

# Probing Equation of State & Bulk Properties Through Heavy-Ion Collisions & Simulations

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Theoretical Justifications and Motivations  
for Early High-Profile FRIB Experiments

FRIB-TA Topical Program

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# Physics Shopping List for HI Collisions

- Bulk isovector properties of nuclear matter up to  $2\rho_0$ 
  - symmetry energy
  - isovector component of proton and neutron mean fields w/ $p$ -dependence or isovector  $m^*$
  - isospin conductance/diffusion
  - uncertainty quantification in conclusions
- Improved constraints on isoscalar properties of nuclear matter and connection to neutron stars
  - pressure as a function of density up to five times the normal density
  - role of momentum dependence of isoscalar mean fields and in-medium cross sections
  - new observables such as triple differential momentum distributions



## Physics Shopping List for HI Collisions Cont.

- Refined understanding of bulk properties
  - pressure and energy at finite temperature and connection to neutron-star mergers
  - connection to effective field theory
  - role of off-shellness in in-medium subthreshold production of mesons
  - role of fluctuations and correlations in central-reaction dynamics
- Advances in transport theory
  - implementation of aspects of chiral effective theory
  - structure effects in initial conditions
  - mechanisms of cluster production
  - off-shell transport
  - fully quantal transport applicable down to Coulomb barrier



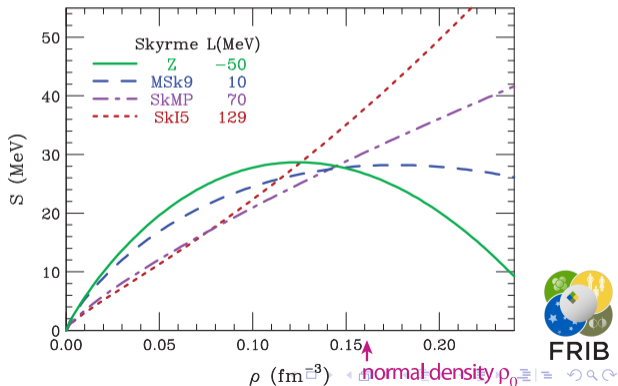
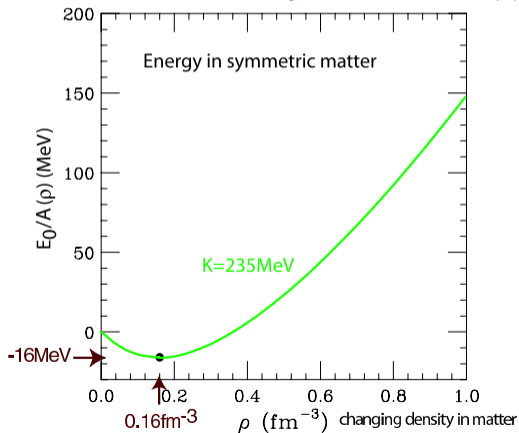
## Cold EOS Breakdown

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 + \mathcal{O}(\dots^4)$$

symmetric matter

(a)symmetry energy

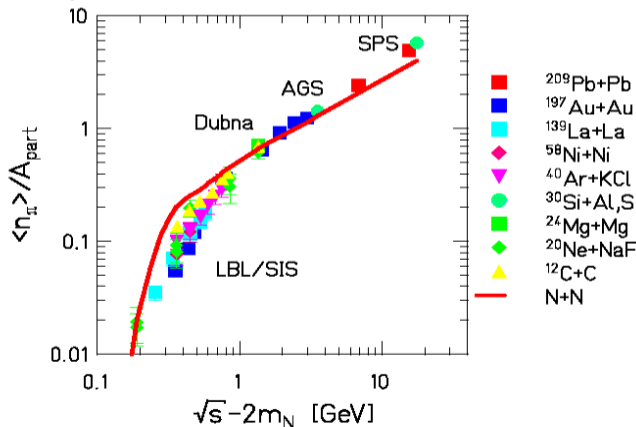
$$\rho = \rho_n + \rho_p$$



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## Heavy-Ion Collisions

- Measurements very inclusive
- Equilibrium reached only late in collisions, so transport theory needed to simulate collisions and extrapolate to equilibrium
- Collision energy changes densities reached, but also excitation above zero temperature & degrees of freedom
- Parallel data analyses carried over time



