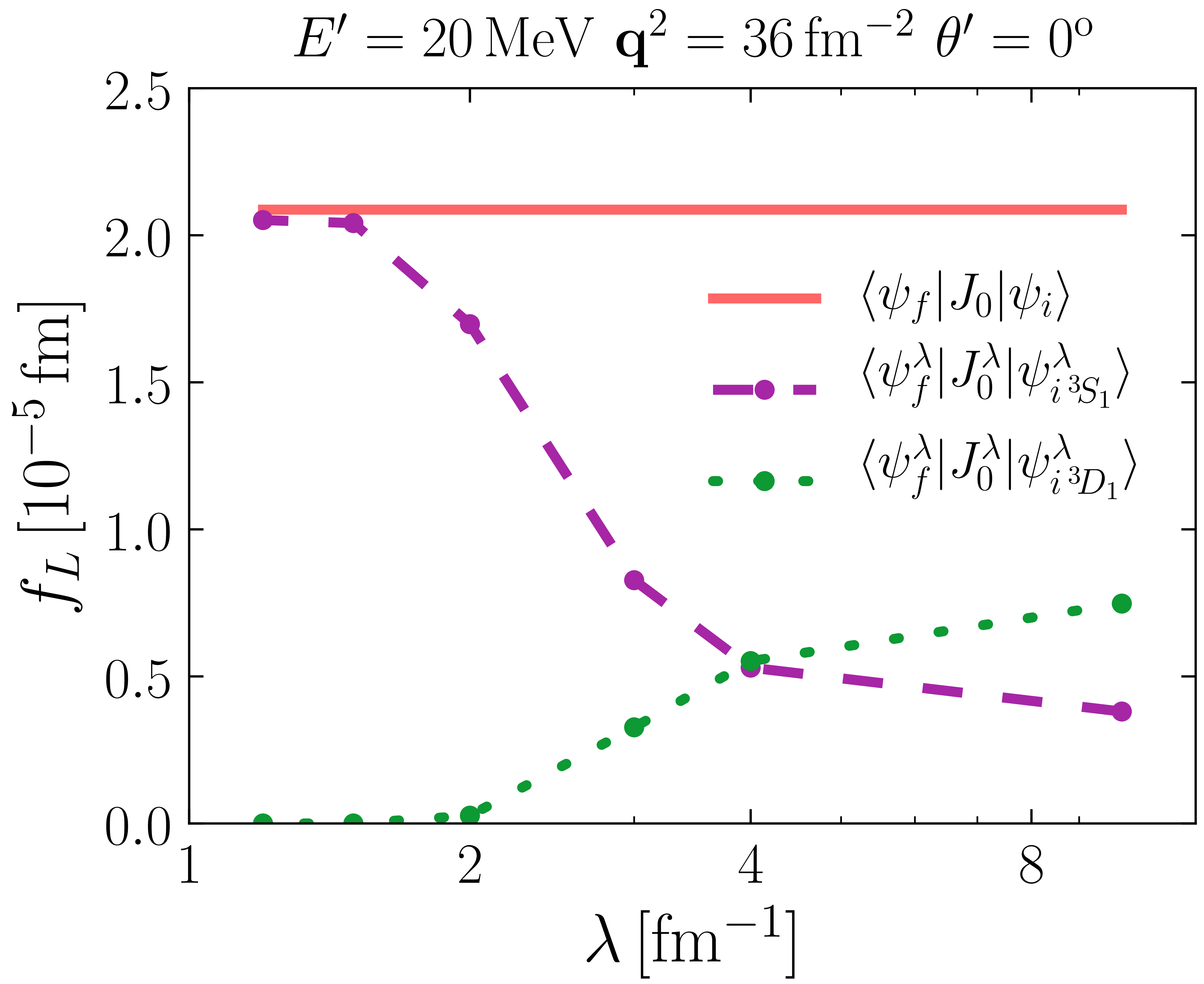
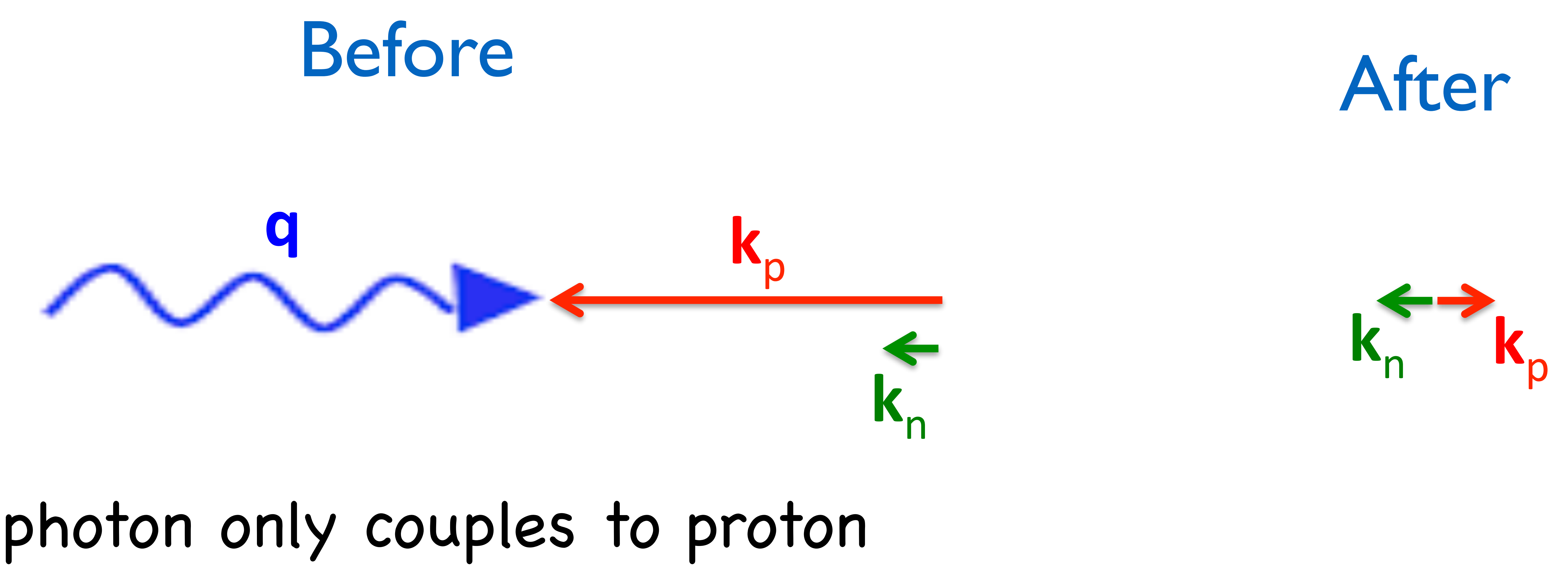




Scale Dependence of Interpretations

- Consider large q^2 near threshold (small p′) for θ=0 in **high-resolution** picture (COM frame of outgoing np)



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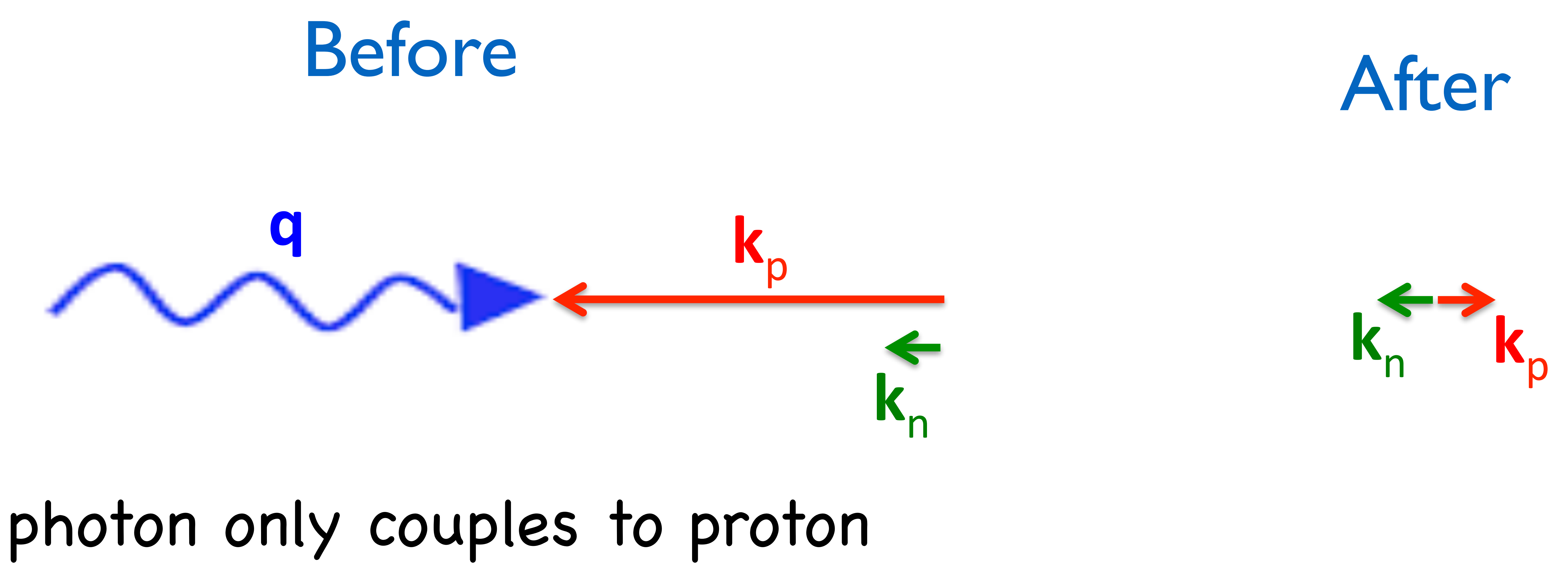

∴ proton has large momentum => initial large relative momentum

(i.e., SRC pair)

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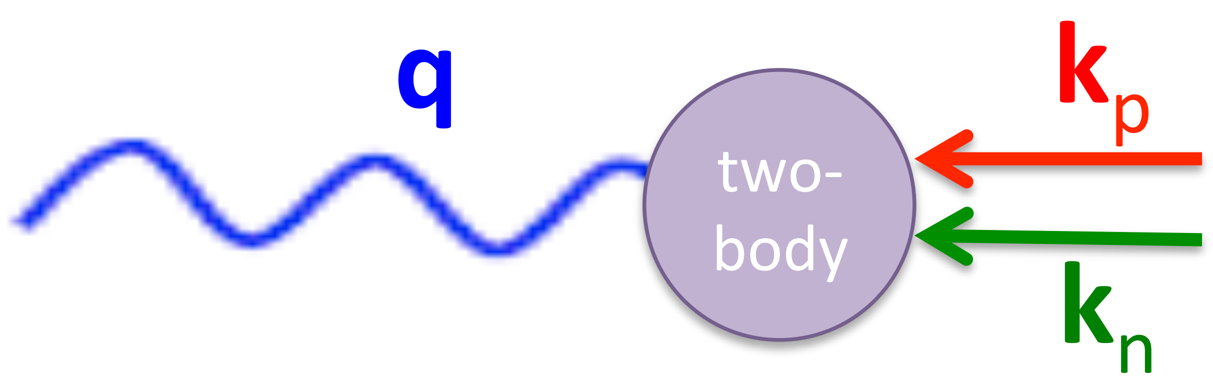
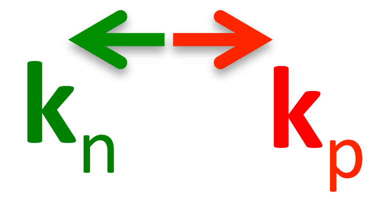
Before



After



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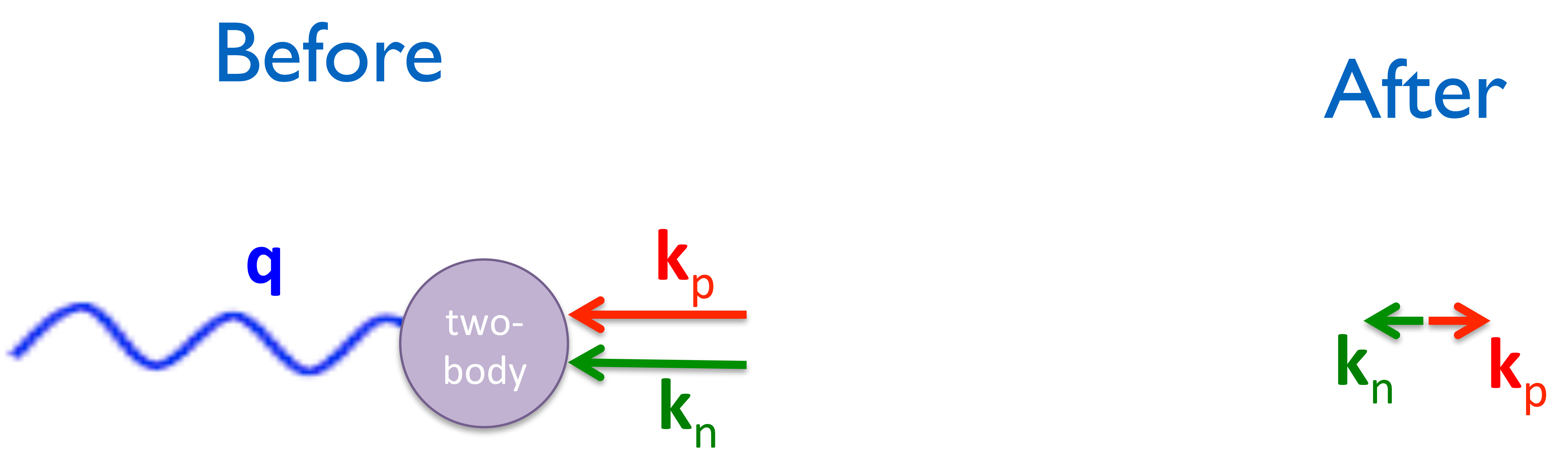

no large relative momentum in evolved deuteron wf

1-body current makes no contribution

∴ 2-body current mostly stops the low-relative momentum np pair

Example 2: Quasi-deuteron model at low resolution

- Introduced by Levinger to explain knock-out of high-energy protons in photo-absorption on nuclei at energies of order 100 MeV

- High RG resolution: emitted protons from *pn* SRCs with deuteron quantum numbers ("quasi-deuterons")

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- So cross section should be proportional to photo-disintegration of deuteron:

$$\frac{\sigma_A(E_\gamma)}{\sigma_d(E_\gamma)} \approx L\frac{NZ}{A}$$

- Defines Levinger constant *L*
- Given by ratio of high-k tails of mom. distributions
- Low RG resolution: take ratio of evolved operators in simple states

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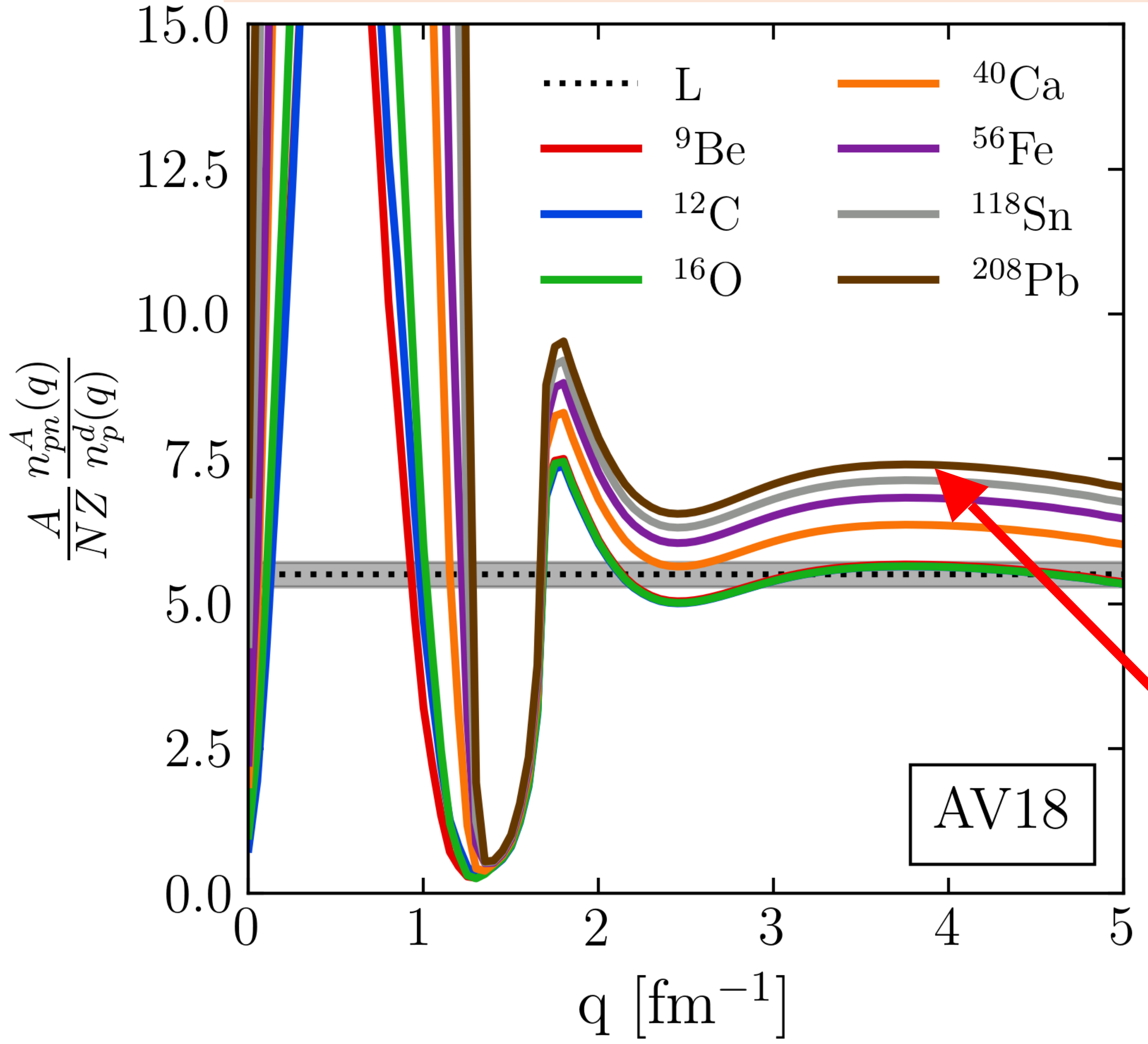
Tropiano, et al. PRC 106 (2022)



Ratios of evolved mom. dists. to *d*

Plateau despite >100 variation in this range of *q*

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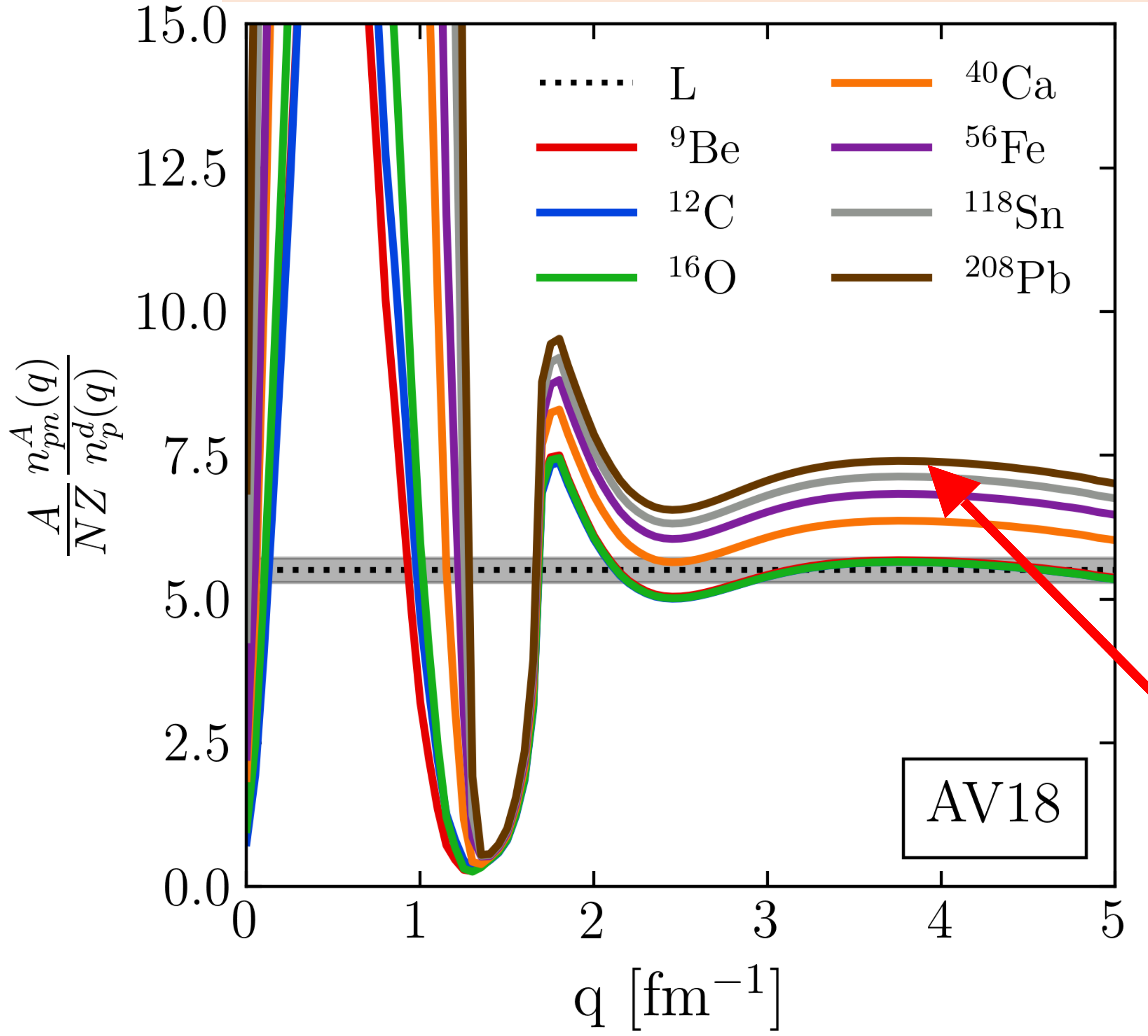

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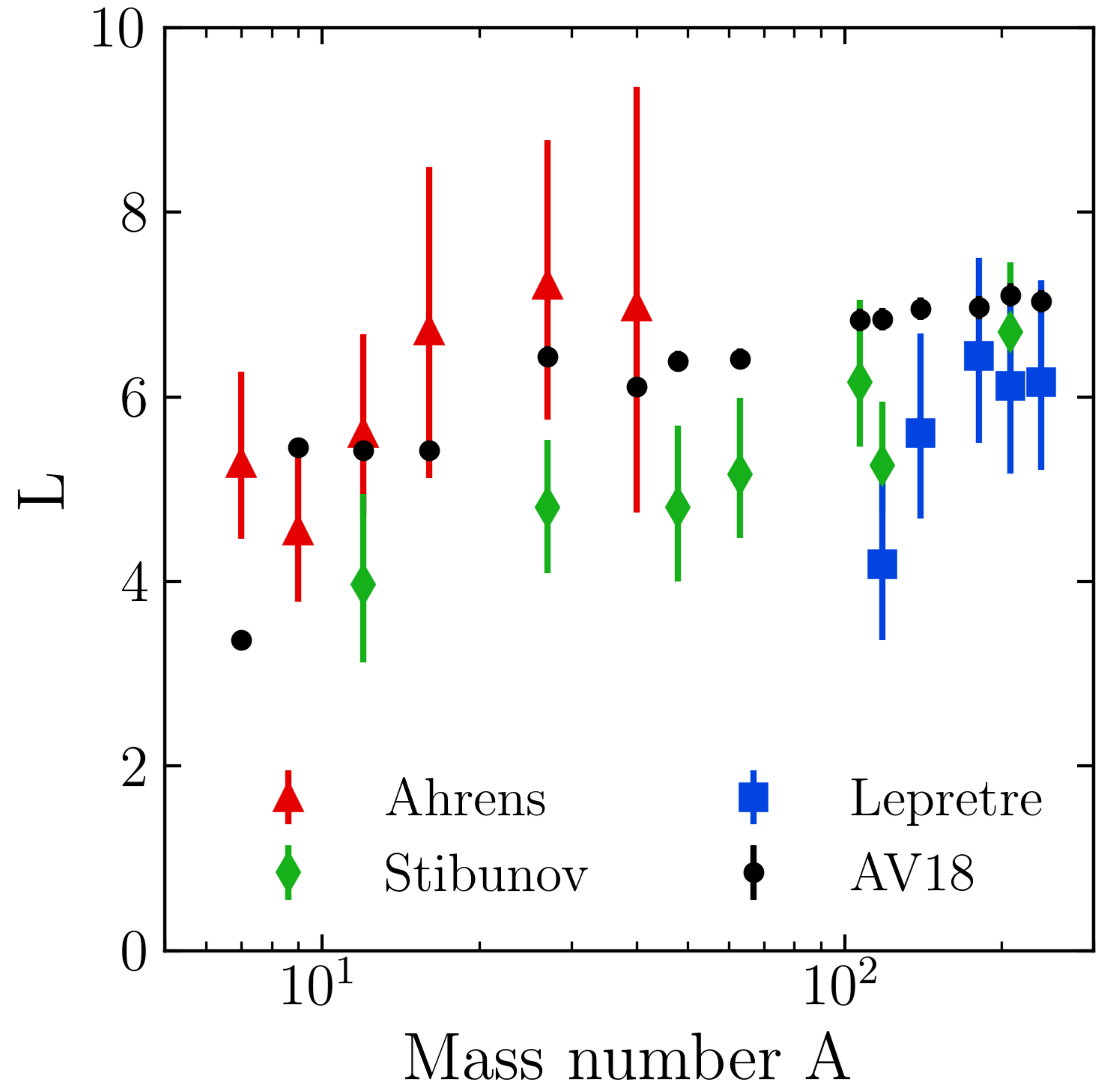

Black points: *L* from evolved mom. dists.

Colored points: *L* extracted from data

Technical note: High-q tails at low RG resolution

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- Counter-intuitively, calculating high-q tails (momentum distributions, etc.) is easy when $q \gg \lambda$

$$\hat{n}_\lambda(q) = U_\lambda a_q^\dagger a_q U_\lambda^\dagger \approx (F^{\mathrm{hi}}(q))^2 \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\lambda} F^{\mathrm{lo}}(\mathbf{k}) F^{\mathrm{lo}}(\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Technical note: High-q tails at low RG resolution

- Counter-intuitively, calculating high-q tails (momentum distributions, etc.) is easy when $q \gg \lambda$

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smeared contact operator
low-k physics
A-dependence in ME's

Universal (A-indep)
Wilson Coeff, fixed by A=2
depends on operator

Technical note: High-q tails at low RG resolution

FRIB

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Universal (A-indep)
Wilson Coeff, fixed by A=2
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smeared contact operator
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A-dependence in ME's

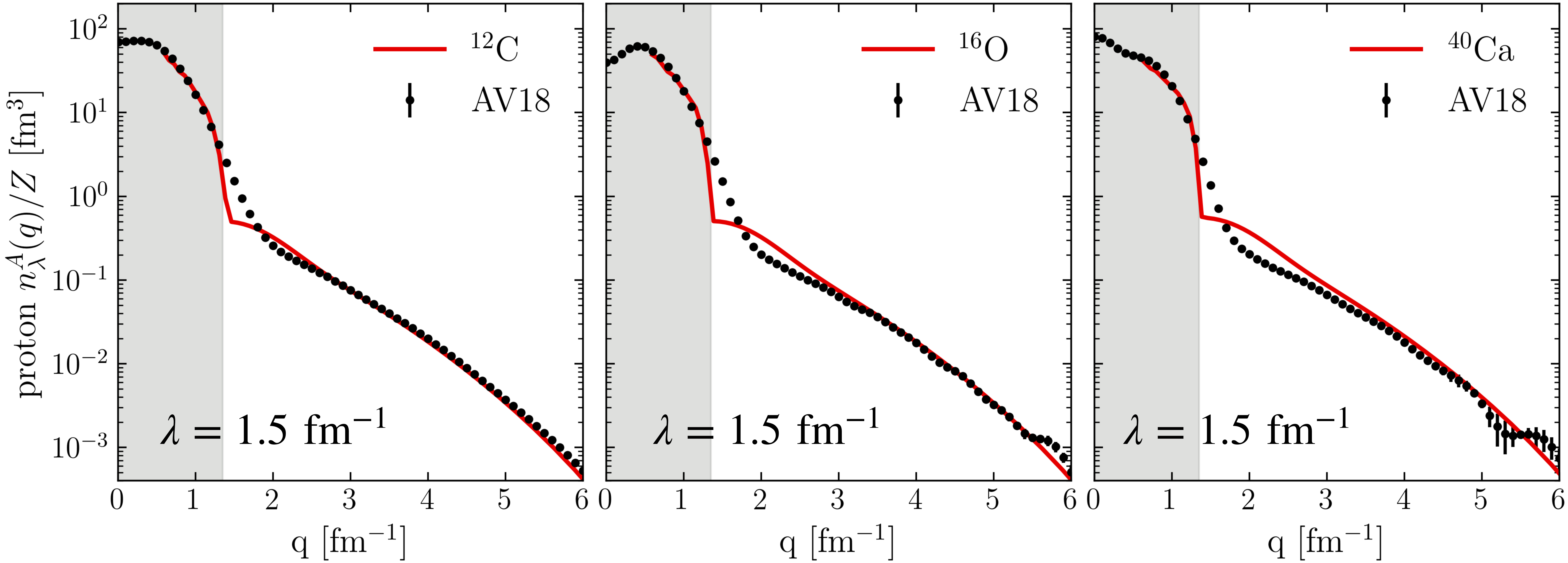
Explains why tails in A-body scale off the deuteron (same q-dependence)

Easy to calculate since only have to evaluate a smeared contact operator in "simple" low resolution wf's (amenable to approximations)

$$n(q) = \langle A_{\mathrm{hi}} | a^\dagger_q a_q | A_{\mathrm{hi}} \rangle \approx (F^{\mathrm{hi}}(q))^2 \langle A^\lambda_{\mathrm{lo}} | \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\lambda} a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | A^\lambda_{\mathrm{lo}} \rangle$$

Tropiano, et al. PRC 104 (2021)

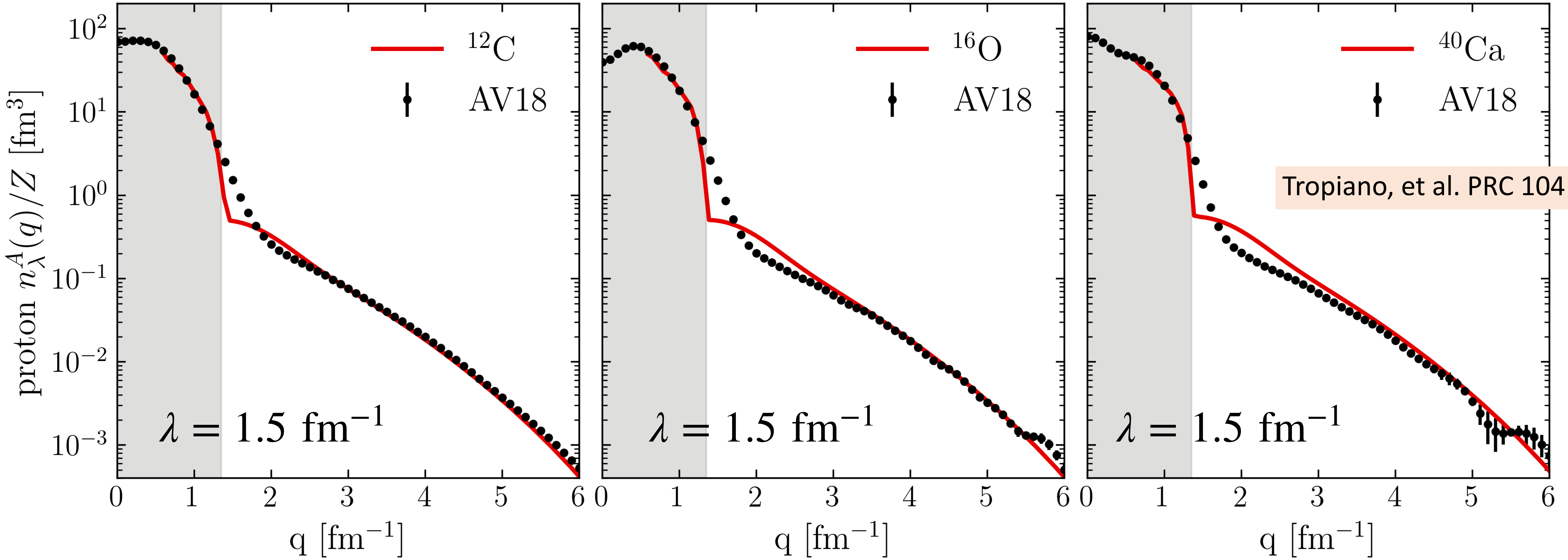
Technical note: High-q tails at low RG resolution



Approximate low-resolution wf as HF treated in LDA

Decent reproduction of full VMC calculations with av18 (note: LDA breaks down at small q)

Technical note: High-q tails at low RG resolution



Tropiano, et al. PRC 104 (2021)

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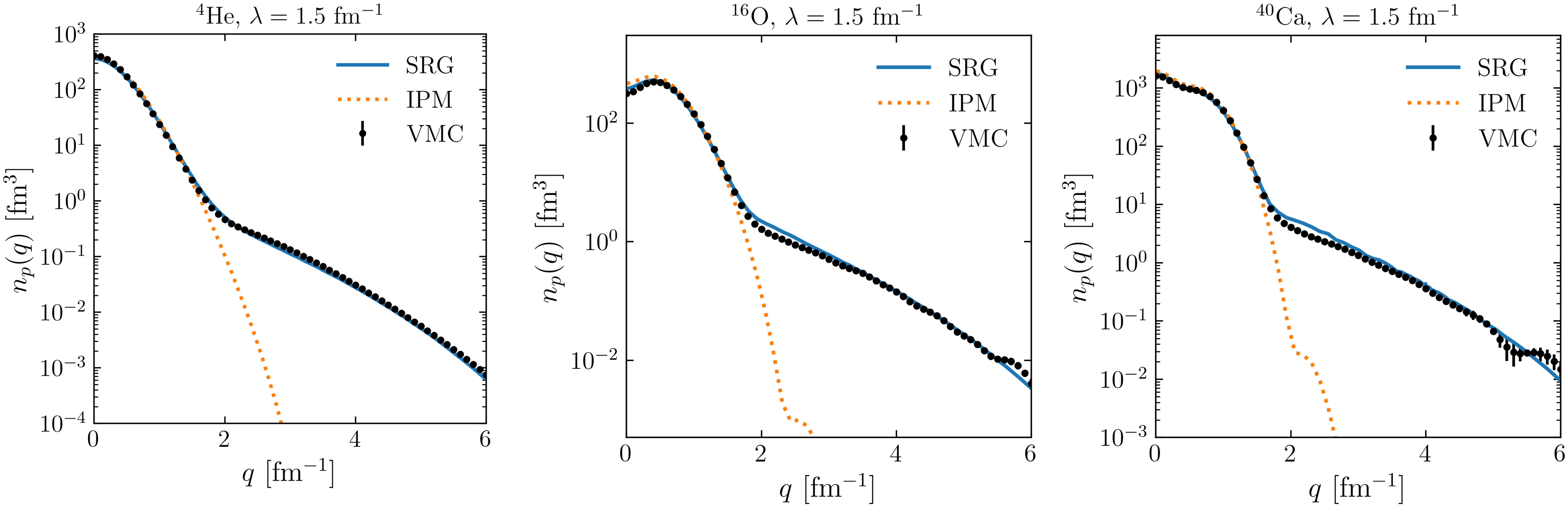
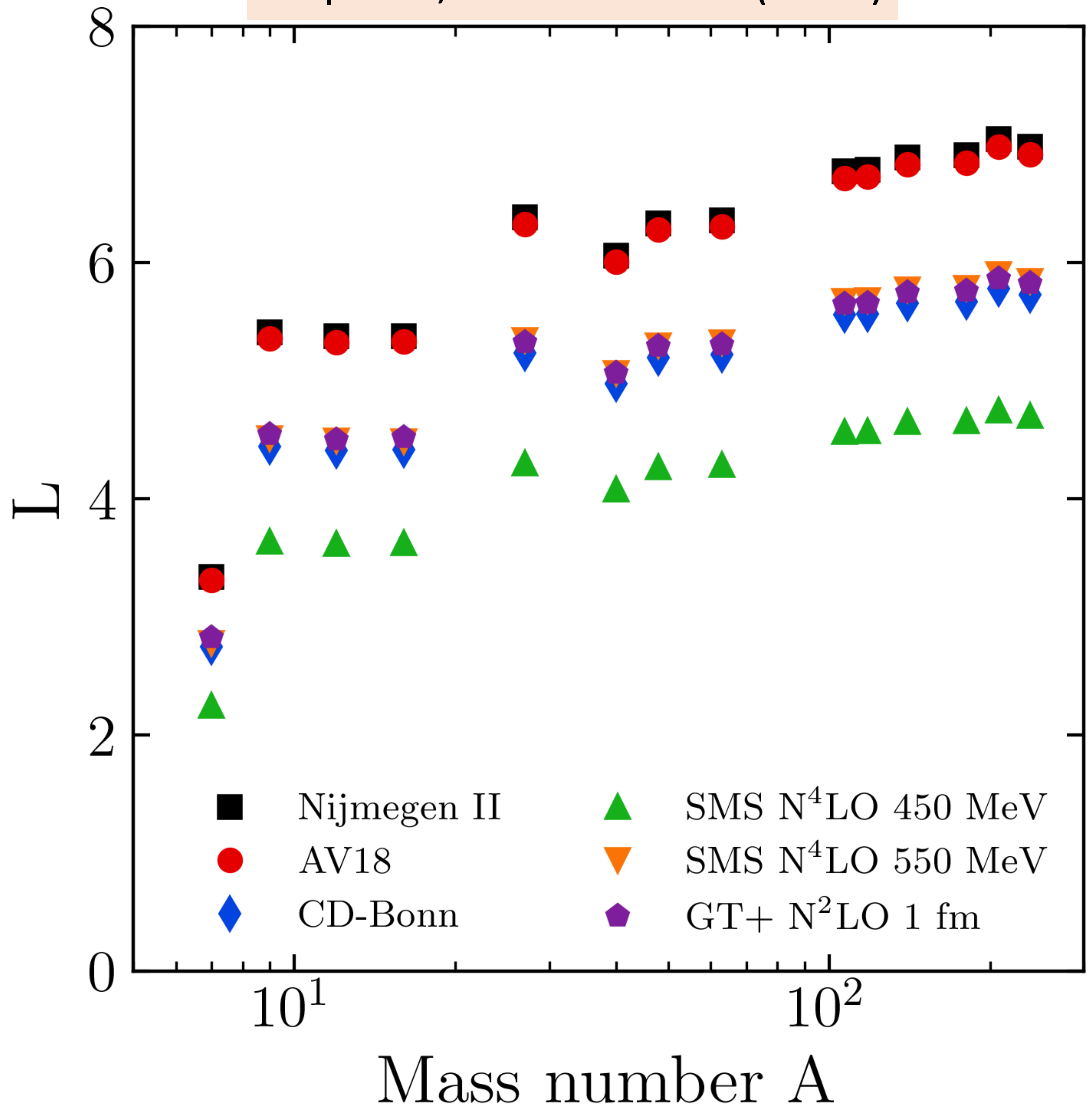


Approximate low-resolution wf as a single Slater determinant of Woods-Saxon s.p. orbitals

Good reproduction of full VMC calculations with av18

Levinger constant: Scale and Scheme dependence

Tropiano, et al. PRC 106 (2022)



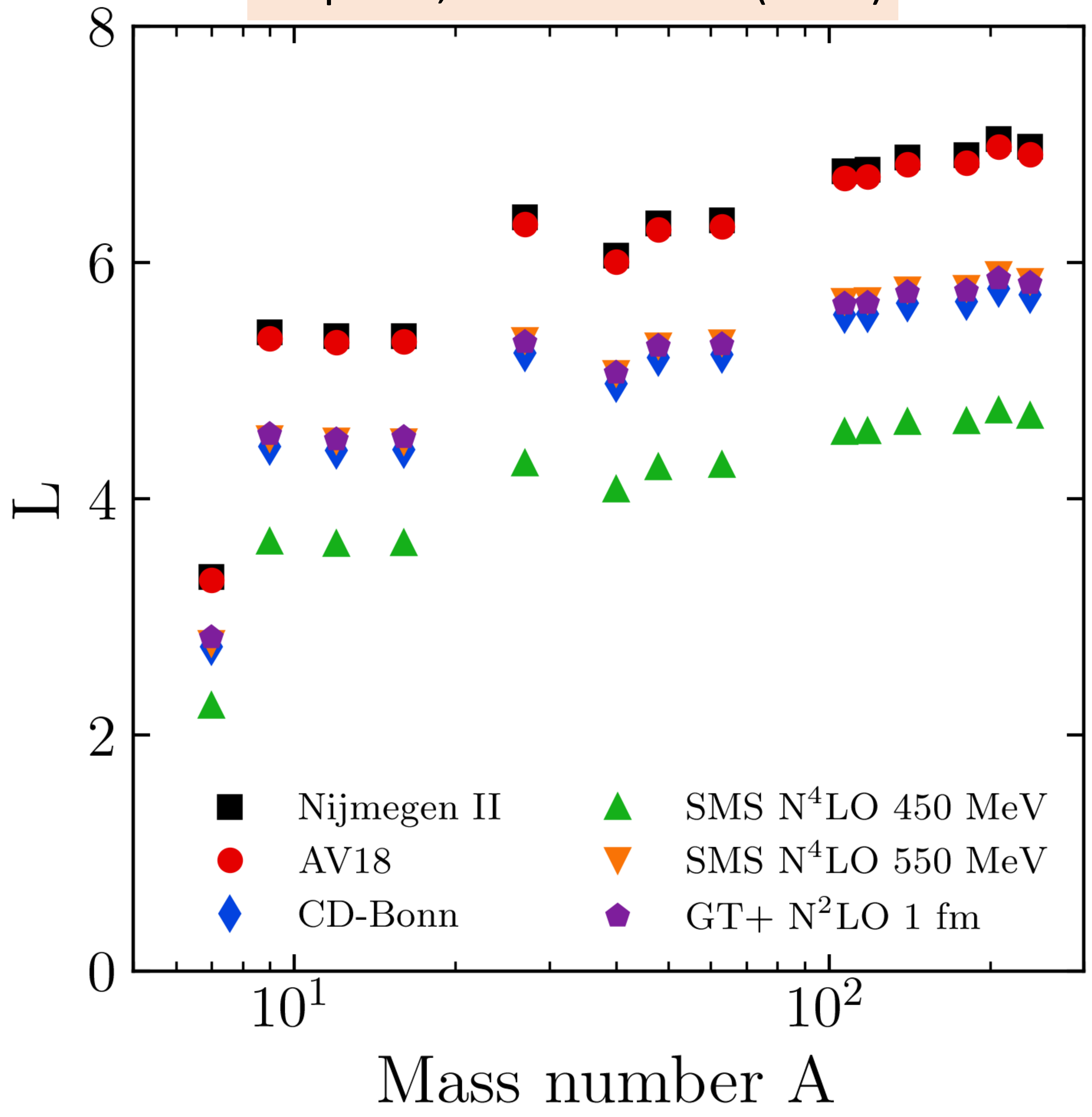
Average Levinger constant for several nuclei comparing different NN interactions.

- Varying the input NN interaction changes the values of L
- Hard interactions give high L values and soft interactions give low L values
- But a ratio of cross sections should be RG invariant! So why is there sensitivity to the interaction?
  - We've assumed only an initial one-body operator for all Hamiltonians!

Levinger constant: Scale and Scheme dependence

FRIB

Tropiano, et al. PRC 106 (2022)



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  - We've assumed only an initial one-body operator for all Hamiltonians!

- **Strategy**: Match results using a reference momentum distribution (AV18)
  - One-body initial operator for AV18
  - Two-body initial operator for soft potentials

RG matching structure and reactions

- Two interactions $H_{\rm hard}$ and $H_{\rm soft}$ (e.g., av18 and SMS N[4]LO 550)

- Find approx. matching scale λ_M by $H_{\rm soft} \approx U_{\rm hard}(\lambda_M) H_{\rm hard} U^\dagger_{\rm hard}(\lambda_M)$ from comparing deuteron wf's

- The initial operator to be used with $H_{\rm soft}$ now has 1- and 2-body components and is given by $\hat{O}_{\rm soft} = U_{\rm hard}(\lambda_M) \hat{O}_{\rm hard} U^\dagger_{\rm hard}(\lambda_M)$

- In this example, find $\lambda_M \approx 4.5 \ {\rm fm}^{-1}$ is optimal

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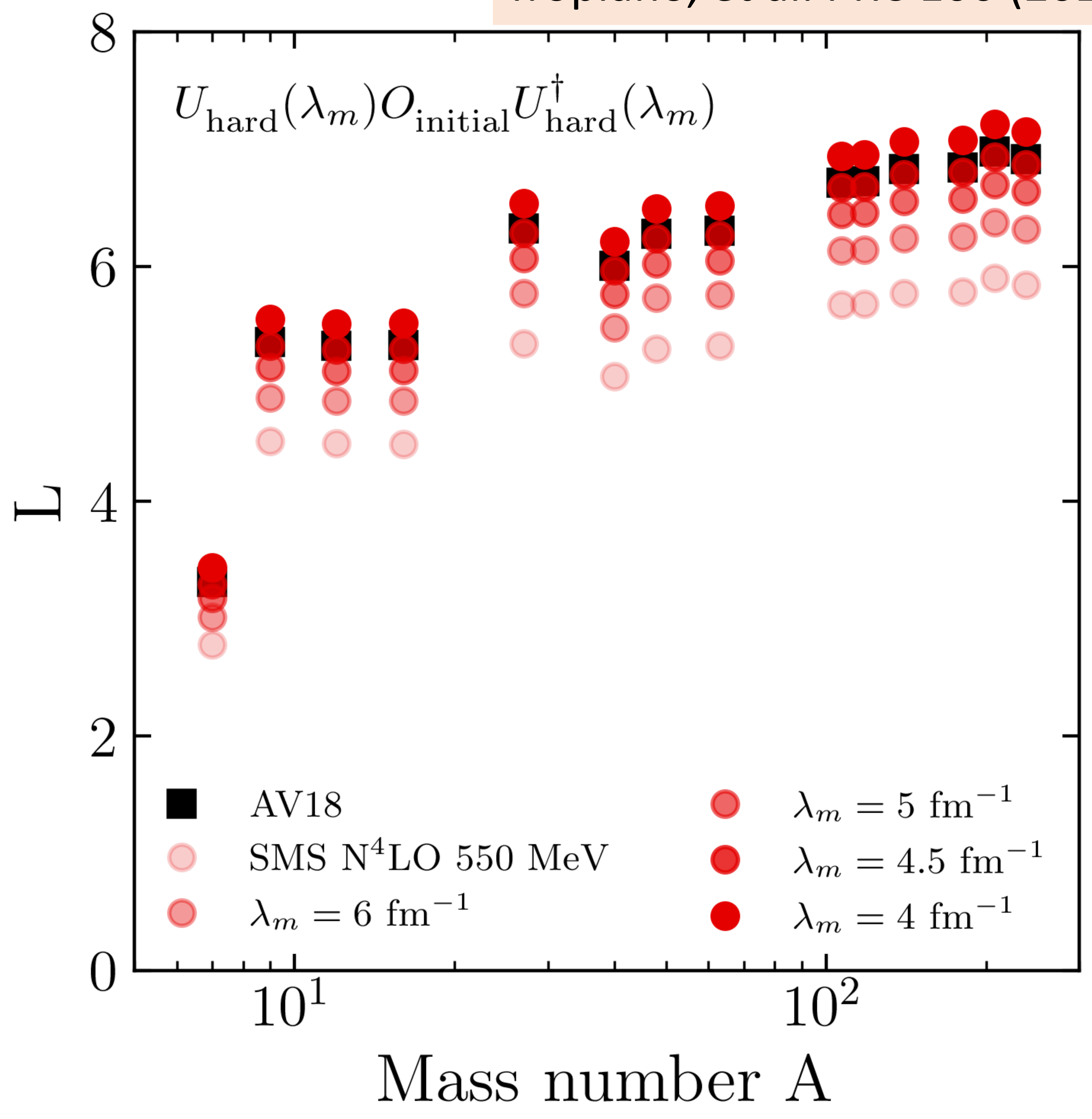
Now av18 and N⁴LO 550 give the same L after RG matching

Average Levinger constant for several nuclei comparing the SMS N⁴LO 550 MeV and AV18 potentials. Results are also shown for the SMS N⁴LO 550 MeV potential with an additional two-body operator due to inverse-SRG transformations from AV18.

Tropiano, et al. PRC 106 (2022)



Summary

- Ab-initio structure theory advances driven by working at low resolutions. Workhorses of structure theory (shell model and DFT) are low-resolution pictures
- Consistent structure and reaction calculations should be matched to the same RG scale and scheme (cf. quenching)
  - Ex 1: high-q SRC reactions amenable to low-resolution picture. Observable cross sections **stay the same**, but components (FSIs, current operators, claims about what components of the wf are being "probed", physical interpretations) **change**
  - Ex 2: Levinger constant example as a simple prototype to match initial reaction operators to appropriate RG scale (e.g., eliminated scale dependence of L, which is a ratio of cross sections)
- Extensions underway to $(e, e'p)$ knockout cross section to test scale/scheme dependence of components
- Extensions to nucleon probes: 1st steps: Hisham et al. *RG Evolution of Optical Potentials,* PRC 106 (2022).

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FRIB

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Extras

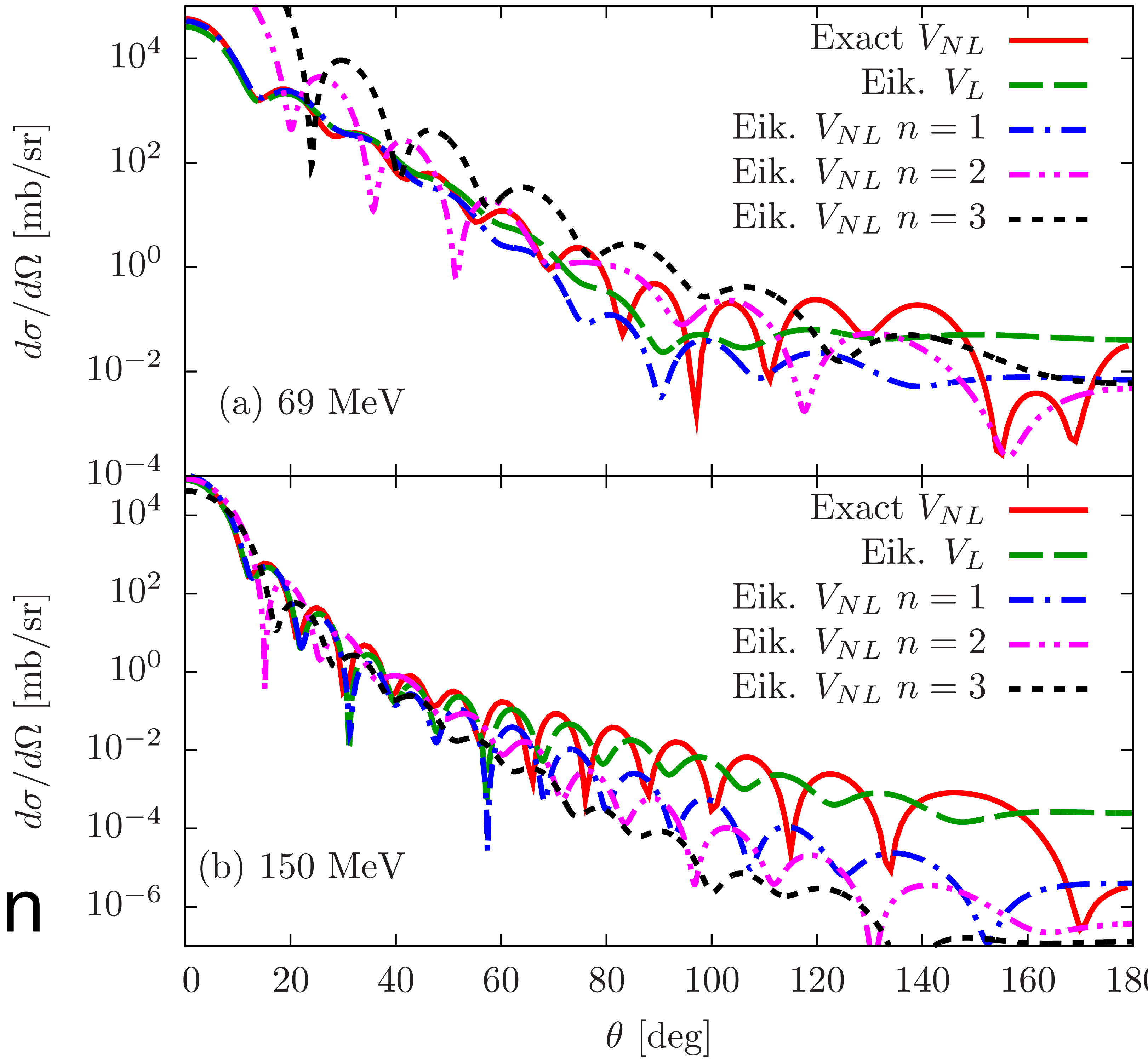
Non-locality and problems with Eikonal?

Considering non-locality in the optical potentials within eikonal models

C. Hebborn[1,2,*] and F. M. Nunes[3,4,†]

Results: Our results show that transfer observables are significantly impacted by non-locality in the high-energy regime. Because knockout reactions are dominated by stripping (transfer to inelastic channels), non-locality is expected to have a large effect on knockout observables too. Three approaches are explored for extending the eikonal method to non-local interactions, including an iterative method and a perturbation theory.

Conclusions: None of the derived extensions of the eikonal model provide a good description of elastic scattering. This work suggests that non-locality removes the formal simplicity associated with the eikonal model.



Optical potentials inherently non-local from their microscopic definition

Moreover:

Hisham et al. find the optical potential "inherits" the non-locality of the SRG-evolved NN interaction, which is rather different than Perey-Buck parameterizations

Computing SRC operators at low-RG resolutions

$\hat{O}_q^{\rm hi}$ = operator that probes high-q components at high-RG resolution

$$\langle A^{\rm hi} | \hat{O}_q^{\rm hi} | A^{\rm hi} \rangle \neq 0$$

e.g., $\hat{n}^{\rm hi}(\mathbf{q}) = a^\dagger_{\mathbf{q}} a_{\mathbf{q}}\,,\ \hat{\rho}^{\rm hi}(\mathbf{q}, \mathbf{Q}) = a^\dagger_{\frac{Q}{2}+q} a^\dagger_{\frac{Q}{2}-q} a_{\frac{Q}{2}-q} a_{\frac{Q}{2}+q}$

Computing SRC operators at low-RG resolutions

FRIB

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SRG evolve to $\lambda \lesssim \mathbf{q}$

$$\langle A^{\text{hi}} | \hat{O}_q^{\text{hi}} | A^{\text{hi}} \rangle = \langle A^{\text{hi}} | \hat{U}_\lambda^{\dagger} \hat{U}_\lambda \hat{O}_q^{\text{hi}} \hat{U}_\lambda^{\dagger} \hat{U}_\lambda | A^{\text{hi}} \rangle = \langle A^{\text{lo}} | \hat{O}_q^{\text{lo}} | A^{\text{lo}} \rangle \leftarrow$$

wf's of **soft** $\hat{H}^{\text{lo}} = \hat{U}_\lambda \hat{H}^{\text{hi}} \hat{U}_\lambda^{\dagger}$

$$\langle A^{\text{lo}} | \hat{O}_q^{\text{hi}} | A^{\text{lo}} \rangle \approx 0$$

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$$\hat{U}_\lambda = \hat{1} + \frac{1}{4} \sum_{K,k,k'} \delta U_\lambda^{(2)}(k,k')\, a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} + \frac{1}{36} \sum \delta U_\lambda^{(3)} a^\dagger a^\dagger a^\dagger aaa + \cdots$$

fixed from SRG evolution on A=2

fixed from SRG evolution on A=3

$\delta U_\lambda(\mathbf{k},\mathbf{k}')$ inherits symmetries of V_{NN} (Galilean, partial wave structure, etc.)

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Wick's theorem to evaluate $\hat{O}_q^{\rm lo} = \hat{U}_\lambda \hat{O}_q^{\rm hi} \hat{U}_\lambda^\dagger = \hat{O}_{1b}^{\rm lo} + \hat{O}_{2b}^{\rm lo} + \cancel{\hat{O}_{3b}^{\rm lo}} + \cdots$

Computing SRC operators at low-RG resolutions

$\hat{O}^{\rm hi}_q$ = operator

SRG $H^{\rm lo}_\lambda$ a "cluster" hierarchy $V^{2N}_\lambda \gg V^{3N}_\lambda \gg V^{4N}_\lambda \ldots$

cancellations of KE/PE "amplify" the importance of 3N for bulk energies

$+q \; a^\dagger_{\frac{Q}{2}-q} \; a_{\frac{Q}{2}-q} \; a_{\frac{Q}{2}+q}$

SRG evol

$\langle A^{\rm hi} | \hat{O}^{\rm hi}_q | A$

$\hat{U}_\lambda \hat{H}^{\rm hi} \hat{U}^\dagger_\lambda$

≈ 0

$\hat{U}_\lambda = \hat{1}$

Wick's

Computing SRC operators at low-RG resolutions

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$_{+q} a^\dagger_{\frac{Q}{2}-q} a_{\frac{Q}{2}-q} a_{\frac{Q}{2}+q}$

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For high-q operators ($\lambda \lesssim q$), evidence that

$\hat{O}^{1b}_q(\lambda) \ll \hat{O}^{2b}_q(\lambda)$ **BUT** $\hat{O}^{2b}_q(\lambda) \gg \hat{O}^{3b}_q(\lambda) \gg \cdots$

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$\hat{U}_\lambda \hat{H}^{\text{hi}} \hat{U}^\dagger_\lambda$

≈ 0

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Wick's

SRG H_λ^{lo} a "cluster" hierarchy $V_\lambda^{2N} \gg V_\lambda^{3N} \gg V_\lambda^{4N} \ldots$

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$\hat{O}_q^{1b}(\lambda) \ll \hat{O}_q^{2b}(\lambda)$ **BUT** $\hat{O}_q^{2b}(\lambda) \gg \hat{O}_q^{3b}(\lambda) \gg \cdots$

Can assess SRG truncations by varying λ (observables don't change if no approximation made)

Computing SRC operators at low-RG resolutions

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SRG evol

$\langle A^{\text{hi}} | \hat{O}_q^{\text{hi}} | A$

$\hat{U}_\lambda \hat{H}^{\text{hi}} \hat{U}_\lambda^\dagger$

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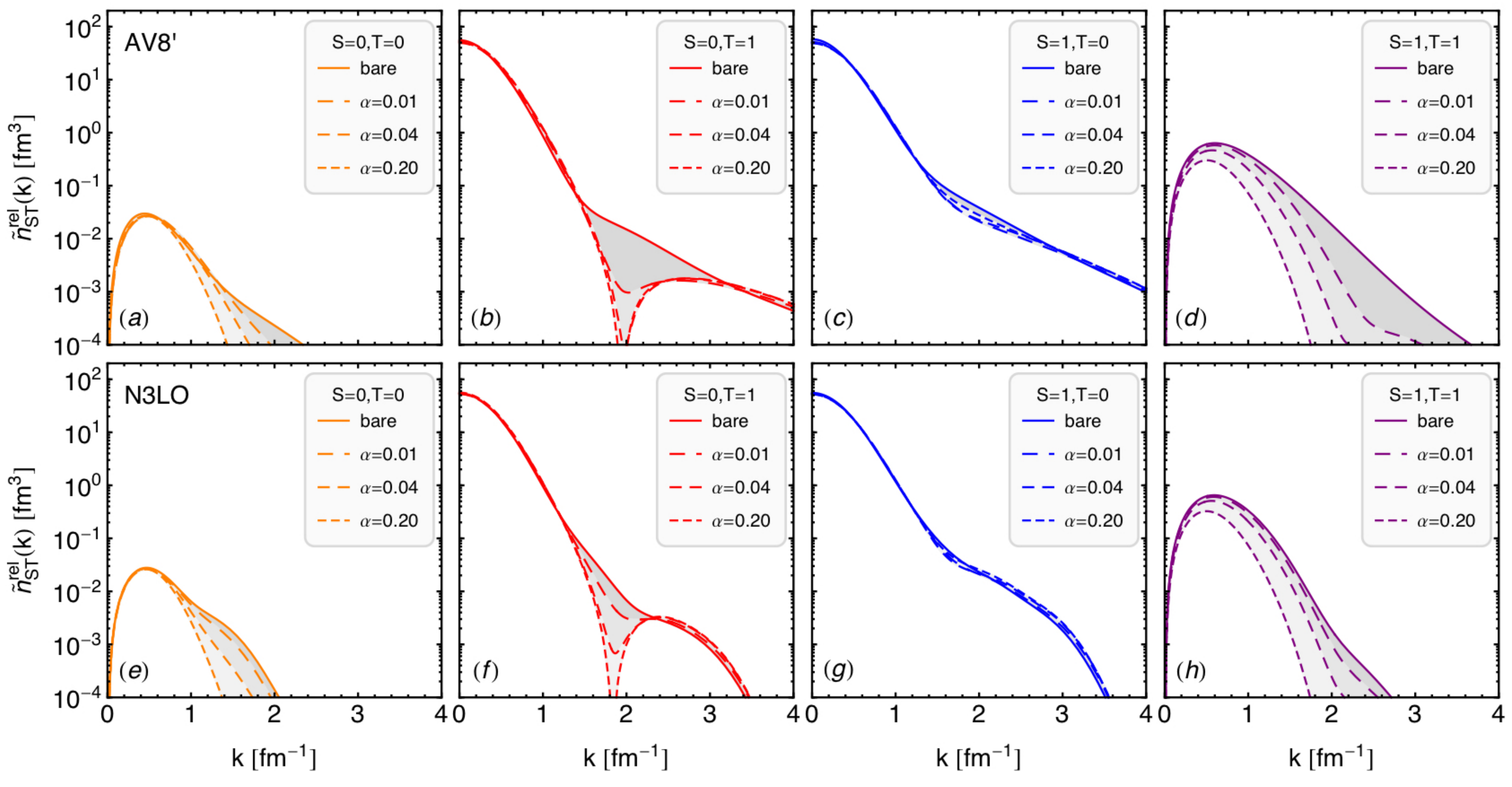
$\hat{U}_\lambda = \hat{1}$

Wick's

Neff, Feldmeier, Horiuchi PRC 92 (2015)



Some λ-dependence for relative momentum dist.

integral over sizable CM ==> non-SRC physics; sensitive to induced 3-body

Computing SRC operators at low-RG resolutions

$\hat{O}^{\mathrm{hi}}_{q}$ = operator

$+q\, a^{\dagger}_{\frac{Q}{2}-q}\, a_{\frac{Q}{2}-q}\, a_{\frac{Q}{2}+q}$

SRG evol

$\langle A^{\mathrm{hi}} | \hat{O}^{\mathrm{hi}}_{q} | A$

$\hat{U}_{\lambda} \hat{H}^{\mathrm{hi}} \hat{U}^{\dagger}_{\lambda}$

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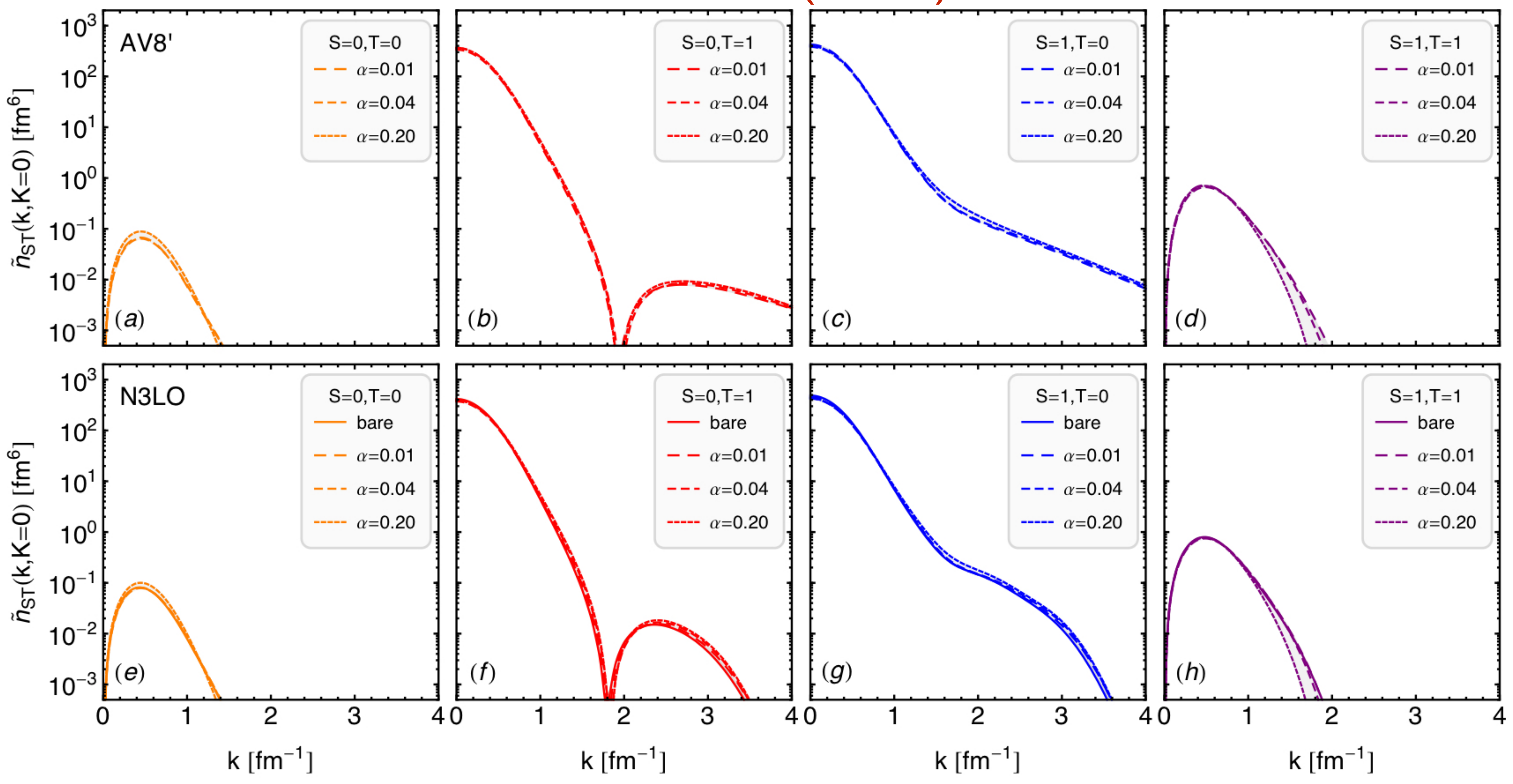

reduced λ-dependence for K=0 pair momentum dist.

induced 3-body negligible <==> SRC pairs 2-body physics

cf. LCA, GCF, leading-order Brueckner, …

Computing SRC operators at low-RG resolutions

momentum distribution

$$\hat{n}^{\mathrm{hi}}(\mathbf{q}) = a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}}$$

$$\hat{n}^{\mathrm{lo}}(\mathbf{q}) = \left(\hat{1} + \delta U_{\lambda}^{(2)}\right) a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \left(\hat{1} + \delta U_{\lambda}^{\dagger(2)}\right)$$

$$\Downarrow$$

Computing SRC operators at low-RG resolutions

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$$\Downarrow$$

$$\hat{n}^{\mathrm{lo}}(\mathbf{q}) = a^\dagger_{\mathbf{q}} a_{\mathbf{q}} + \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k},\mathbf{k}')\, a^\dagger_{\mathbf{q}-\mathbf{k}+\mathbf{k}'} a^\dagger_{\mathbf{q}+\mathbf{k}-\mathbf{k}'} a_{\mathbf{q}-2\mathbf{k}'} a_{\mathbf{q}} \;+\; h.c.$$

$$+ \frac{1}{4}\sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k},\mathbf{q}-\mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{q}-\mathbf{K}/2,\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

$$+ (\cdots)\, a^\dagger a^\dagger a^\dagger aaa \;+\; (\cdots) a^\dagger a^\dagger a^\dagger a^\dagger aaaa \;\cdots$$

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Consider $\mathbf{q} \gg \lambda$

momenta $\gg \lambda$ absent in $|A^{\rm lo}\rangle$

$$\hat{n}^{\rm lo}(\mathbf{q}) = \cancel{a^\dagger_{\mathbf{q}} a_{\mathbf{q}}} + \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k},\mathbf{k}')\, \cancel{a^\dagger_{\mathbf{q}-\mathbf{k}+\mathbf{k}'} a^\dagger_{\mathbf{q}+\mathbf{k}-\mathbf{k}'} a_{\mathbf{q}-2\mathbf{k}'} a_{\mathbf{q}}} + h.c.$$

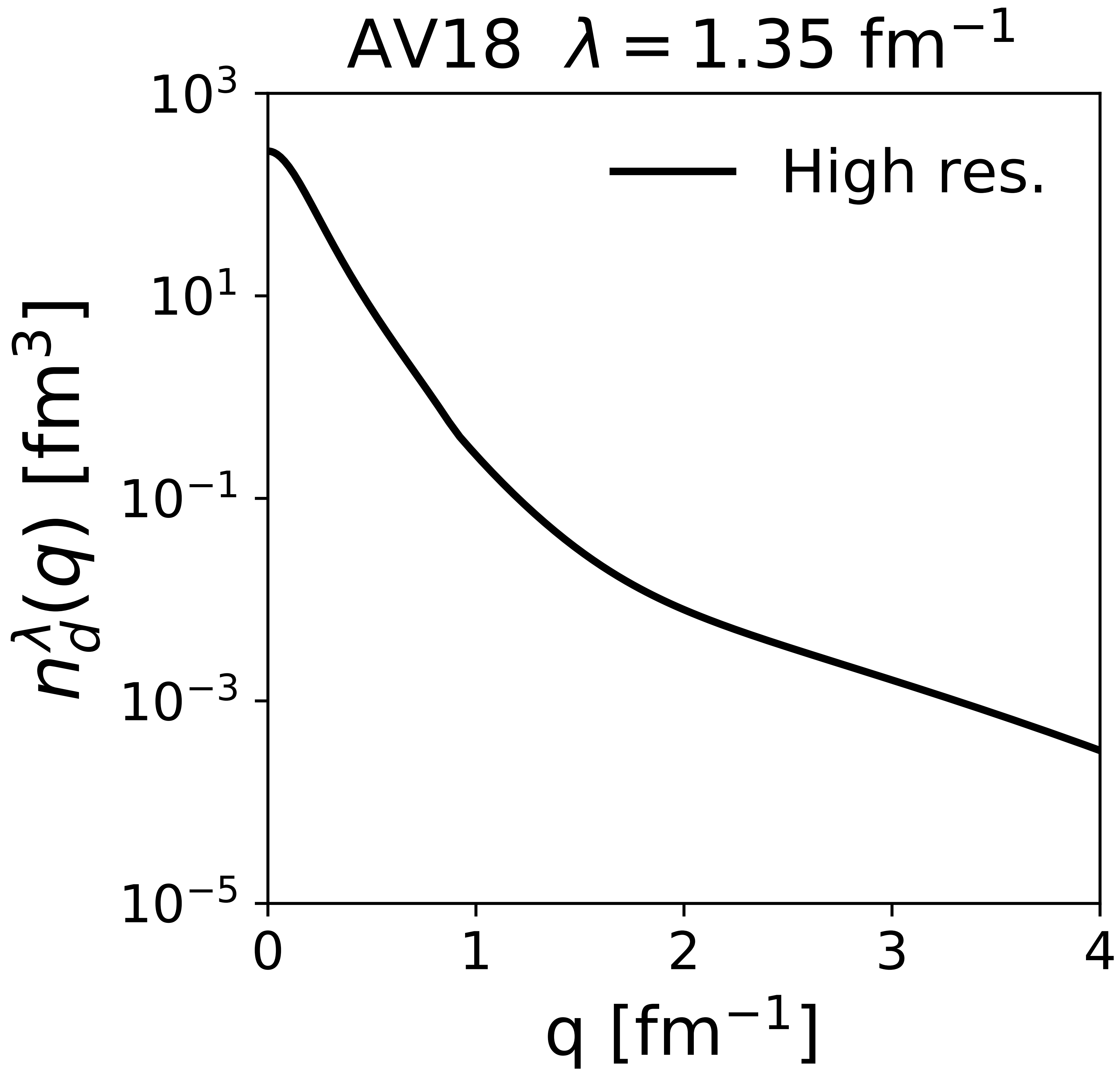
$$+ \frac{1}{4}\sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k},\mathbf{q}-\mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{q}-\mathbf{K}/2,\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

$$+ \cancel{(\cdots)\, a^\dagger a^\dagger a^\dagger aaa} + \cancel{(\cdots) a^\dagger a^\dagger a^\dagger a^\dagger aaaa} \;\cdots$$

Computing SRC operators at low-RG resolutions

Deuteron illustration

$$\hat{n}^{\text{lo}}(\mathbf{q}) = \left(\hat{1} + \delta U_\lambda^{(2)}\right) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \left(\hat{1} + \delta U_\lambda^{\dagger(2)}\right)$$



$$\langle D^{\text{hi}} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | D^{\text{hi}} \rangle$$

Computing SRC operators at low-RG resolutions

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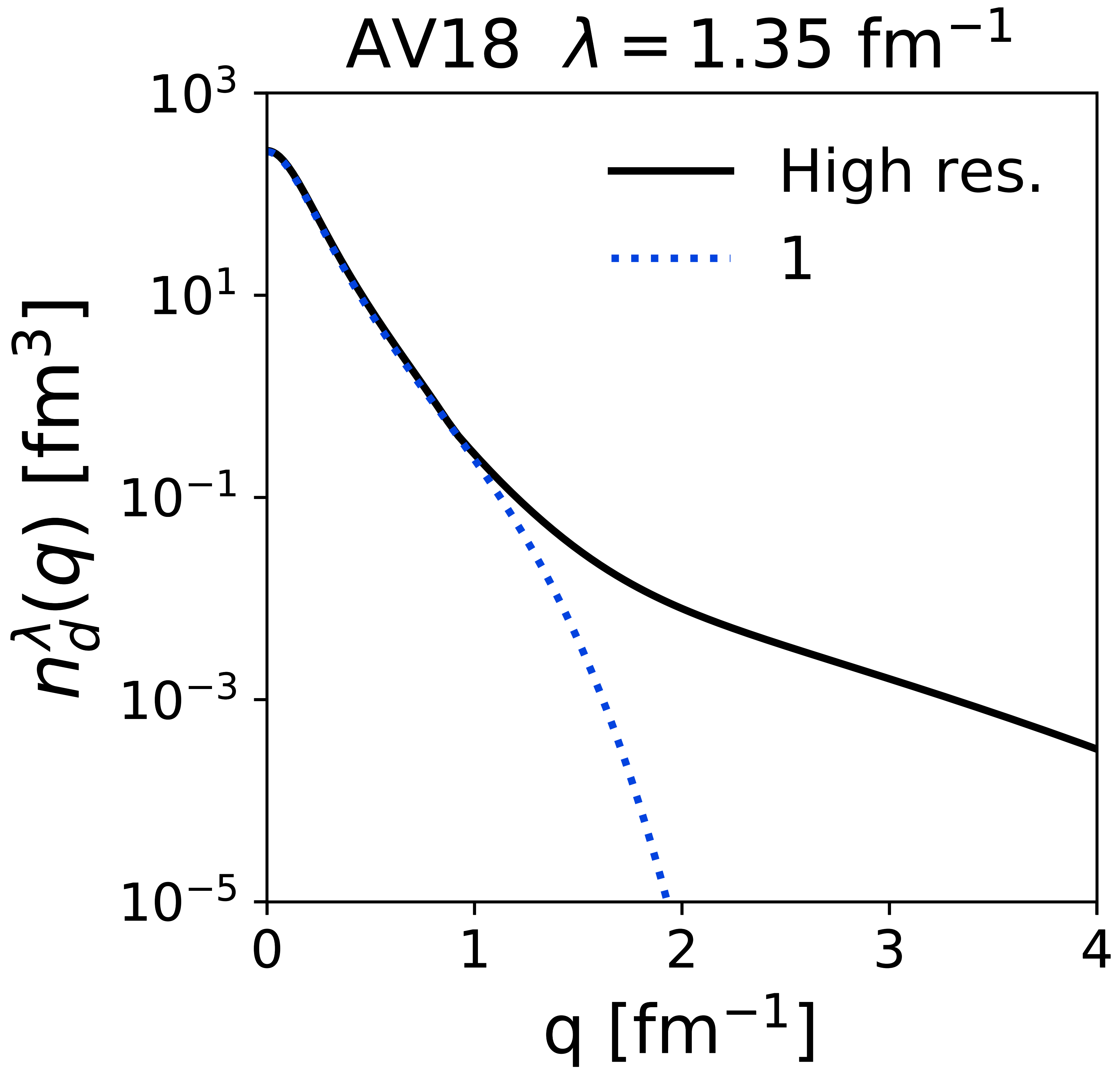

$$\langle D^{\rm hi} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | D^{\rm hi} \rangle$$

$$\langle D^{\rm lo} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | D^{\rm lo} \rangle$$

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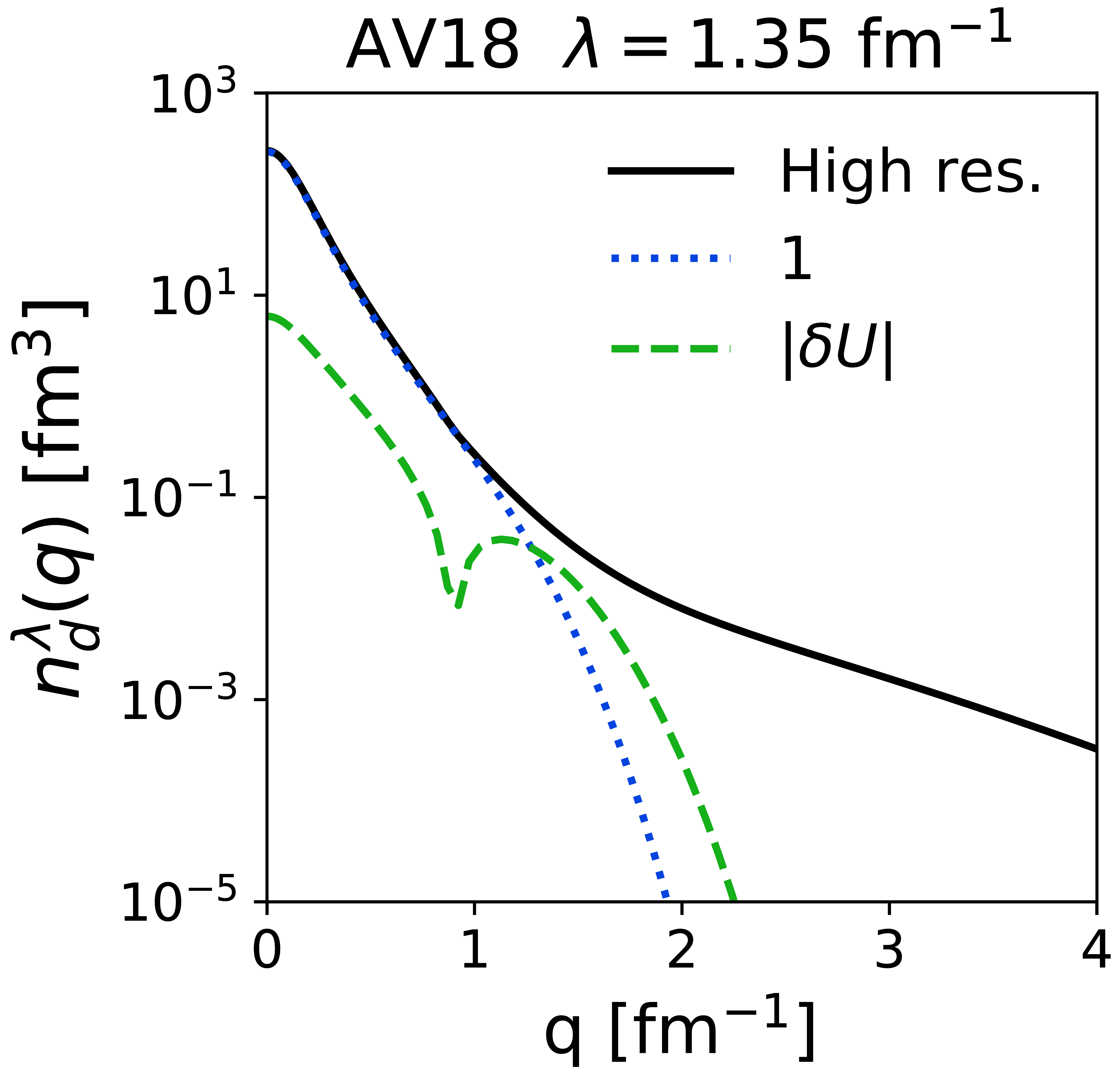


$$\langle D^{\mathrm{hi}} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | D^{\mathrm{hi}} \rangle$$

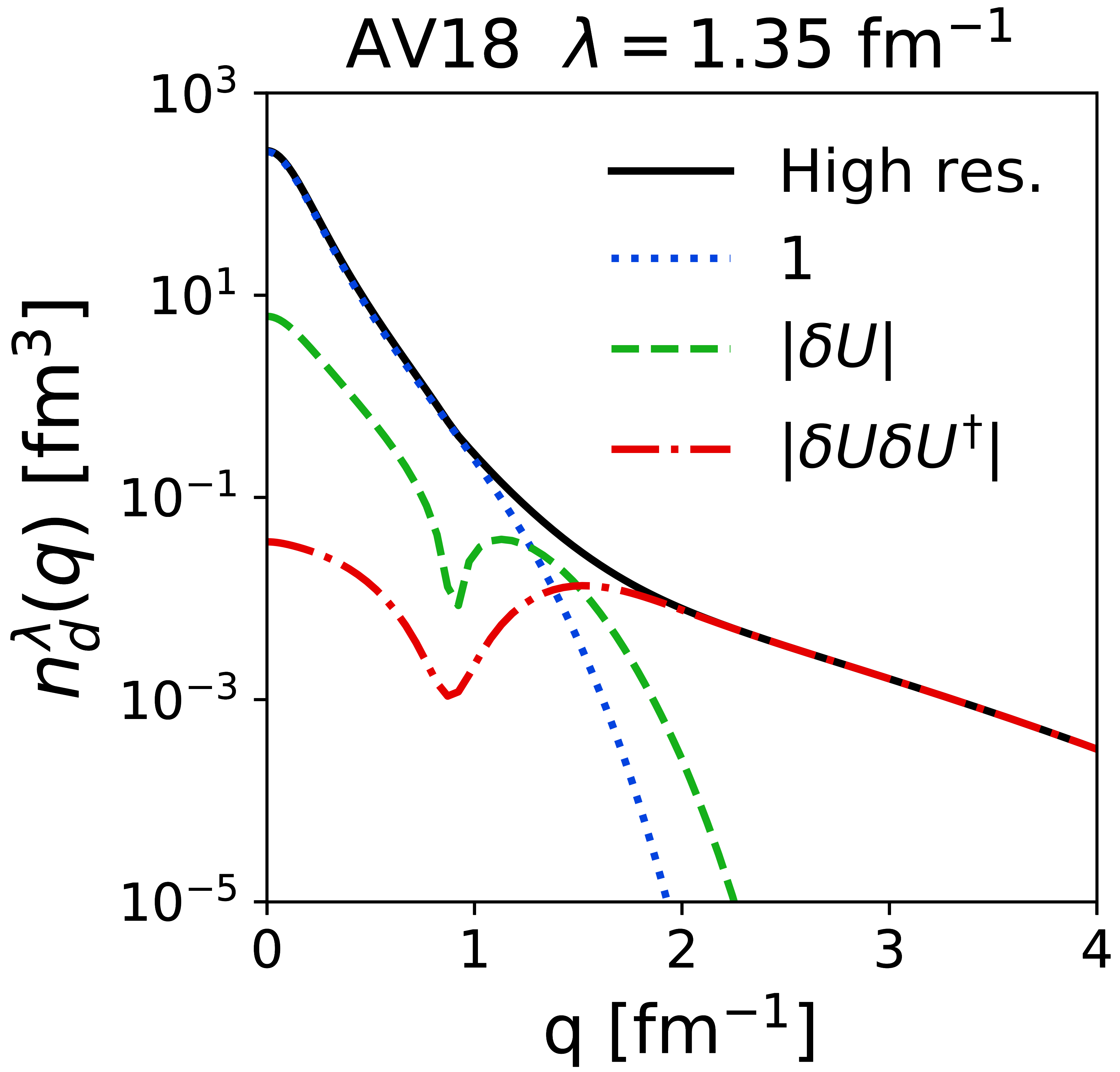
$$\langle D^{\mathrm{lo}} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | D^{\mathrm{lo}} \rangle$$

$$\langle D^{\mathrm{lo}} | \delta U a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \delta U^{\dagger} | D^{\mathrm{lo}} \rangle$$

Computing SRC operators at low-RG resolutions

Deuteron illustration

$$\hat{n}^{\mathrm{lo}}(\mathbf{q}) = \left(\hat{1} + \delta U_\lambda^{(2)}\right) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \left(\hat{1} + \delta U_\lambda^{\dagger(2)}\right)$$



$$\langle D^{\mathrm{hi}} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | D^{\mathrm{hi}} \rangle$$

$$\langle D^{\mathrm{lo}} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | D^{\mathrm{lo}} \rangle$$

$$\langle D^{\mathrm{lo}} | \delta U a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \delta U^\dagger | D^{\mathrm{lo}} \rangle$$

$$\langle D^{\mathrm{lo}} | \delta U a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \delta U^\dagger | D^{\mathrm{lo}} \rangle$$

Computing SRC operators at low-RG resolutions

Consider $\mathbf{q} \gg \lambda$

momenta $\gg \lambda$
absent in $|A^{\text{lo}}\rangle$

$$n^{\text{lo}}(\mathbf{q}) \approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_{\lambda}(\mathbf{k}, \mathbf{q} - \mathbf{K}/2)\, \delta U^{\dagger}_{\lambda}(\mathbf{q} - \mathbf{K}/2, \mathbf{k}')\, a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}}\, a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}}\, a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}\, a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

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Expectation value in $|A^{\mathrm{lo}}\rangle$ ==> only "soft" $\mathbf{K}, \mathbf{k}', \mathbf{k} \lesssim \lambda$ contribute

$$\approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q})\, \delta U_\lambda^\dagger(\mathbf{q}, \mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

$K \ll q$

Computing SRC operators at low-RG resolutions

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Scale separation
$k, k' \ll q$

$$\delta U_\lambda(k, q) \approx F^{\mathrm{lo}}_\lambda(k) F^{\mathrm{hi}}_\lambda(q)$$

Computing SRC operators at low-RG resolutions

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$K \ll q$

$$\approx (F^{\mathrm{hi}}(q))^2 \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\lambda} F^{\mathrm{lo}}(\mathbf{k}) F^{\mathrm{lo}}(\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Computing SRC operators at low-RG resolutions

FRIB

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Leading-order
Operator Product Expansion

smeared local operator
low-k physics
A-dependence in ME's

$$\approx (F^{\mathrm{hi}}(q))^2 \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\lambda} F^{\mathrm{lo}}(\mathbf{k}) F^{\mathrm{lo}}(\mathbf{k}')\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

Universal (A-indep)
Wilson Coeff, fixed by A=2
depends on operator

Computing SRC operators at low-RG resolutions

Similar factorized forms for other SRC operators

$$\hat{\rho}^{\mathrm{hi}}(\mathbf{q},\mathbf{Q}) = a^{\dagger}_{\frac{\mathbf{Q}}{2}+\mathbf{q}} a^{\dagger}_{\frac{\mathbf{Q}}{2}-\mathbf{q}} a_{\frac{\mathbf{Q}}{2}-\mathbf{q}} a_{\frac{\mathbf{Q}}{2}+\mathbf{q}} \xrightarrow{\mathbf{q} \gg \lambda} \approx (F^{\mathrm{hi}}(q))^2 \sum_{\mathbf{k},\mathbf{k}'}^{\lambda} F^{\mathrm{lo}}(\mathbf{k}) F^{\mathrm{lo}}(\mathbf{k}')\, a^{\dagger}_{\frac{\mathbf{Q}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{Q}}{2}-\mathbf{k}} a_{\frac{\mathbf{Q}}{2}-\mathbf{k}'} a_{\frac{\mathbf{Q}}{2}+\mathbf{k}'}$$

Computing SRC operators at low-RG resolutions

FRIB

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Scaling of high-q tails

$$\frac{\langle A^{\rm hi} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | A^{\rm hi} \rangle}{\langle D^{\rm hi} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | D^{\rm hi} \rangle} \approx \cancel{\frac{|F^{\rm hi}(q)|^2}{|F^{\rm hi}(q)|^2}} \times \frac{\sum^{\lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} \langle A^{\rm lo} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | A^{\rm lo} \rangle}{\sum^{\lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} \langle D^{\rm lo} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | D^{\rm lo} \rangle}$$

Computing SRC operators at low-RG resolutions

FRIB

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$$F^{\text{hi}}(\mathbf{q}) \propto \Psi^{\text{hi}}_{A=2}(\mathbf{q})$$

ratio of (smeared) contacts
only sensitive to low-k/mean-field physics
approx. independent of resolution scale

(see GCF talks of Diego/Ronan)

Computing SRC operators at low-RG resolutions

RG-evolved SRC operators

links few- and A-body systems (Operator Product Expansion)

RG "derivation" of the GCF

Correlations/scaling for 2 observables w/same leading OPE

Subleading OPE ==> deviations from scaling calculable in principle?

(see GCF talks of Diego/Ronan)

Options for treating wf's at low-RG resolutions

All the hard q physcis factorized in A-indep Wilson Coeffs

SRC calculations amount to computing matrix elements of

$$\sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\lambda} F^{\mathrm{lo}}(\mathbf{k}) F^{\mathrm{lo}}(\mathbf{k}') \left\langle A^{\mathrm{lo}} \middle| a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} \middle| A^{\mathrm{lo}} \right\rangle$$

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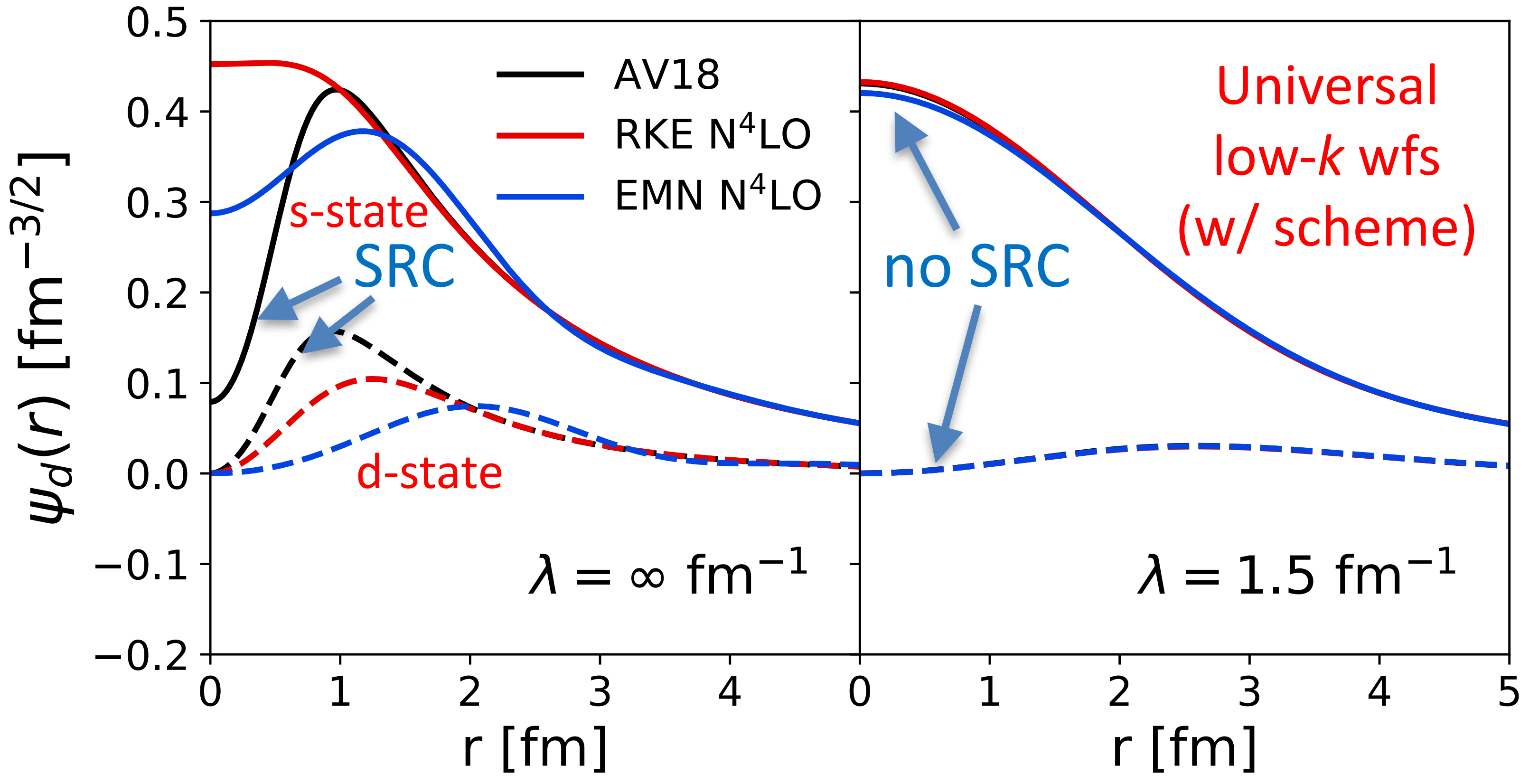
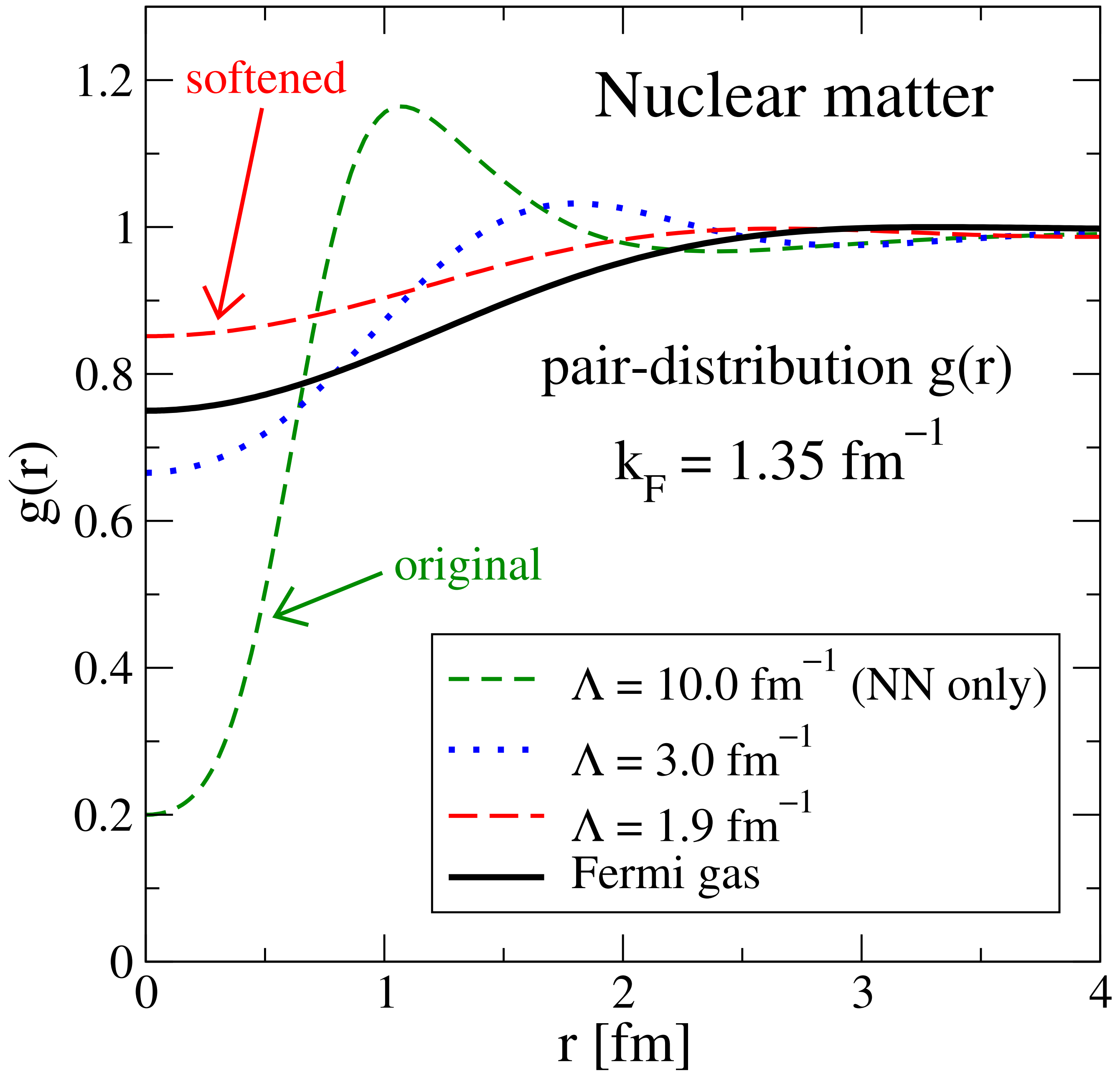
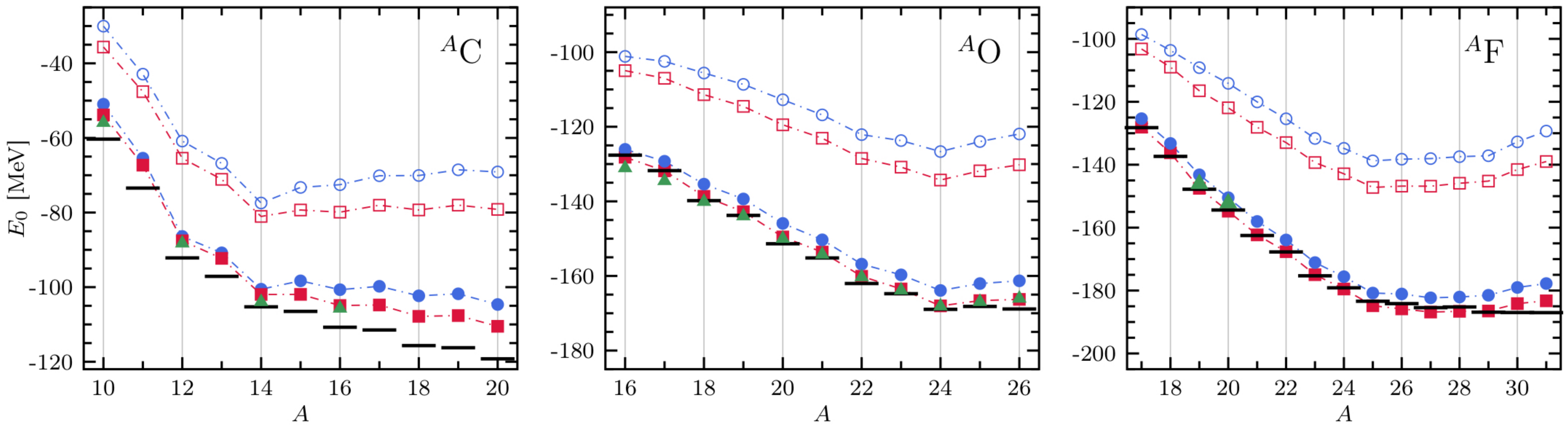


no explicit SRCs at low resolution



Options for treating wf's at low-RG resolutions



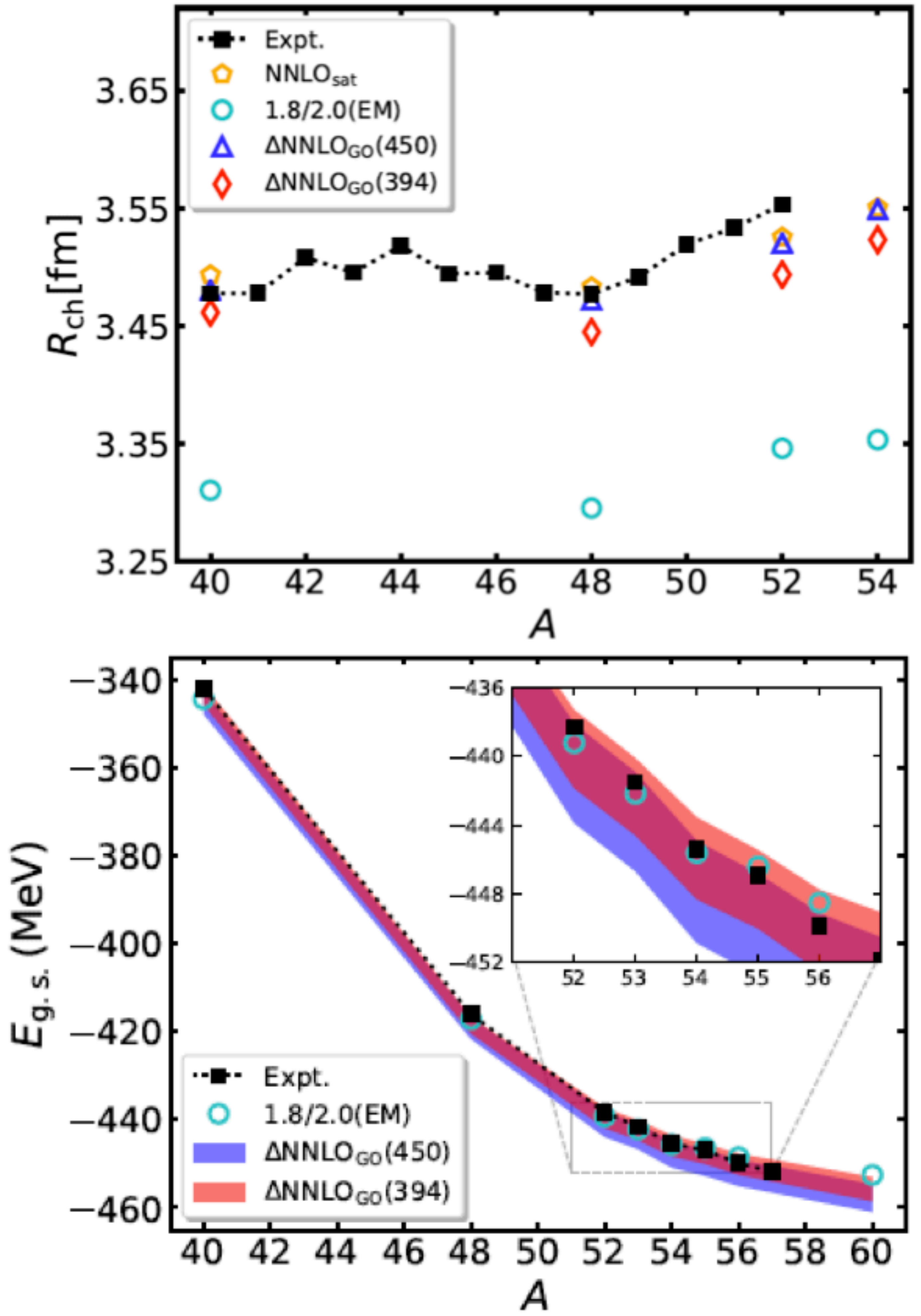
Tichai et al., Frontiers in Physics (2021)

Figure 7. Reference (○/□) and second-order NCSM-PT (●/■) energies with $N^{\mathrm{ref}}_{\mathrm{max}} = 0$ and 2, respectively, for the ground states of $^{11-20}$C, $^{16-26}$O and $^{17-31}$F using the Hamiltonian described in Sec. 3. All calculations are performed using 13 oscillator shells and an oscillator frequency of $\hbar\omega = 20\,\mathrm{MeV}$. The SRG parameter is set to $\alpha = 0.08\,\mathrm{fm}^4$. Importance-truncated NCSM calculations (▲) are shown for comparison. Experimental values are indicated by black bars. Figure taken from Ref. [36].

Simple methods "work"

- MBPT
- shell model
- polynomially scaling methods (IMSRG, CC, SCGF, etc.)

Options for treating wf’s at low-RG resolutions



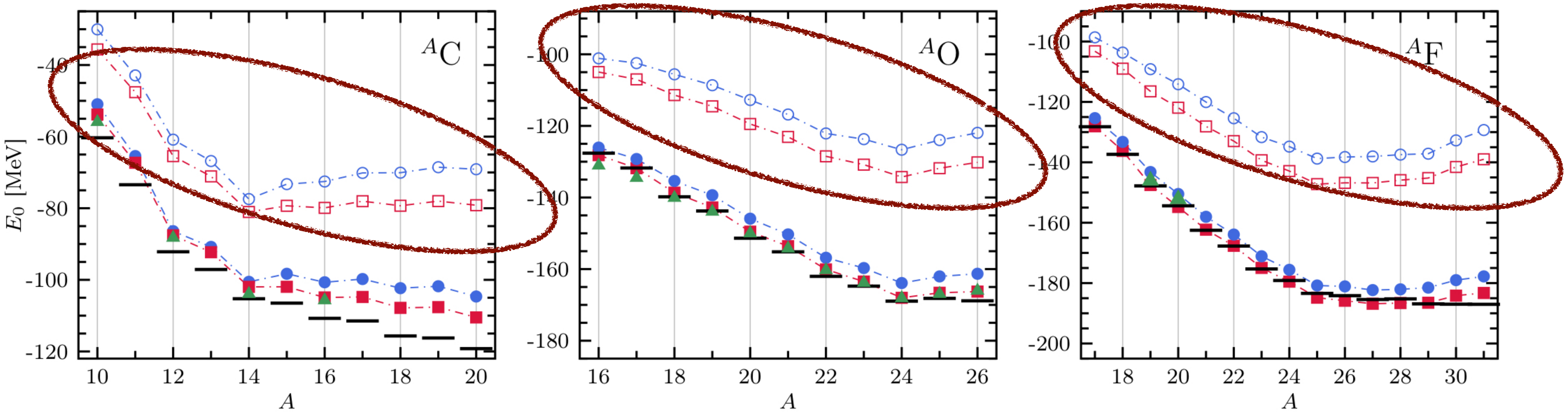
Charge radii (top) and ground-state energies (bottom) of calcium isotopes with A nucleons computed with new potentials ΔNNLO$_{GO}$.

Ongoing developments:

“soft” interactions w/good saturation properties in medium mass

e.g., ΔNNLO$_{GO}$ chiral EFT (with Δ’s)

Options for treating wf's at low-RG resolutions



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Need beyond HF for precision energetics/radii

Can we use HF for SRC studies at low resolution?

Or HF treated in LDA? Let's find out!...

Options for treating wf's at low-RG resolutions

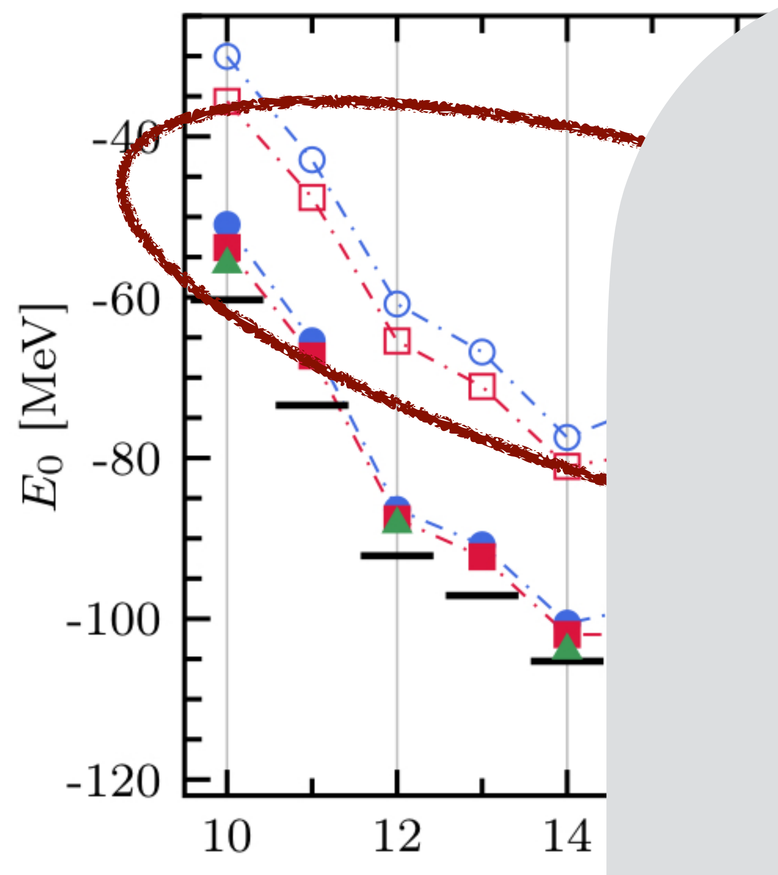


Figure 7. Referen
respectively, for the
All calculations are
The SRG paramete
comparison. Exper

Strategy for SRC calcs. at low-RG scales $\lambda \ll q$

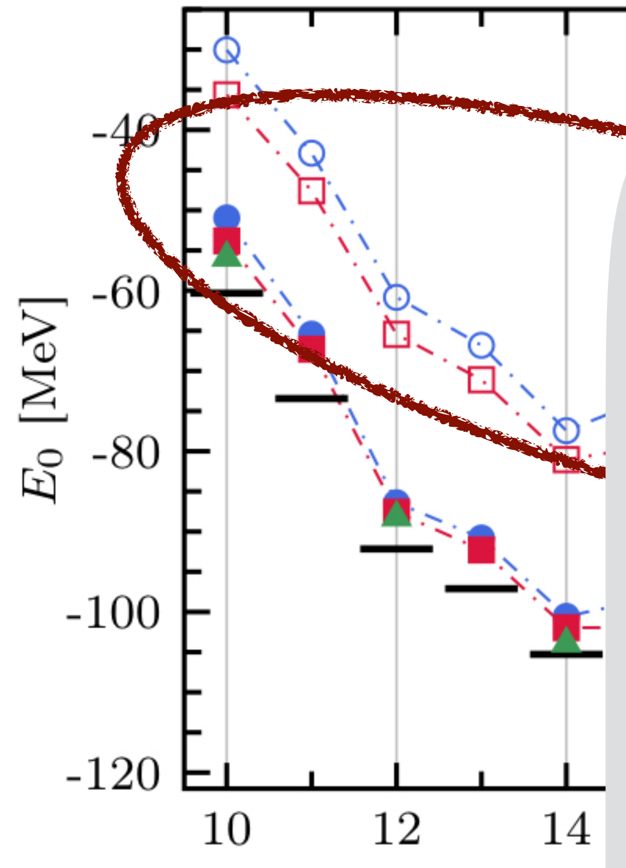
$$\widehat{n}^{\mathrm{lo}}(\mathbf{q}) \approx \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \delta U_\lambda(\mathbf{k}, \mathbf{q}-\mathbf{K}/2)\, \delta U^\dagger_\lambda(\mathbf{k}', \mathbf{q}-\mathbf{K}/2)\, a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}\, a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}\, a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}\, a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}$$

adii

lution?

out!...

Options for treating wf's at low-RG resolutions



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fixed from A=2

What SRC phenomenology can this (ridiculously) simple approach reproduce?

evaluate matrix elements in A-body states using LDA (free fermi gas)

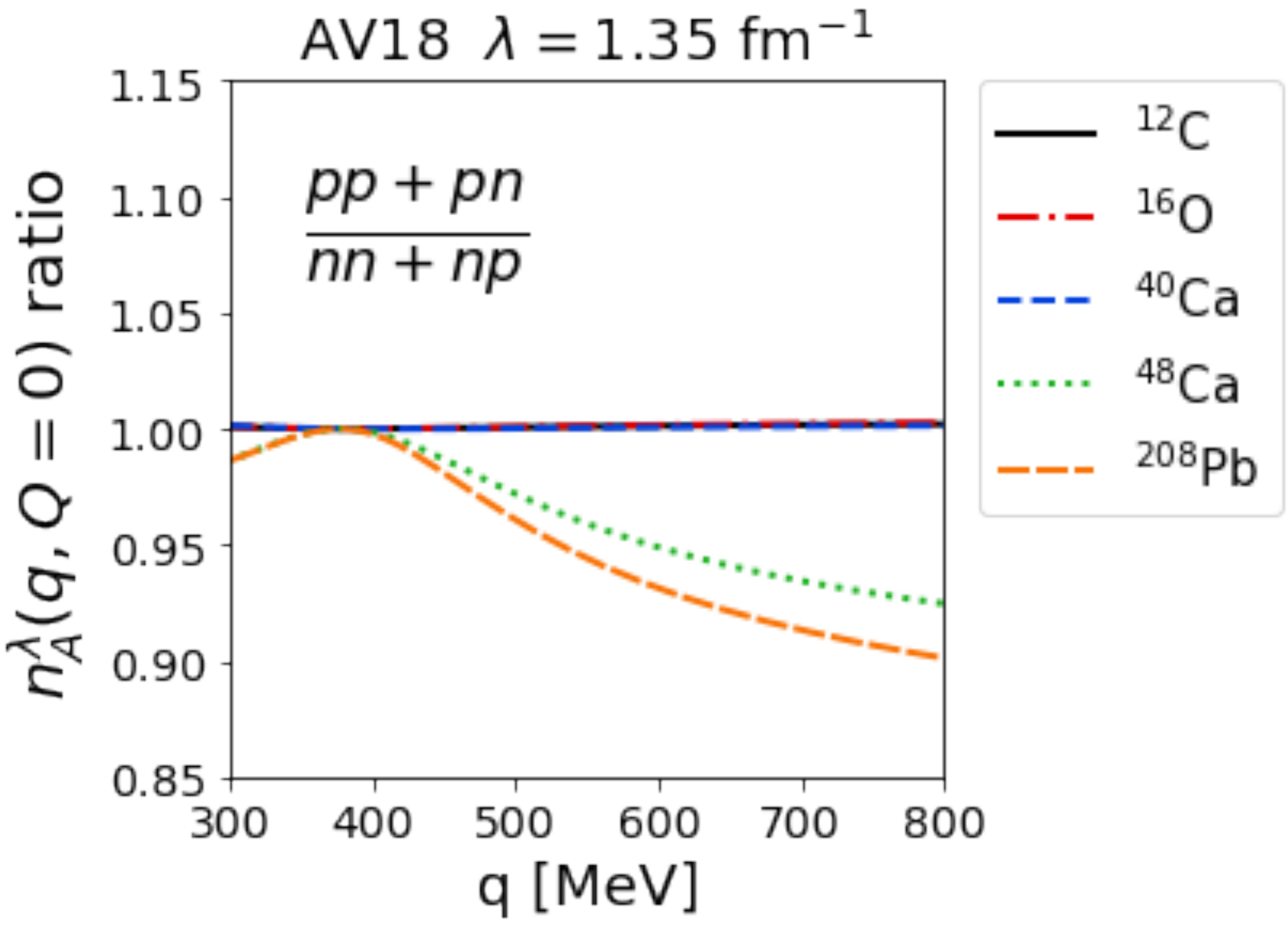
adii

lution?

out!...

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)



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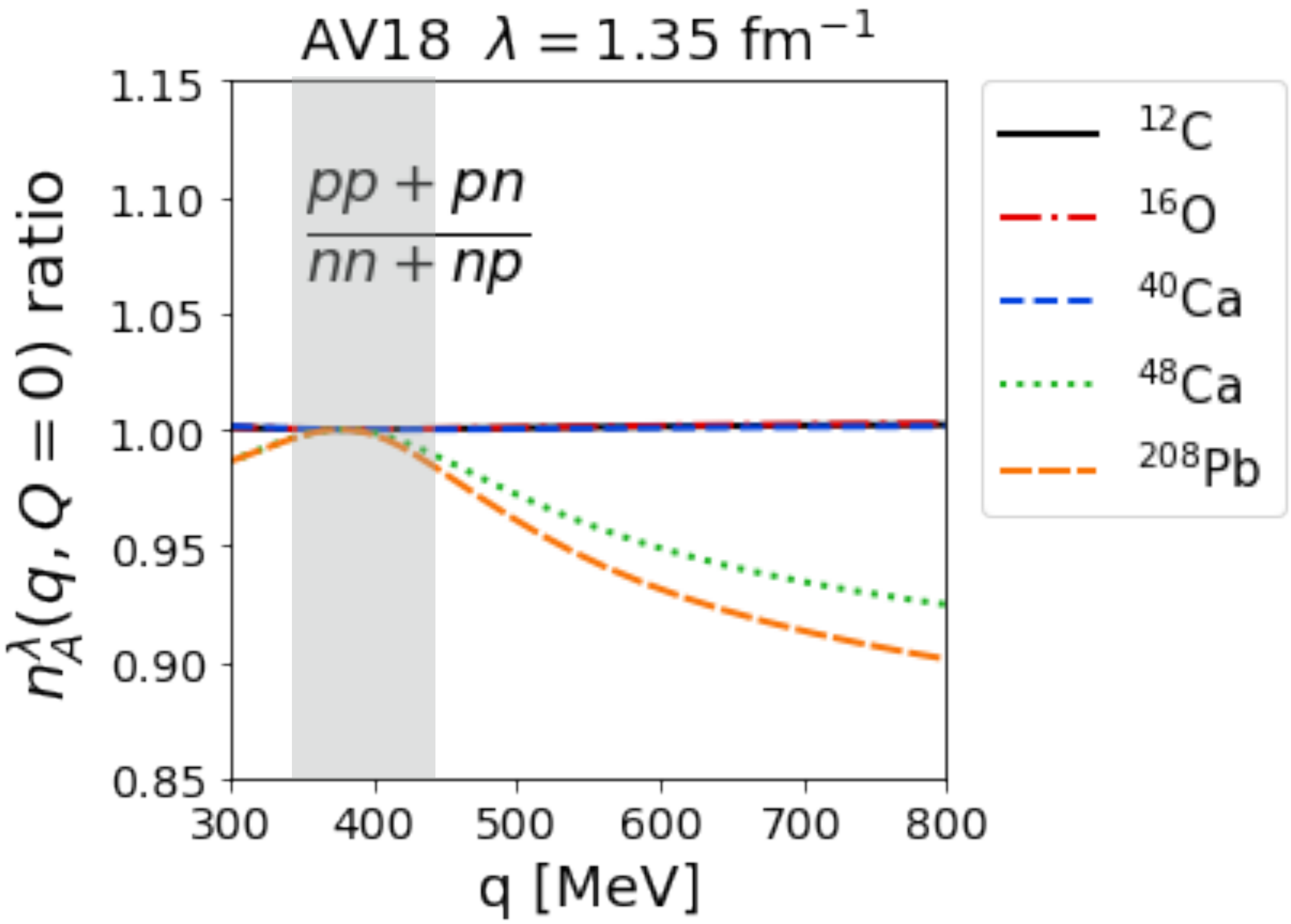

np dominance => ratio should be ~ 1 irrespective of N/Z

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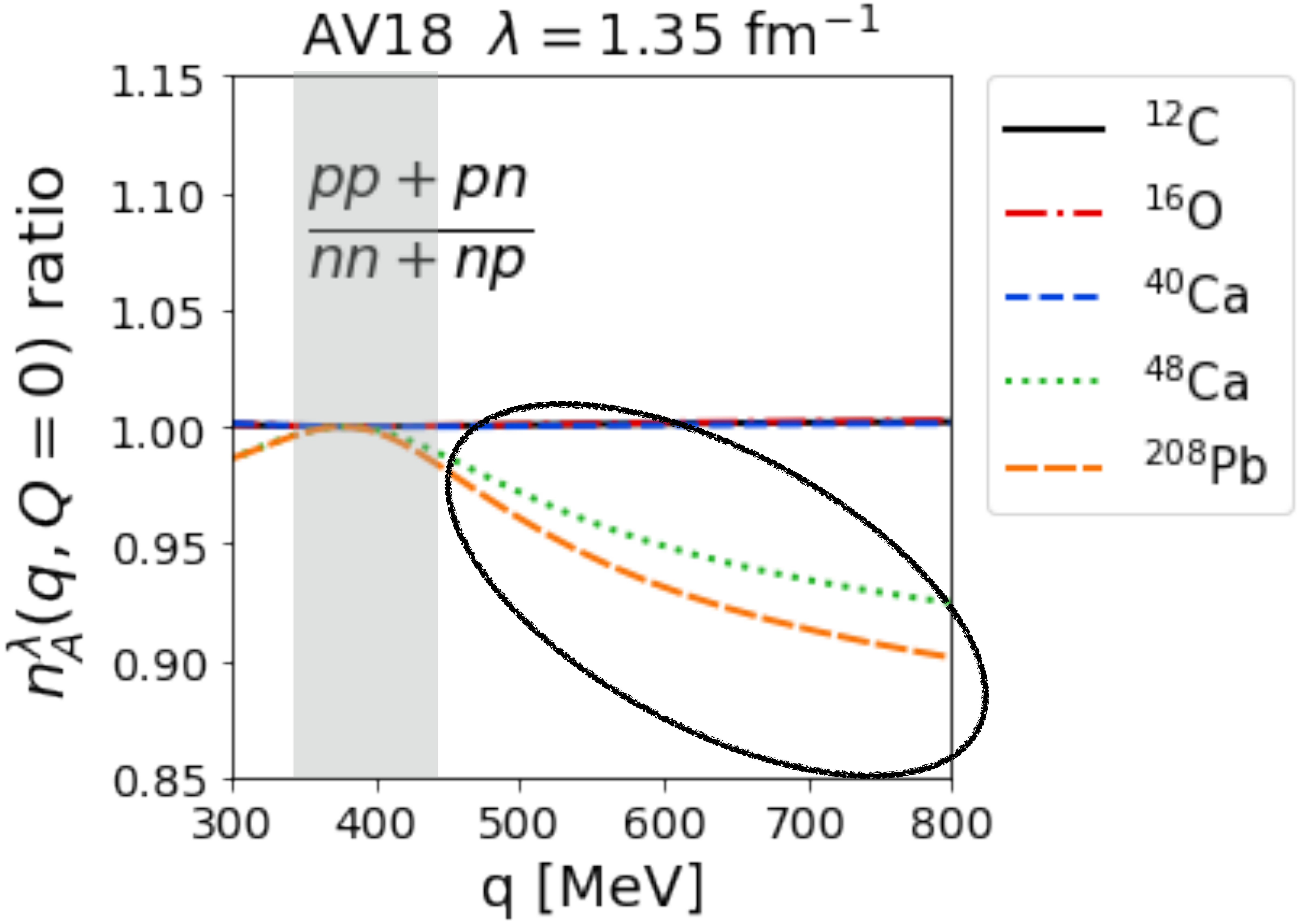
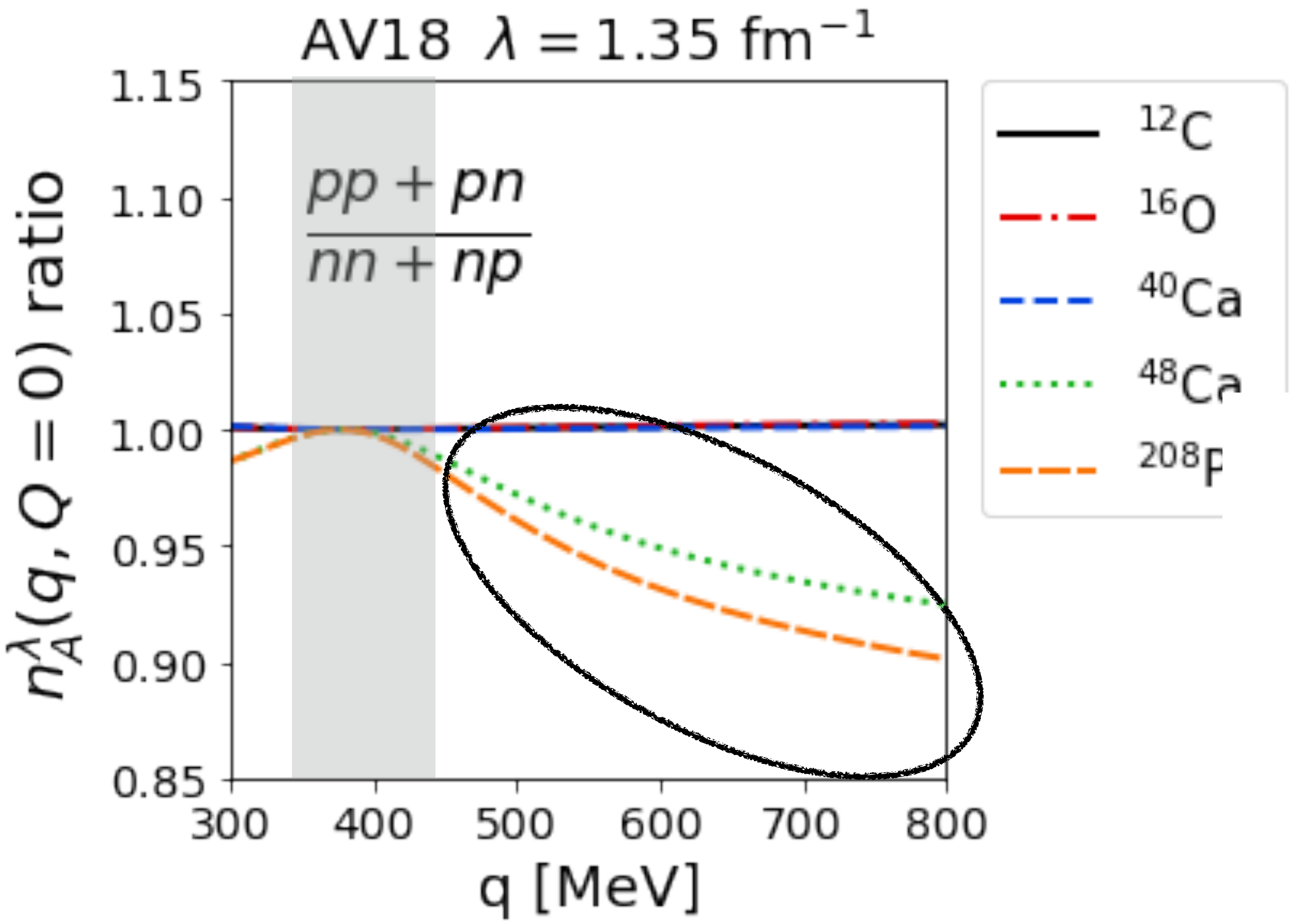


np dominance => ratio should be ~ 1 irrespective of N/Z

transition towards scalar counting at higher relative q

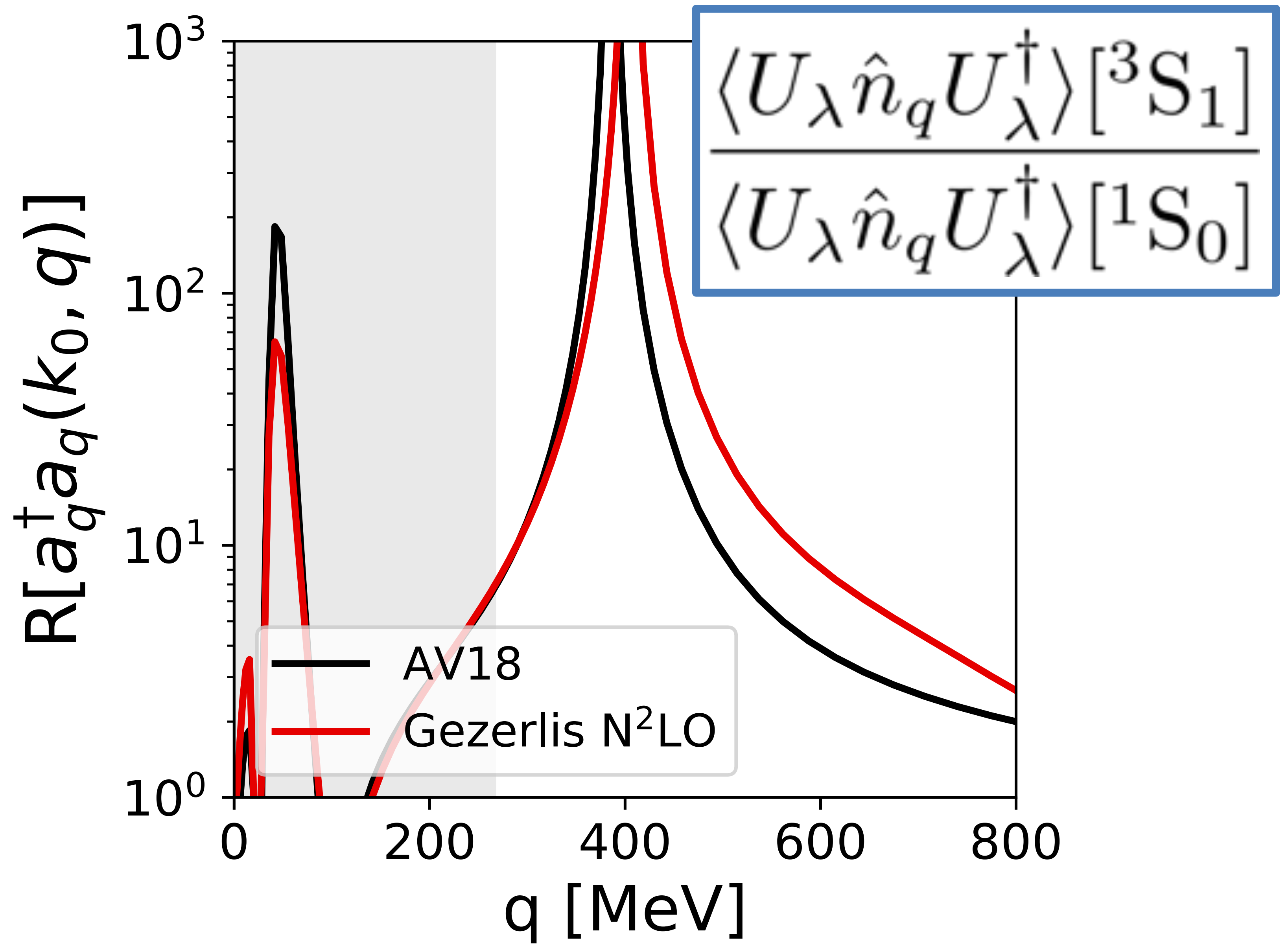
Preliminary SRC low-resolution LDA calculations



Tropiano, SKB, Furnstahl (in progress)

np dominance => ratio should be ~ 1 irrespective of N/Z

Ratio of *evolved* high-mom. distributions in a low-mom. state (insensitive to details!)



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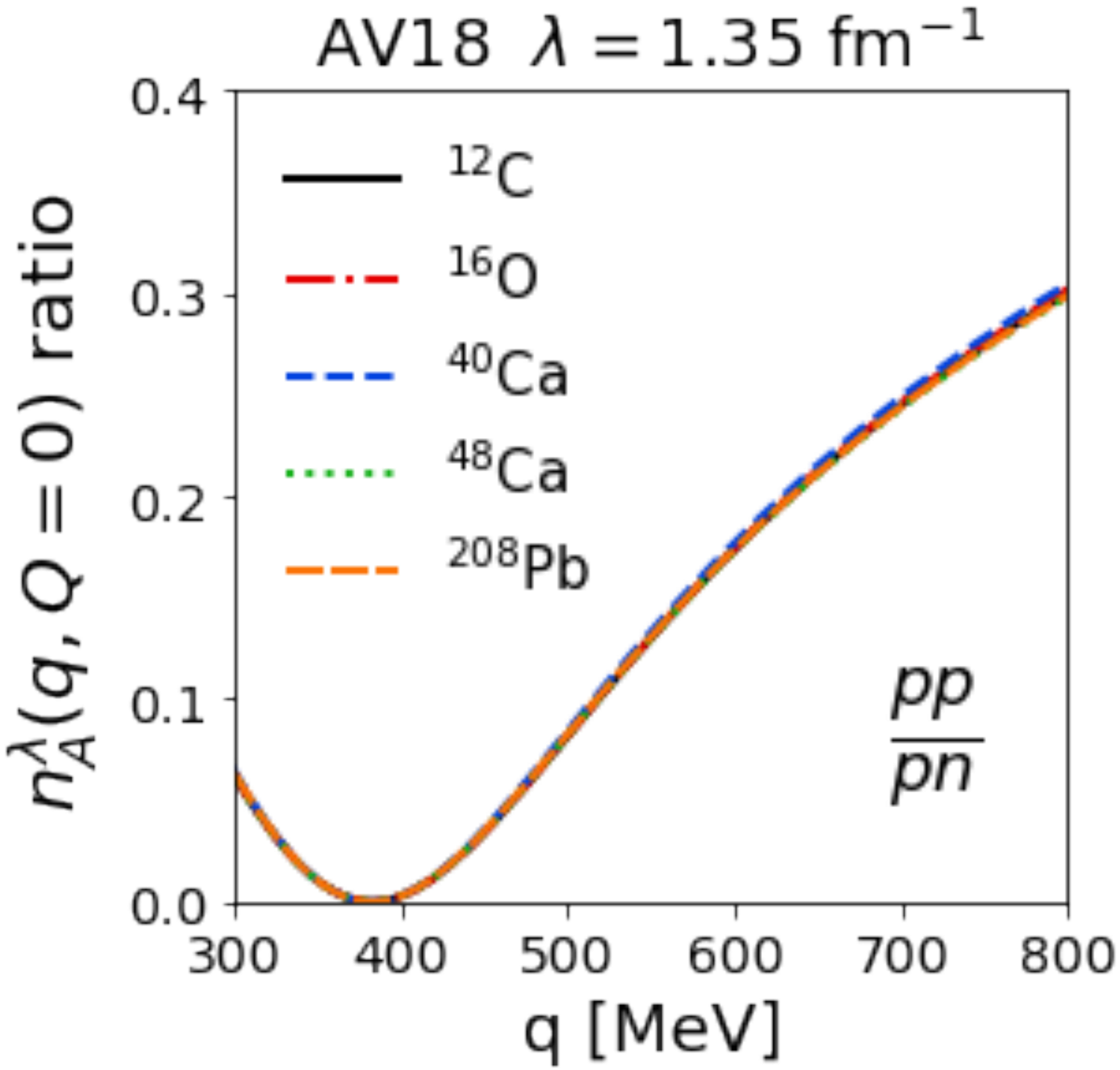

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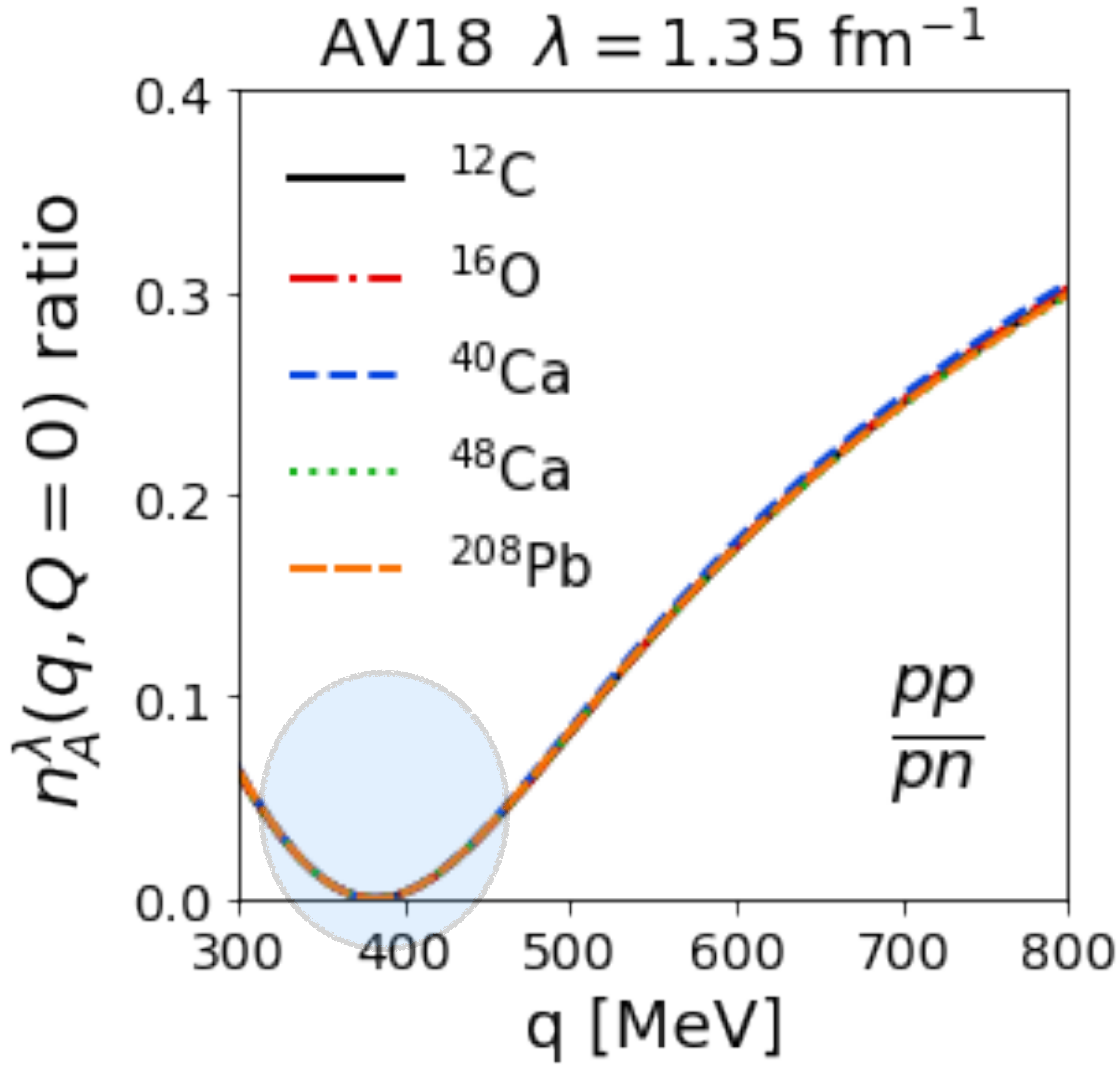

np pair (tensor force) dominance

Preliminary SRC low-resolution LDA calculations

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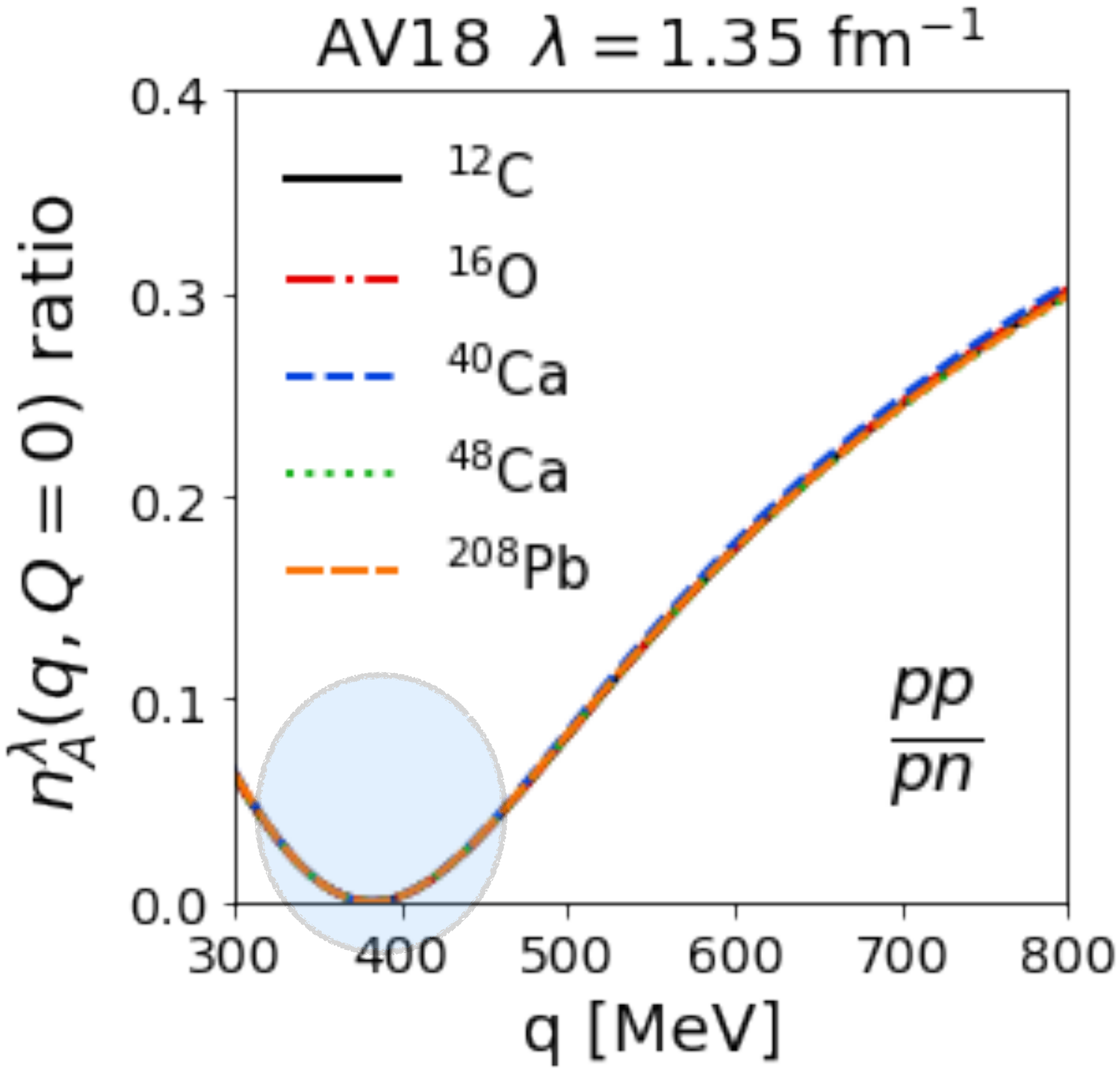

np pair (tensor force) dominance

weak nucleus dependence follows from factorization

Preliminary SRC low-resolution LDA calculations

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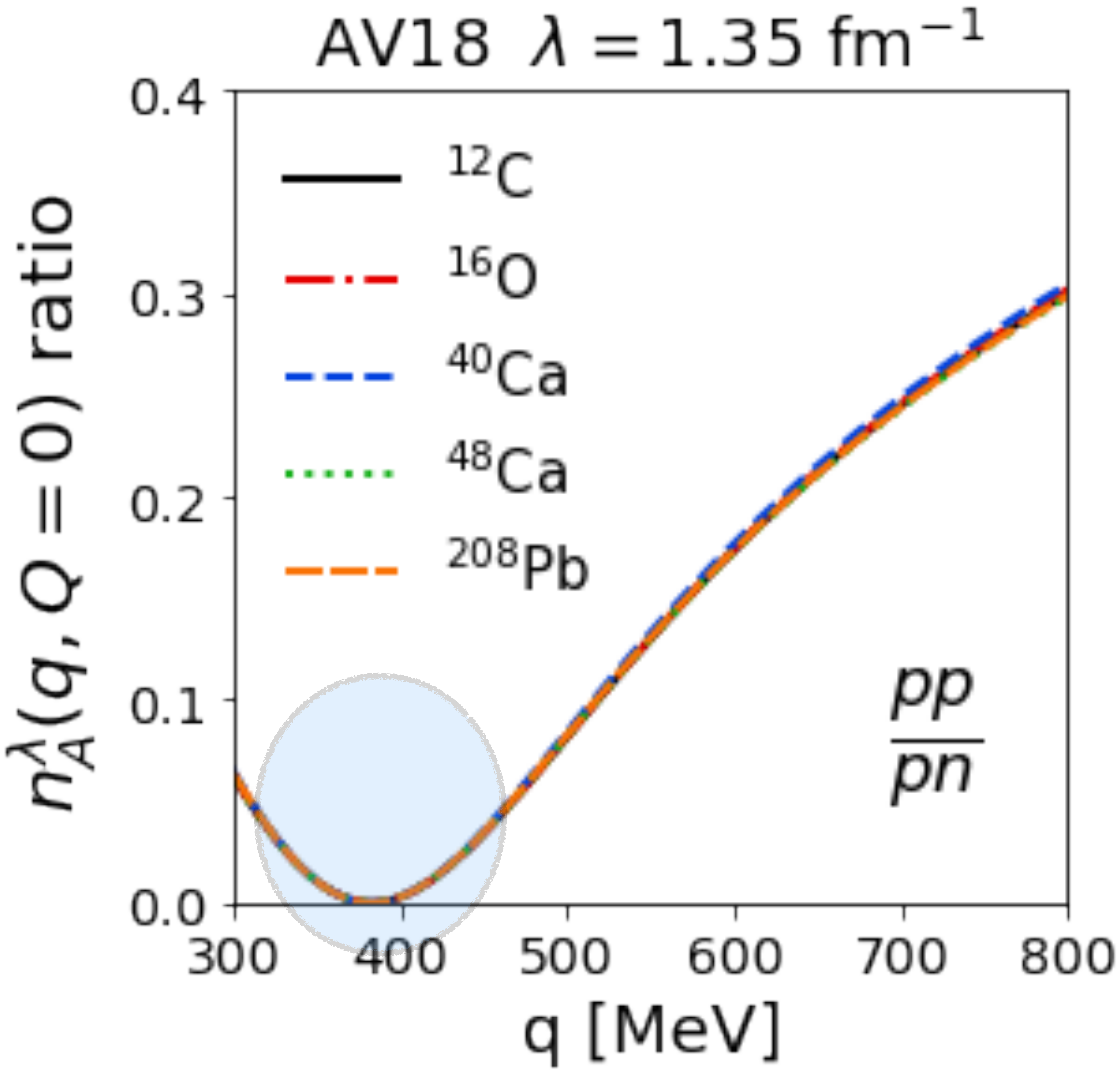


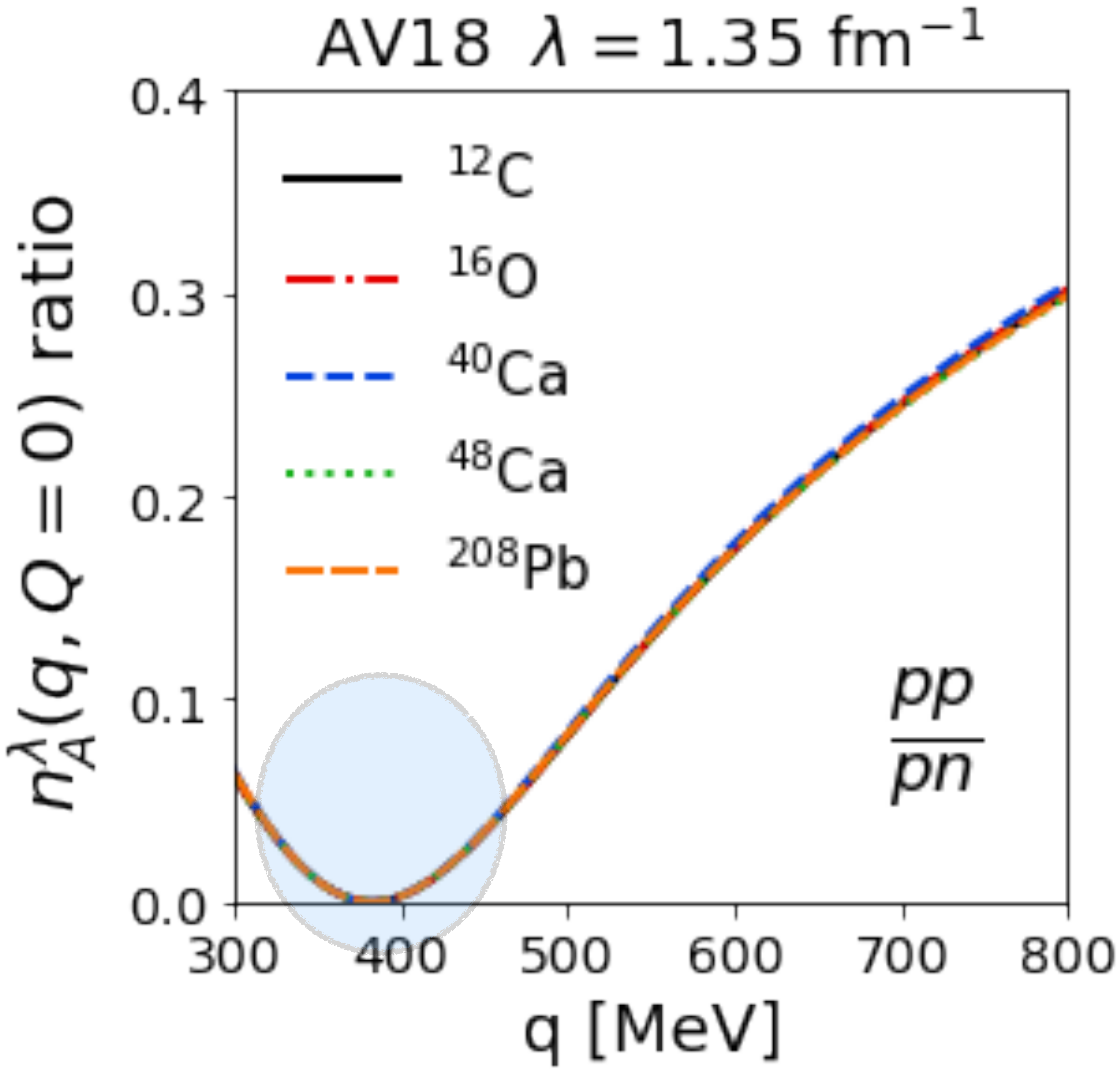
np pair (tensor force) dominance

weak nucleus dependence follows from factorization

$$\text{Ratio} \approx \frac{(F^{hi}_{pp}(q))^2 \left\langle A^{lo} \middle| \sum^{\lambda}_{\mathbf{k},\mathbf{k}'} a^\dagger_{\frac{\mathbf{Q}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{Q}}{2}-\mathbf{k}} a_{\frac{\mathbf{Q}}{2}-\mathbf{k}'} a_{\frac{\mathbf{Q}}{2}+\mathbf{k}'} \middle| A^{lo} \right\rangle}{(F^{hi}_{np}(q))^2 \left\langle A^{lo} \middle| \sum^{\lambda}_{\mathbf{k},\mathbf{k}'} a^\dagger_{\frac{\mathbf{Q}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{Q}}{2}-\mathbf{k}} a_{\frac{\mathbf{Q}}{2}-\mathbf{k}'} a_{\frac{\mathbf{Q}}{2}+\mathbf{k}'} \middle| A^{lo} \right\rangle}$$

Preliminary SRC low-resolution LDA calculations

Tropiano, SKB, Furnstahl (in progress)



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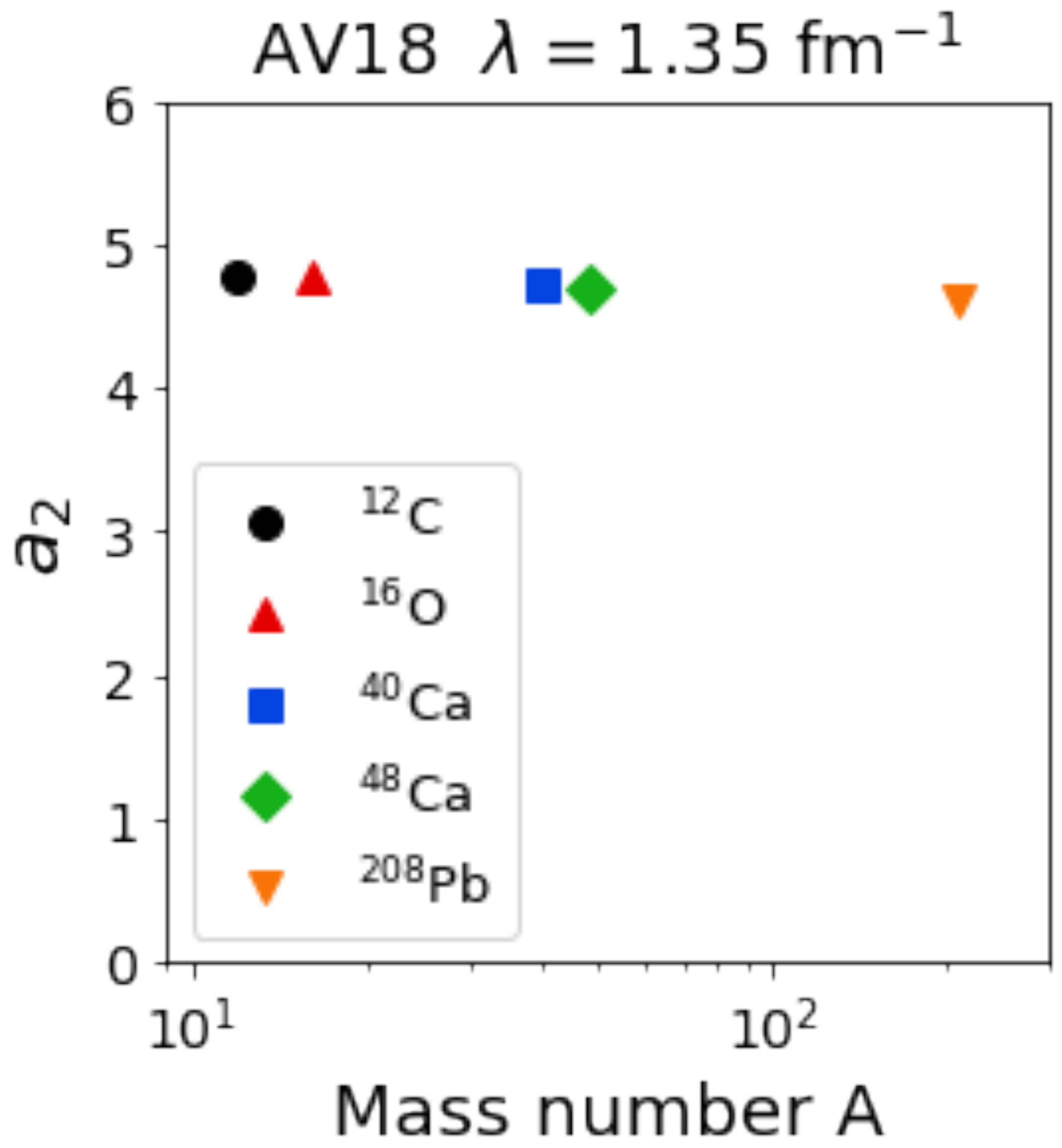
Preliminary SRC low-resolution LDA calculations



Tropiano, SKB, Furnstahl (in progress)

Followed Ryckebusch et al. prescription

$$a_2(A) = \lim_{\text{high } p} \frac{P^A(p)}{P^d(p)} \approx \frac{\int_{\Delta p^{\text{high}}} dp\, P^A(p)}{\int_{\Delta p^{\text{high}}} dp P^d(p)}.$$

$$\Delta p^{\text{high}} = [3.8 \ldots 4.5] \text{ fm}^{-1}$$

Decent agreement w/LCA calcs
(flatter A-dependence)

But systematics need to be explored more!

Looking ahead

Can we use low-RG scale pictures to directly compute cross sections, etc?

$$\underbrace{\langle\psi_f|}_{\text{structure}}\;\overbrace{\widehat{O}(q)}^{\text{reaction}}\;\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger \widehat{O}(q) U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\;\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\;\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

Looking ahead

FRIB

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cf deuteron electrodisintegration studies **More, SKB, Furnstahl PRC96 (2017)**

$$\frac{d}{d\mu_F}\left[\boxed{\sigma} = \boxed{\text{reaction}} \underbrace{\otimes}_{\mu_F} \boxed{\text{structure}}\right] = 0$$

Factorization is scale-dependent (not unique)!!

Looking ahead

Can we use low-RG scale pictures to directly compute cross sections, etc?

$$\underbrace{\langle\psi_f|}_{\text{structure}}\overbrace{\widehat{O}(q)}^{\text{reaction}}\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger \widehat{O}(q) U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

cf deuteron electrodisintegration studies **More, SKB, Furnstahl PRC96 (2017)**

$$\frac{d}{d\mu_F}\left[\sigma = \text{reaction}\underbrace{\otimes}_{\mu_F}\text{structure}\right] = 0$$

Factorization is scale-dependent (not unique)!!

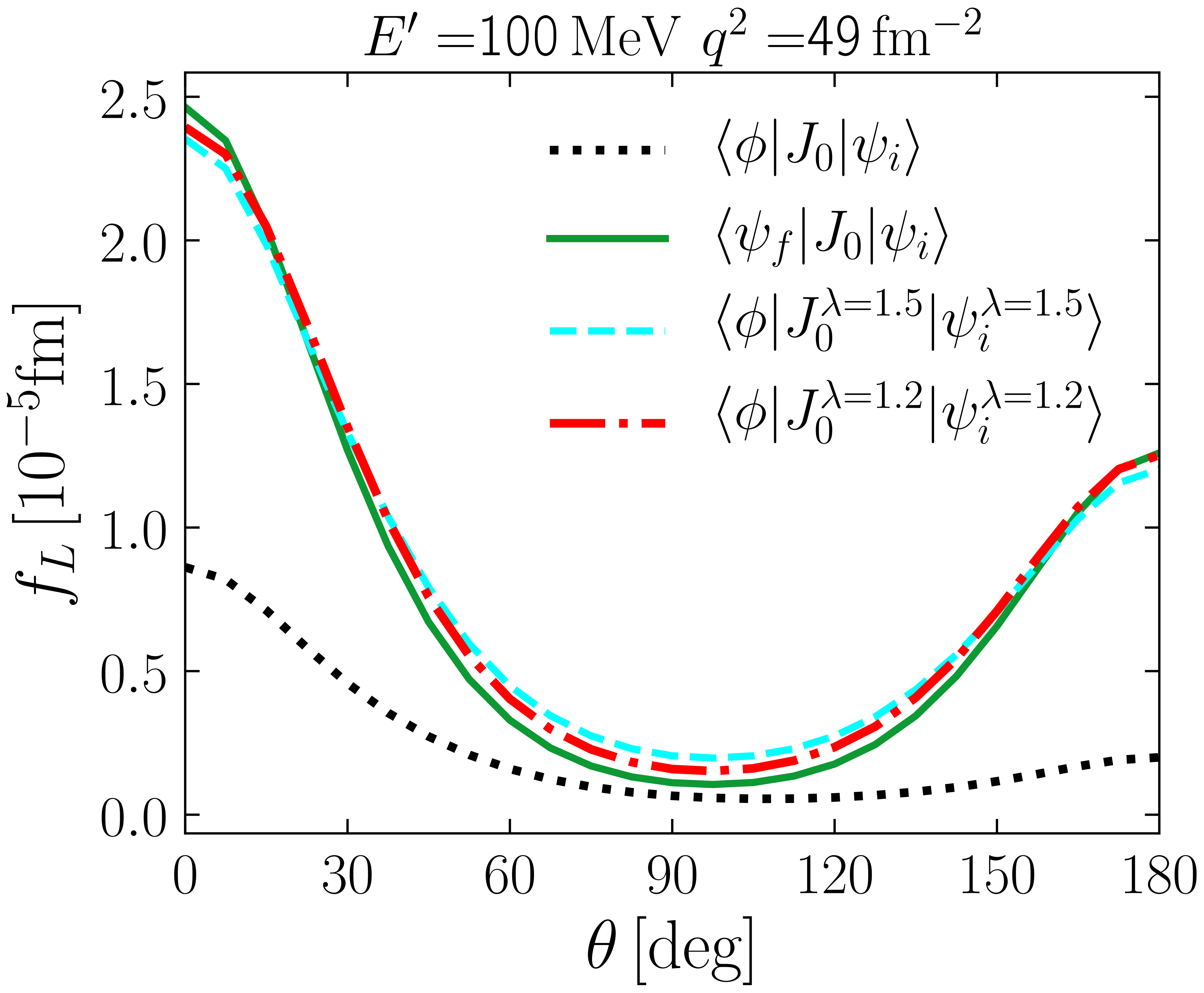
scale/scheme dependence of extracted properties? (e.g., SFs)

extract at one scale, evolve to another? (like PDFs)

how do FSIs, physical interpretations, etc. depend on RG scale?

Scale Dependence of Final State Interactions

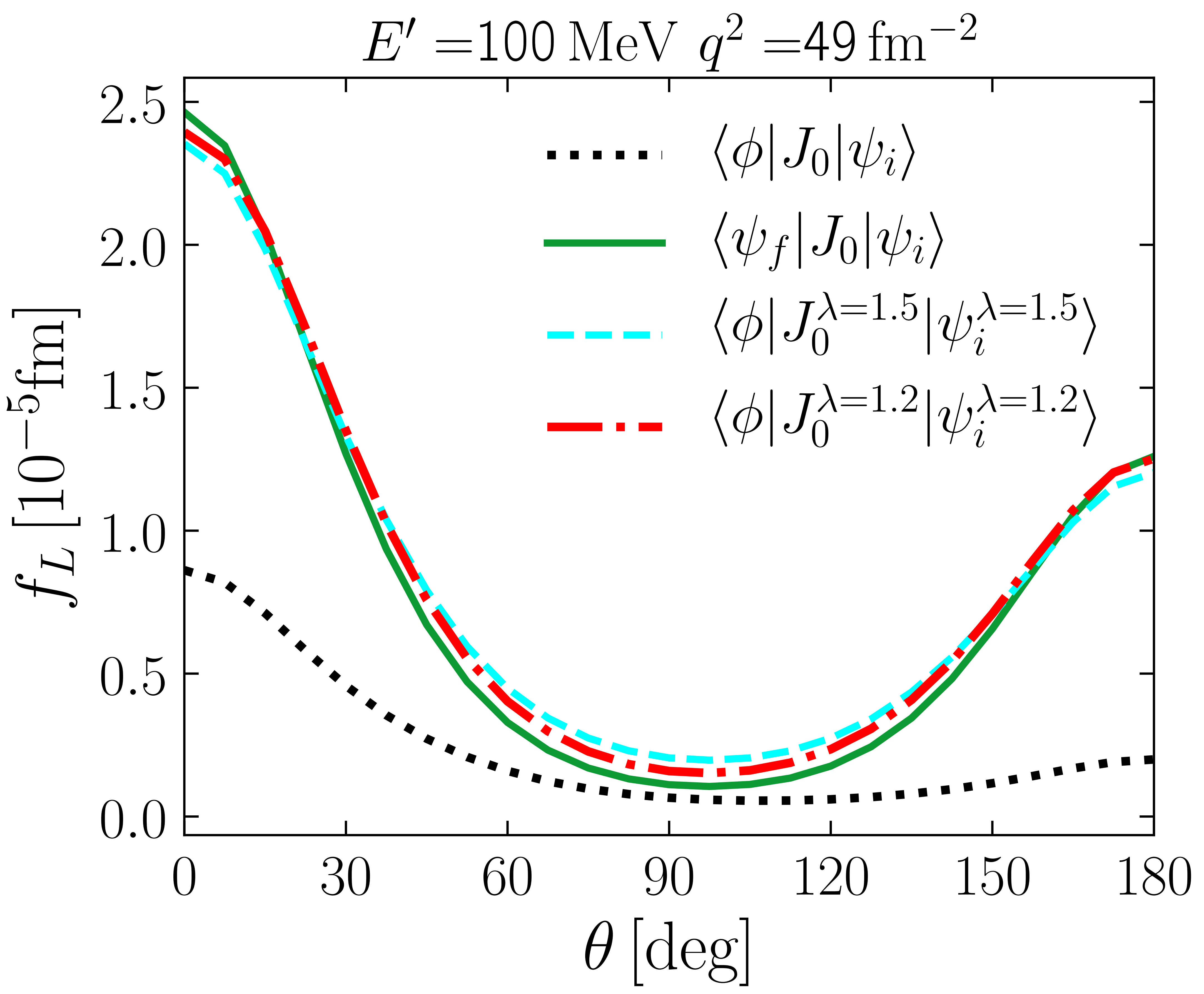
Deuteron Electrodisintegration



x_B=1.64, Q^2= 1.78 GeV^2

Scale Dependence of Final State Interactions

Deuteron Electrodisintegration



x_B=1.64, Q^2= 1.78 GeV^2

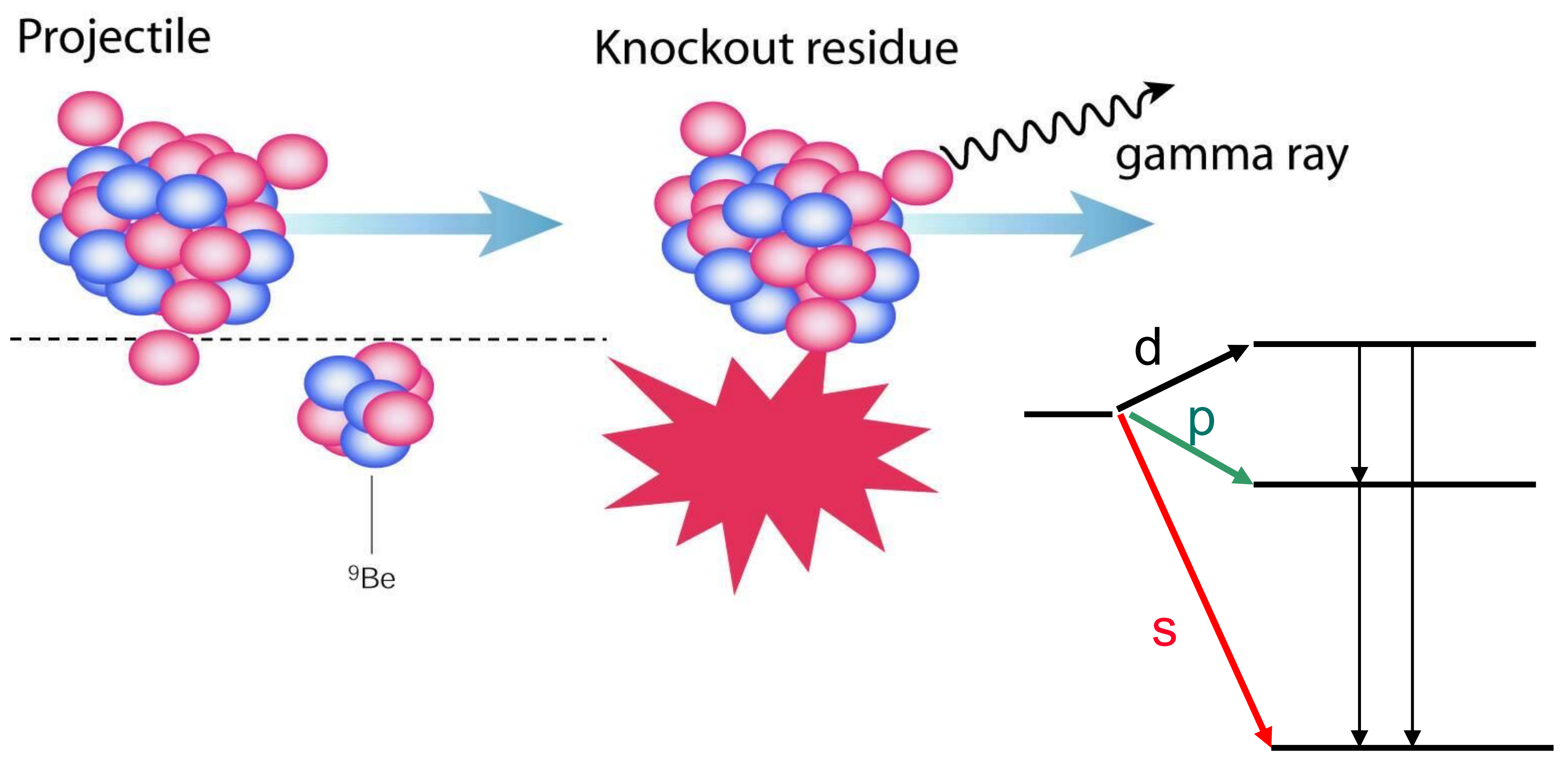
FSI sizable at large λ
but negligible at low-resolution!

Takeaway point:

Size of FSI depends on RG scale/scheme

Ditto physical interpretations

Other exclusive knock-out reactions [pictures from A. Gade]



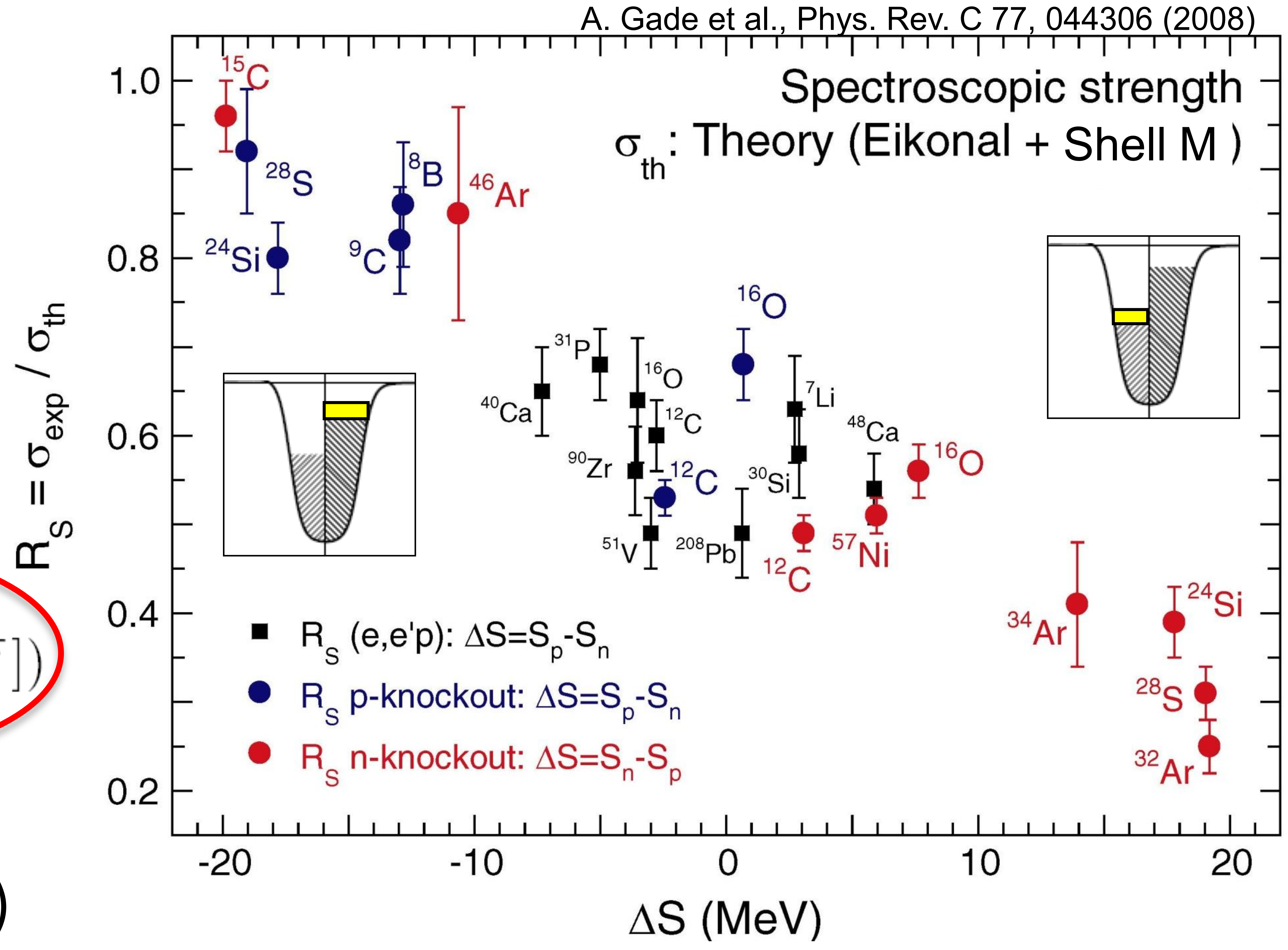
Exclusive reactions, theory vs. experiment

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2S(j^\pi)\,\sigma_{sp}(j, S_N + E_x[j^\pi])$$

Structure theory

Reaction theory

Origin and systematics of R = σ_{exp} / σ_{th} < 1 are not understood (includes e,e'p results)



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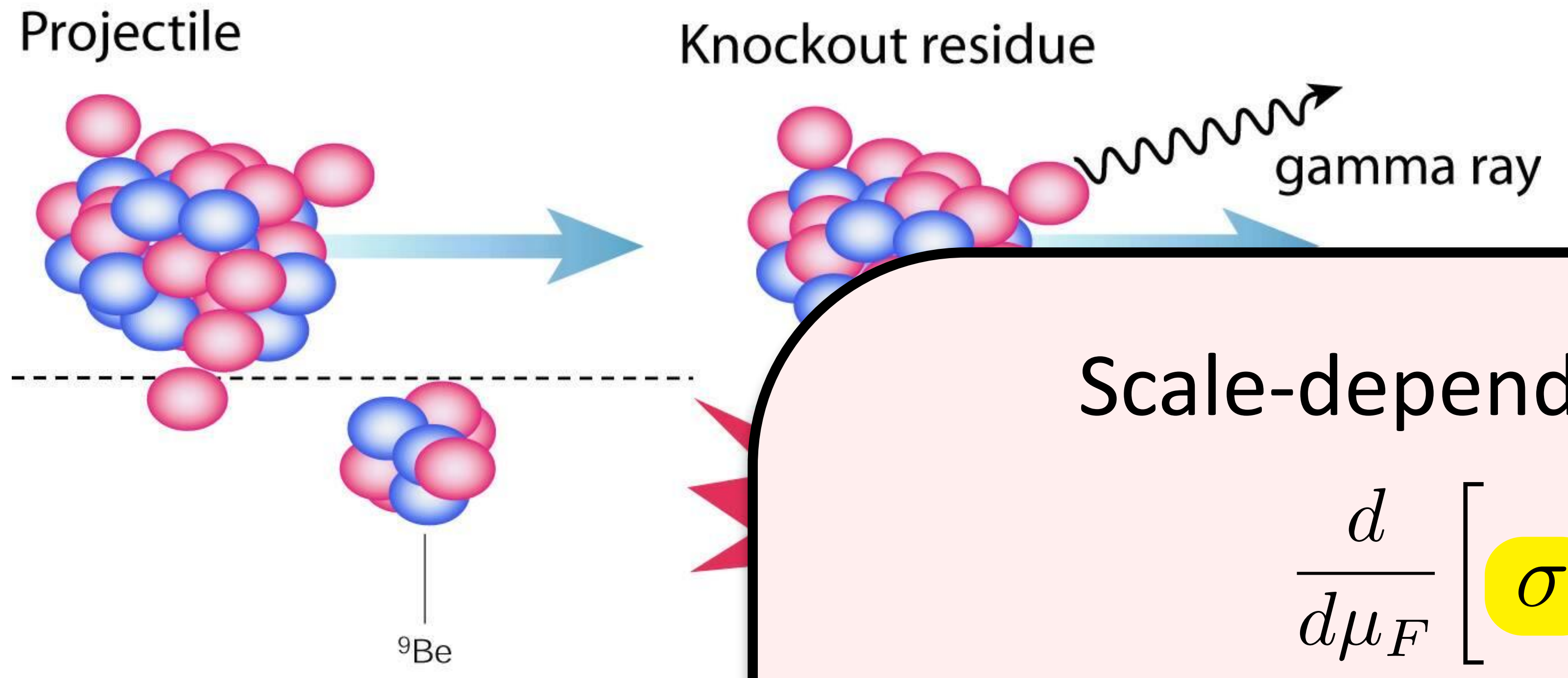

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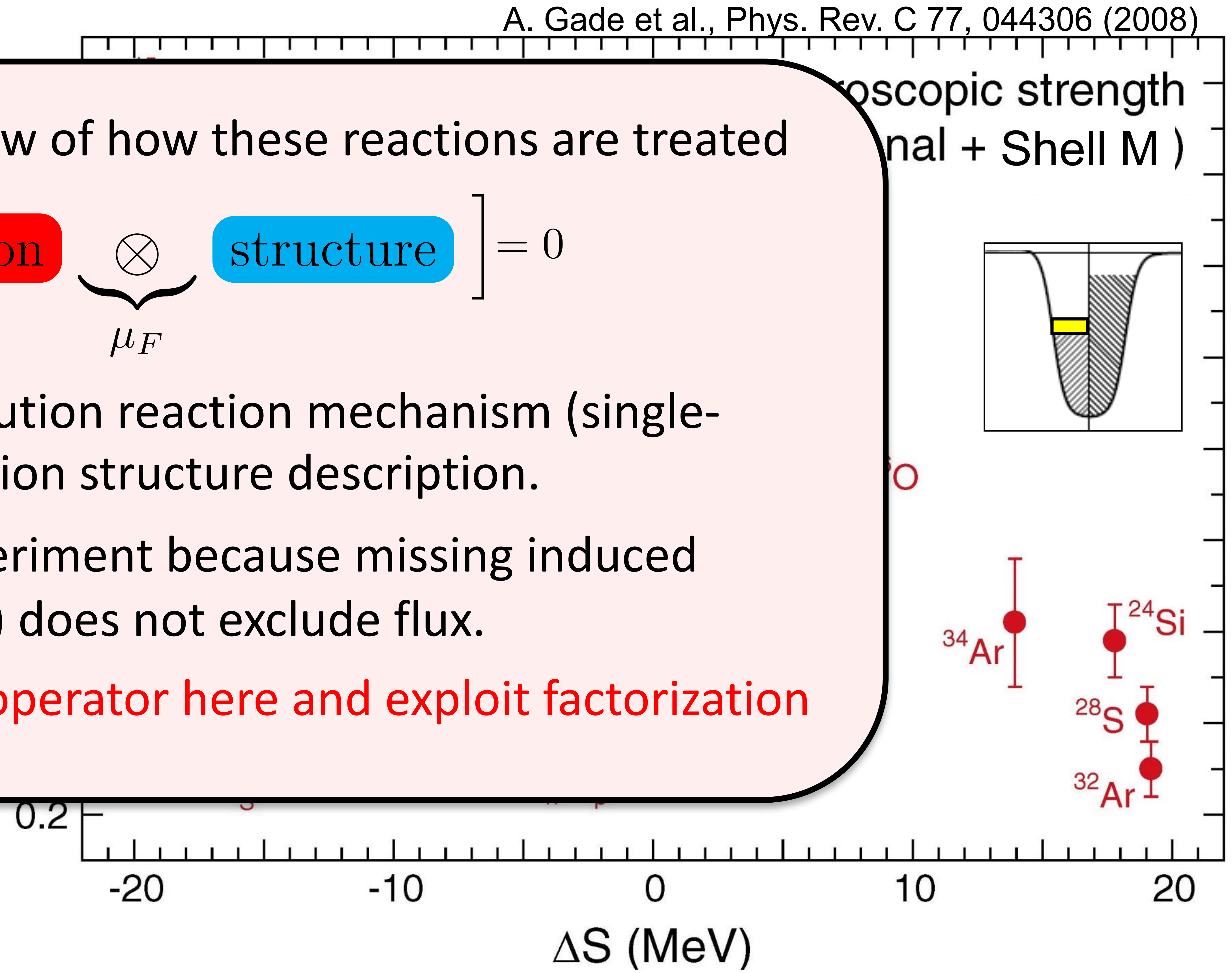


Scale-dependent (RG) view of how these reactions are treated

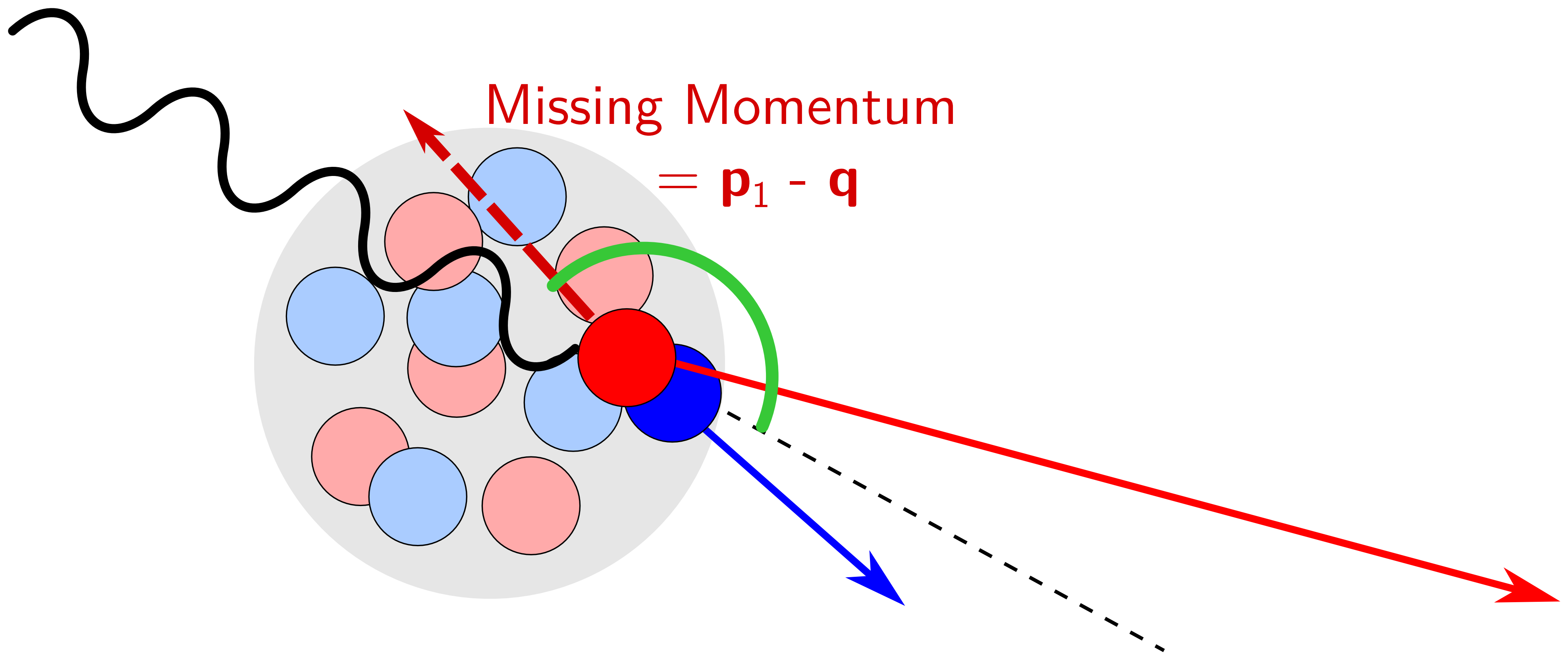
$$\frac{d}{d\mu_F}\left[\sigma = \text{reaction} \underbrace{\otimes}_{\mu_F} \text{structure}\right] = 0$$

- Analysis mixes a high-resolution reaction mechanism (single-particle) with a low-resolution structure description.
- Theory is greater than experiment because missing induced current (e.g., 2-body for *e*⁻) does not exclude flux.
- Plan: use SRG on reaction operator here and exploit factorization

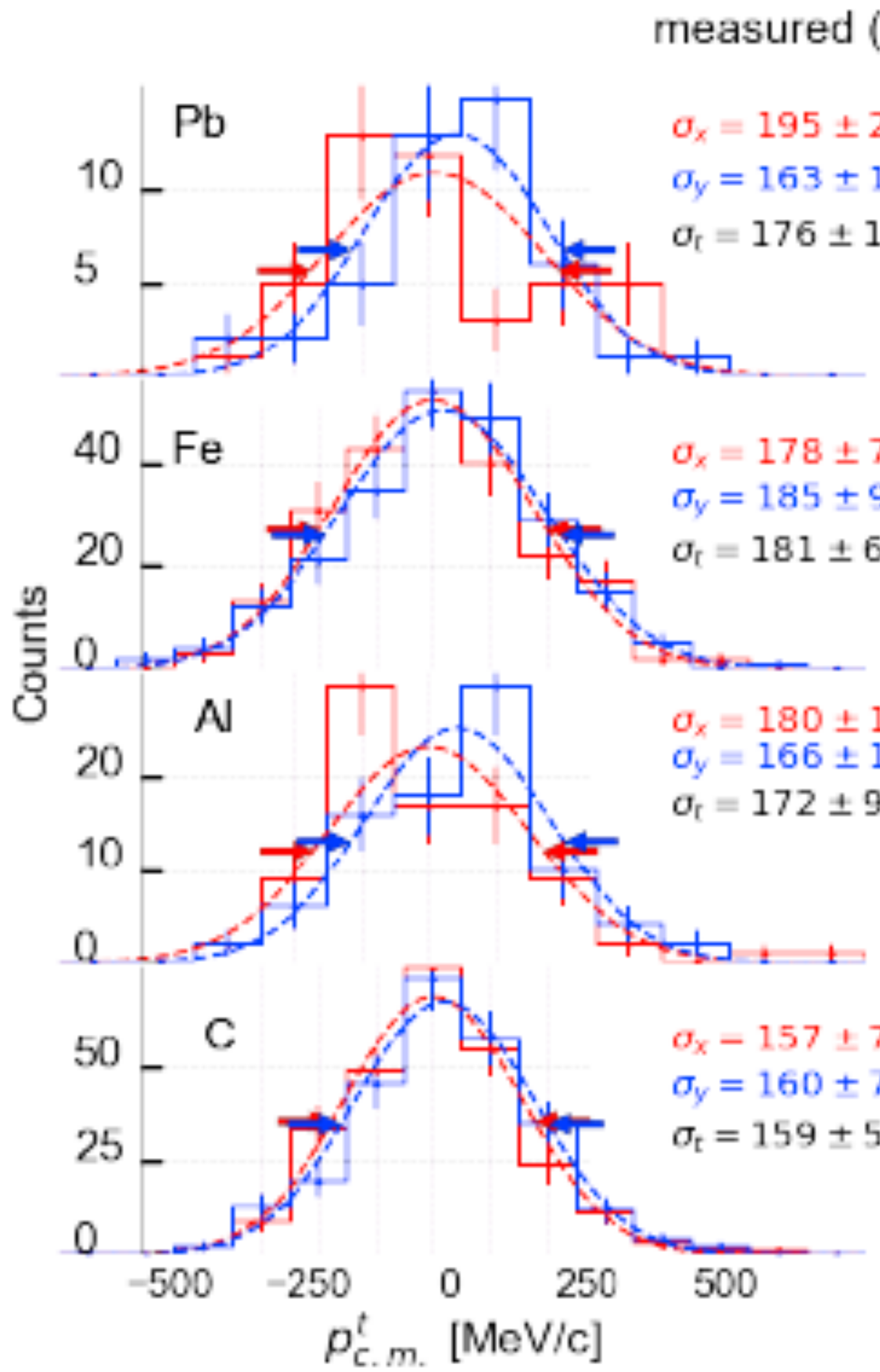
SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

2) Kinematics of knocked-out nucleons



knocked out SRC nucleons fly out almost back-to-back (relative s-wave pairs)



pair CM momentum distribution gaussian of width ~ k_F

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

2) Kin

evolved pair momentum distribution $(\lambda \sim k_F << q)$

$$\rho_{NN,\alpha}(Q,q) \sim \gamma_\alpha^2(q;\Lambda) \sum_{k,k'} |\langle\psi^A(\Lambda)|\left[a^\dagger_{\frac{Q}{2}+k} a^\dagger_{\frac{Q}{2}-k} a_{\frac{Q}{2}-k'} a_{\frac{Q}{2}+k'}\right]_\alpha |\psi^A(\Lambda)\rangle$$

kn

alm

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m.e. of smeared contact operator ==>
high q pairs dominated relative s-waves

evolved $\psi(\Lambda)$ "soft", dominated by MFT configs ==>
CM Q distribution smooth/gaussian with width ~ k_F

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3) np dominance at intermediate (300-500 MeV) relative momenta

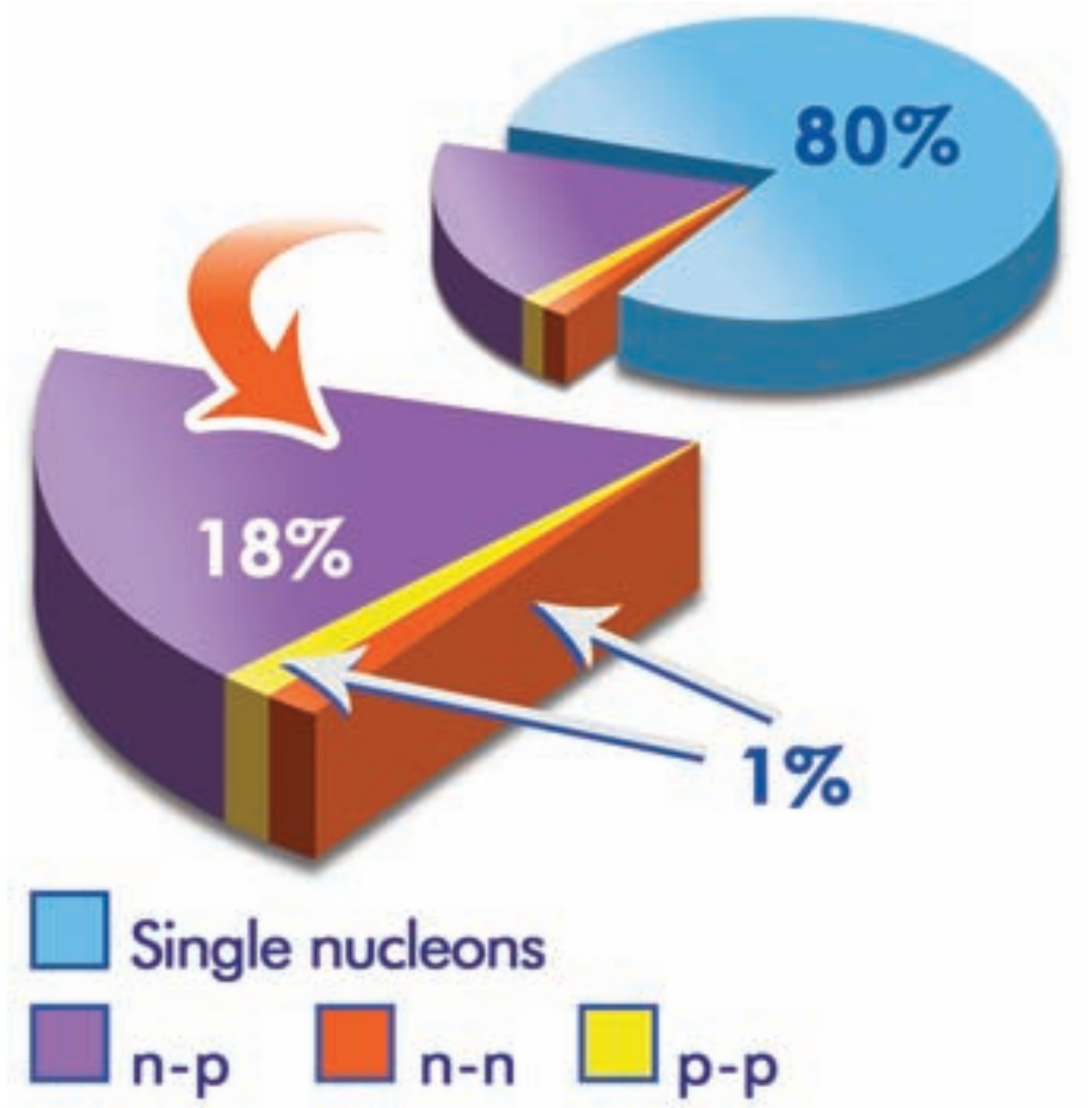

Fig. 3. The average fraction of nucleons in the various initial-state configurations of ^{12}C.

R. Subedi et al., Science (2008)

20% of nucleons in SRC pairs but mostly neutron-proton

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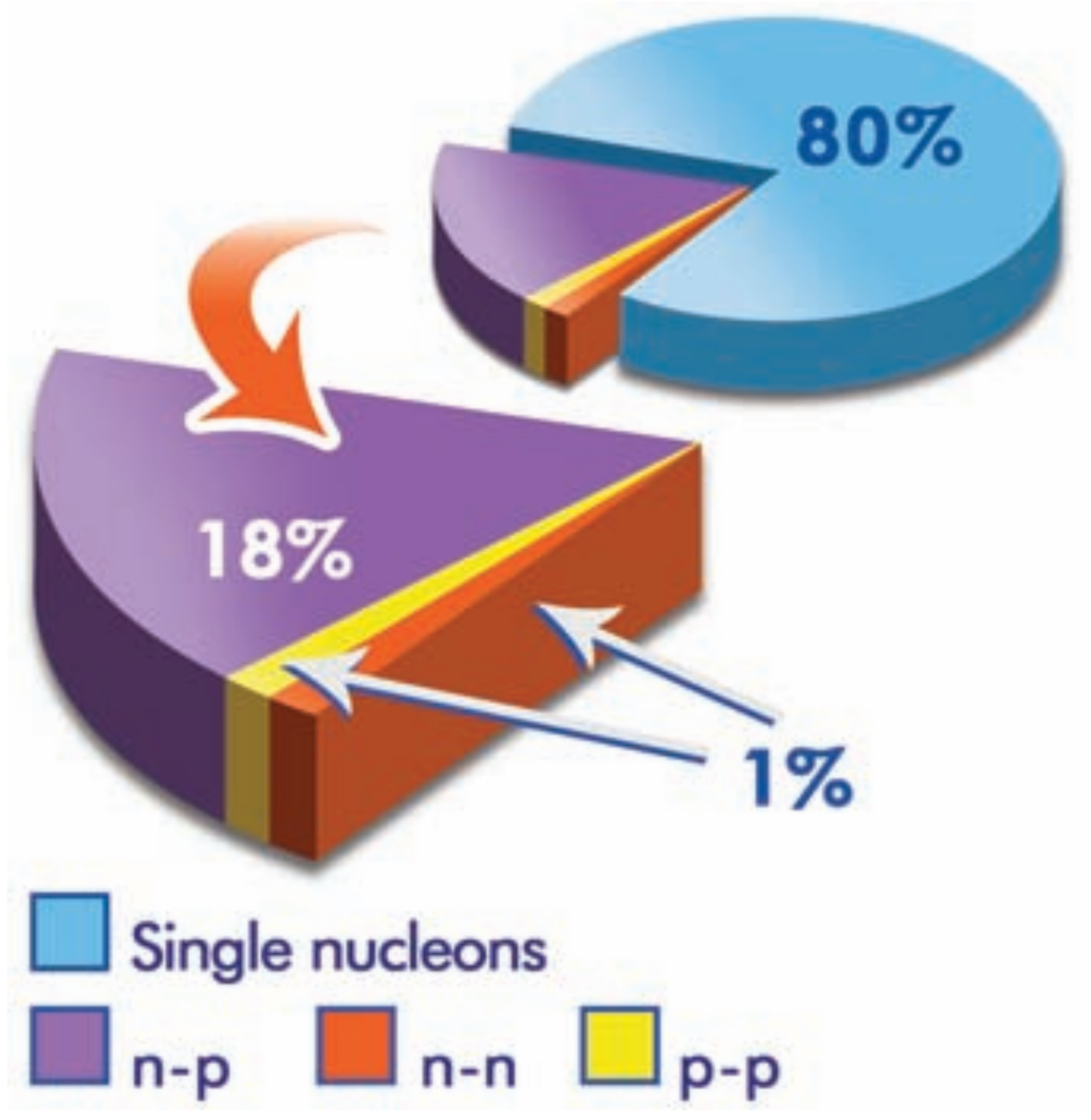

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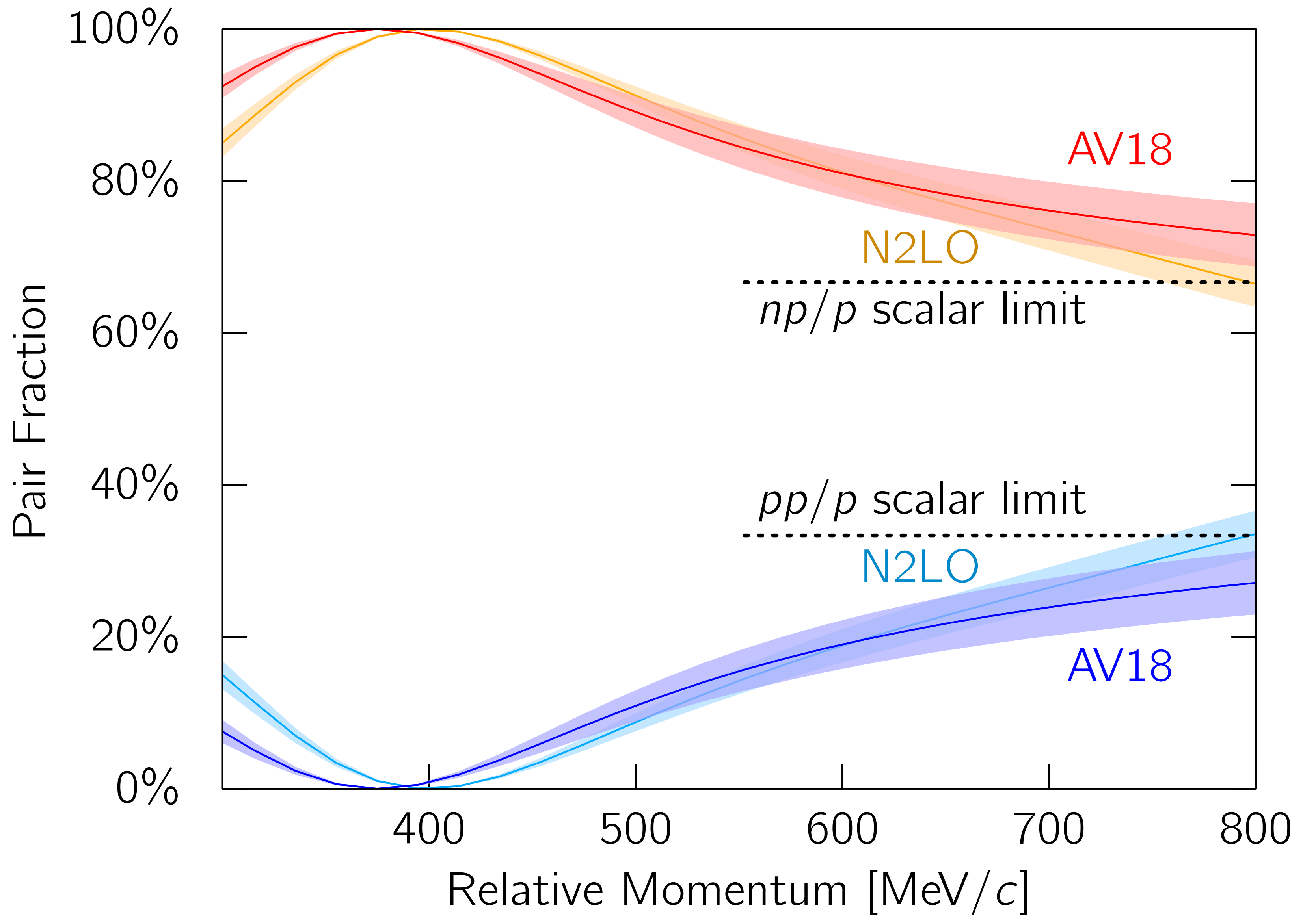

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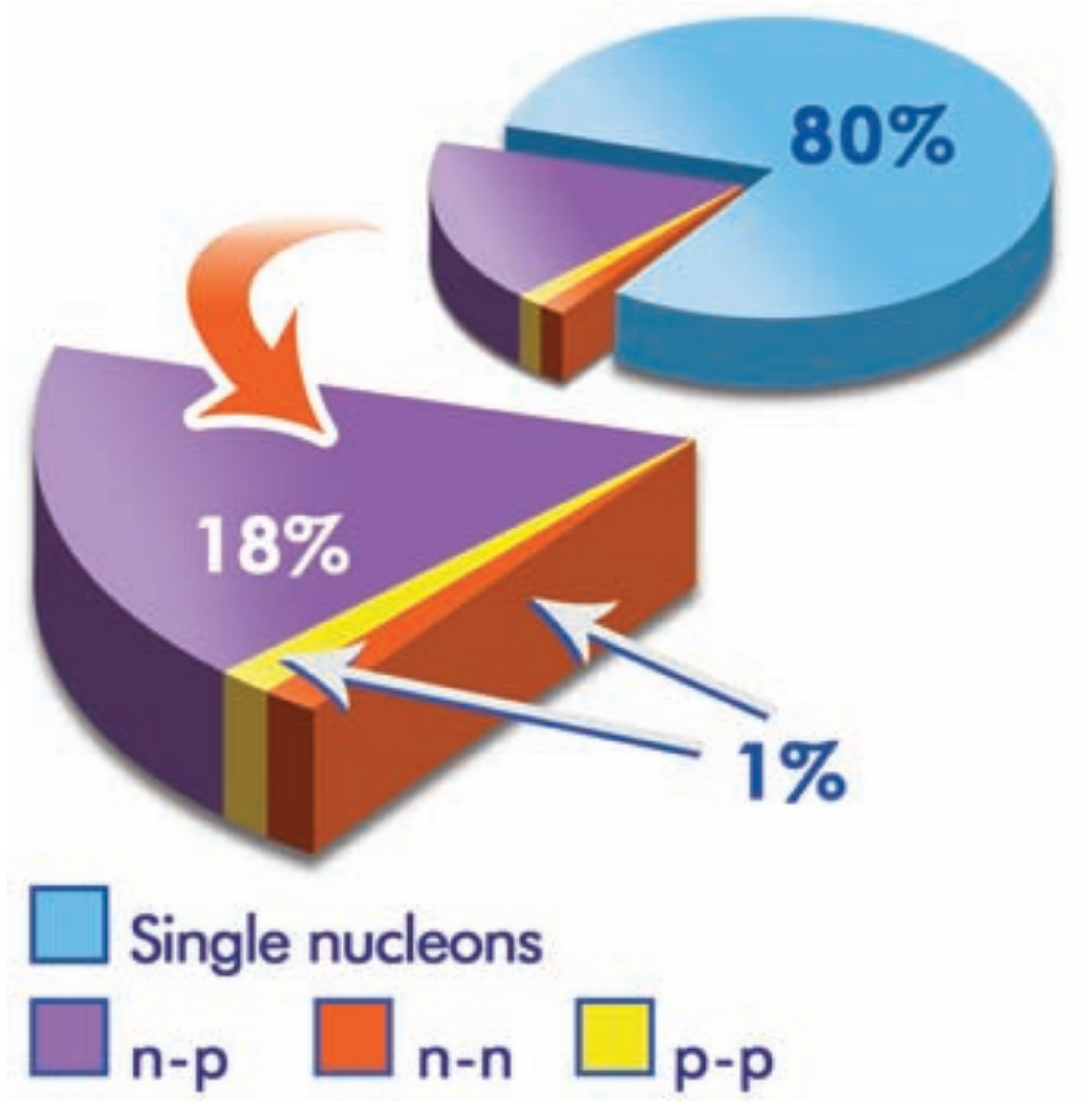

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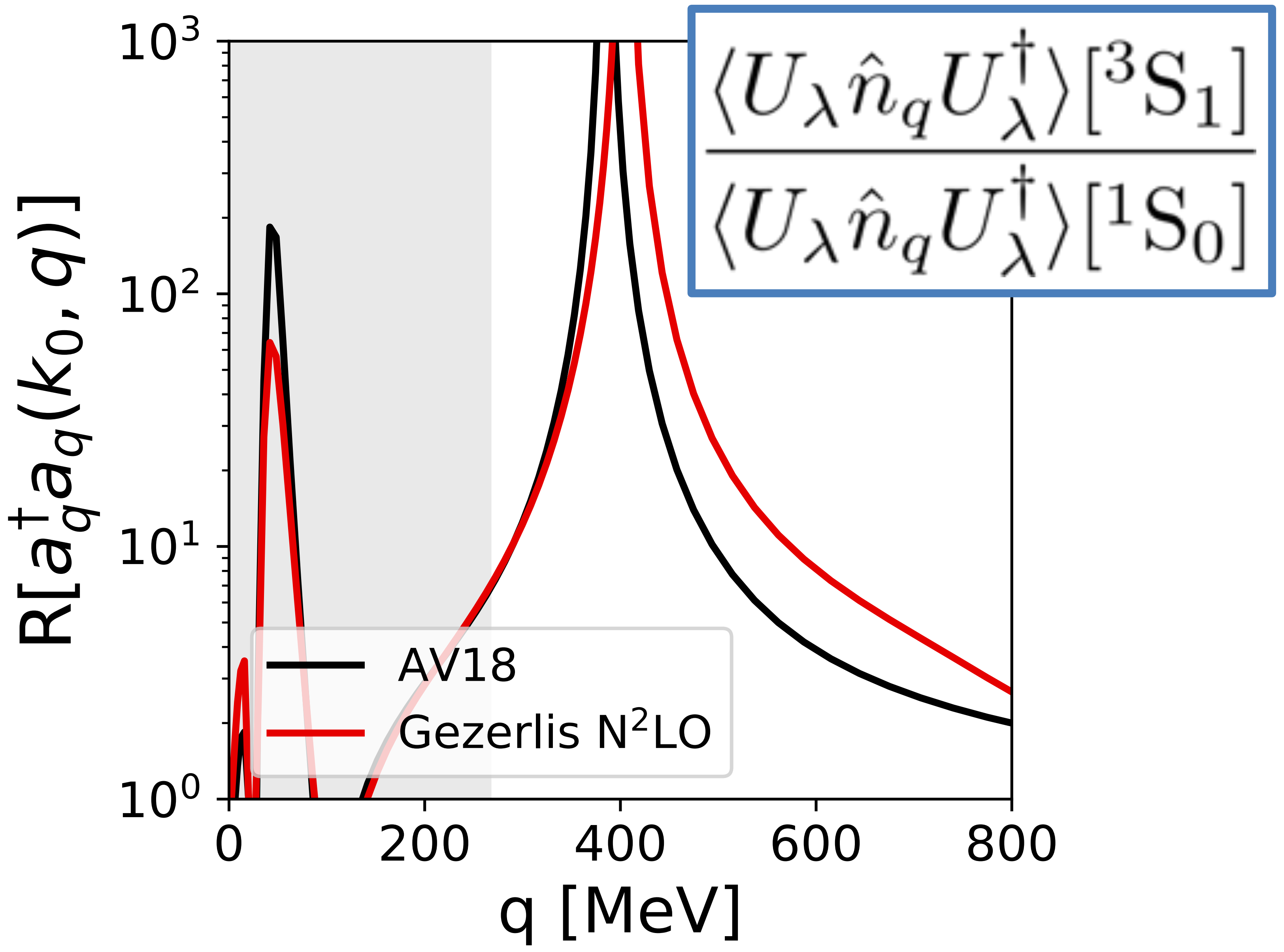
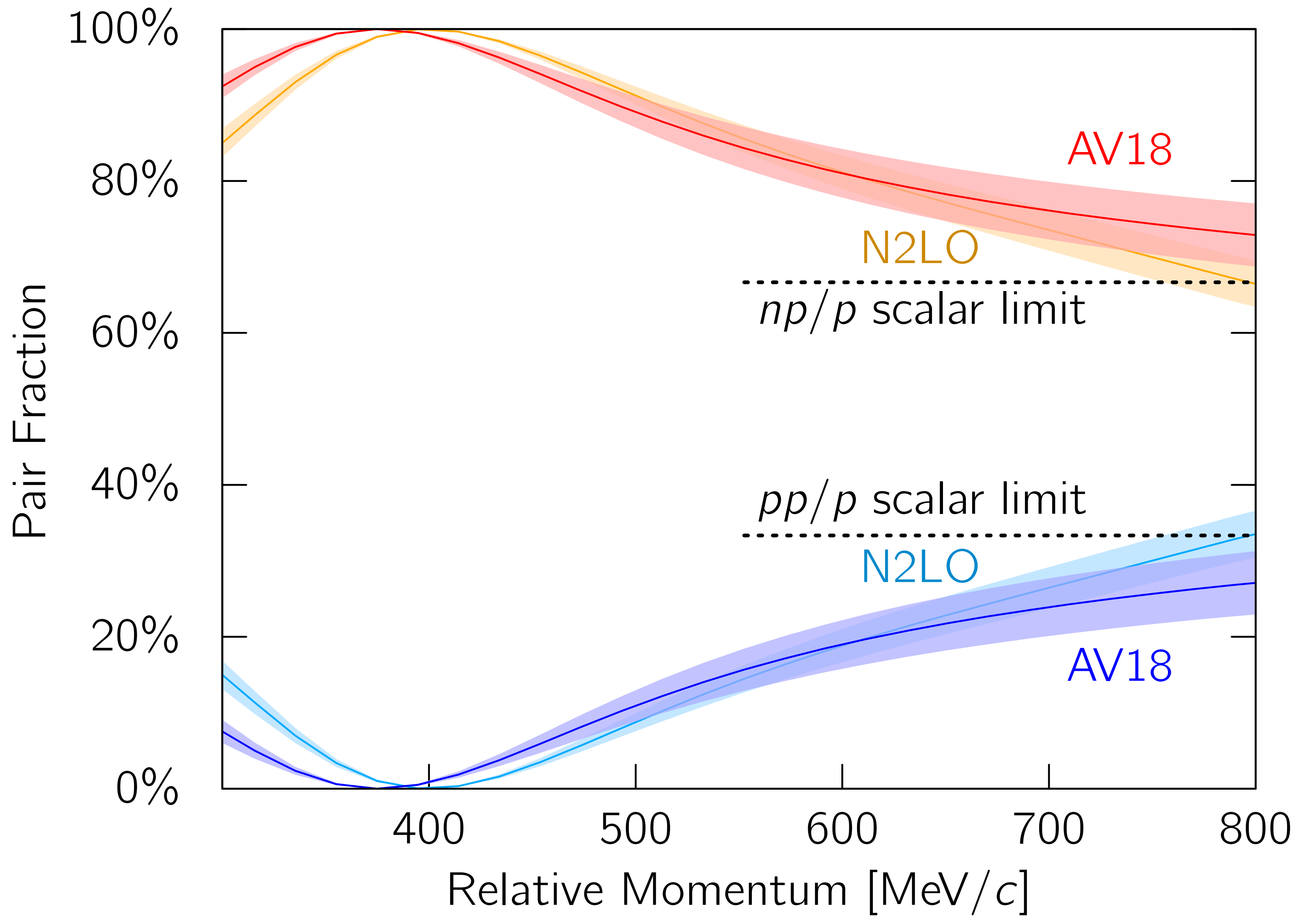
Ratio of *evolved* high-mom. distributions in a low-mom. state (insensitive to details!)





SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Generalized Contact Formalism (GCF)

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Generalized Contact Formalism (GCF)

$$\rho_A^{NN,\alpha}(r) = C_A^{NN,\alpha} \times |\varphi_{NN}^{\alpha}(r)|^2$$
$$n_A^{NN,\alpha}(q) = C_A^{NN,\alpha} \times |\varphi_{NN}^{\alpha}(q)|^2$$

A-dep scale factors (“nuclear contacts”) $C_A \sim <\chi|\chi>$

Universal (same all A, **not** V_{NN}) shape from
two-body zero energy wf ϕ

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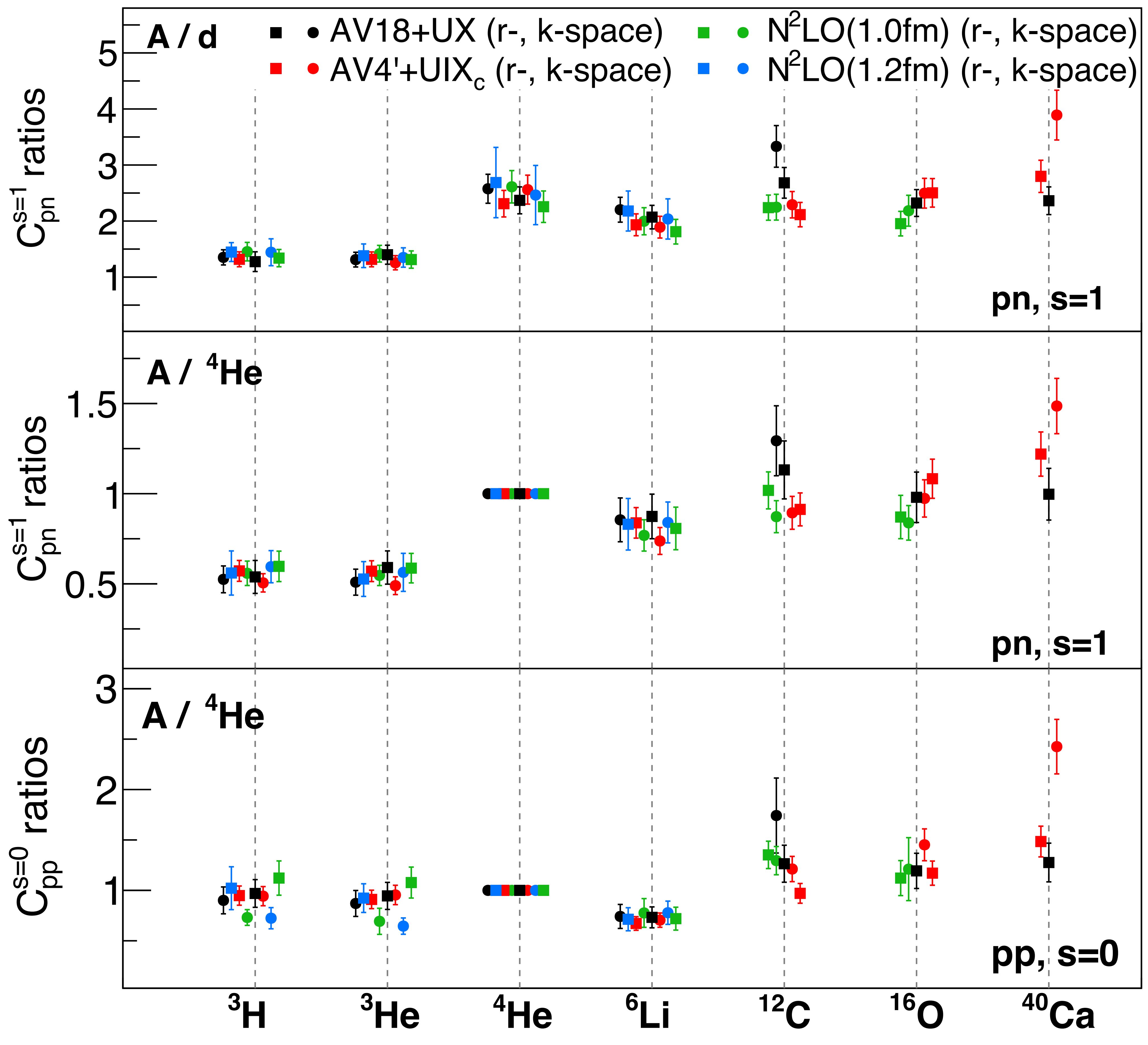
Universal (same all A, **not** V_{NN}) shape from two-body zero energy wf ϕ

But φ_{NN} is scale and scheme dependent. Ratios are independent but only probe "mean field" part



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6) Gene

$\rho_A^{NN,\alpha}($

$n_A^{NN,\alpha}($

A-dep scale

Universal (s
two-body z

But
schem
are in
probe

Contacts **not** RG invariant

$$C_A = \sum_{K,k',k}^{\Lambda_0} \langle \psi^A_{\Lambda_0} | a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} | \psi^A_{\Lambda_0} \rangle \quad \Rightarrow \quad f(\Lambda) \sum_{K,k',k}^{\Lambda} \langle \psi^A_{\Lambda} | a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} | \psi^A_{\Lambda} \rangle$$

A-independent

…But ratios in different A approx. RG invariant