Senger ProgPartNuclPhy 53(04)1

# Transport Theory

Phase-space characteristics of hadronic degrees of freedom followed in semiclassical transport theory

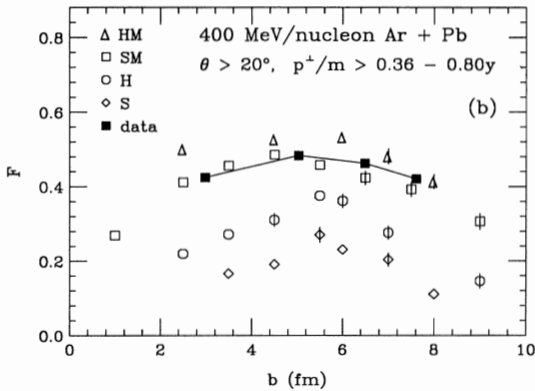
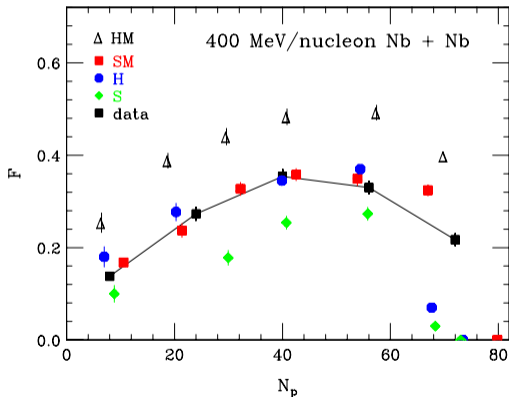
Besides EOS uncertainties include:

- Dependence of mean fields on density, momentum and nonequilibrium features of phase-space distributions
- In-medium interaction rates
- Space-time nonlocalities in collisions → impact on entropy
- Off-shell effects
- Optimal observables for testing individual uncertainties



## Example: Mean-Field $\rho$ vs $p$ Dependence

Impact of centrality and momentum bracket on flow may be used to resolve these dependencies:



Qiubao Pan & PD PRL70(93)2063

## EOS and Flow Anisotropies

EOS assessed through reaction plane anisotropies characterizing particle collective motion

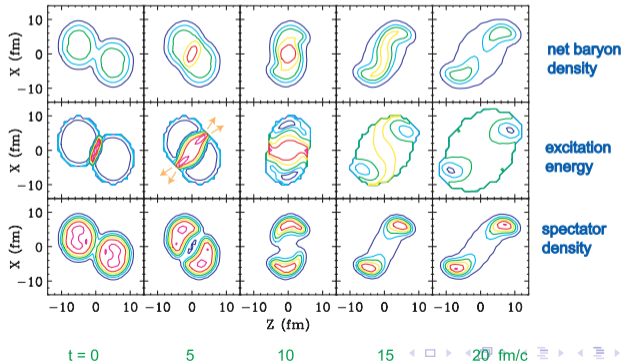
Hydro? Euler eq. in  $\vec{v} = 0$  frame:  $m_N \rho \frac{\partial}{\partial t} \vec{v} = -\vec{\nabla} p$

where  $p$  - pressure From features of  $v$ , knowing  $\Delta t$ , we may learn about  $p$  in relation to  $\rho$

$\Delta t$  fixed by spectator motion

For high  $p$ , expansion rapid and much affected by spectators

For low  $p$ , expansion sluggish and completes after spectators gone





# EOS and Flow Anisotropies

EOS assessed through reaction plane anisotropies characterizing particle collective motion

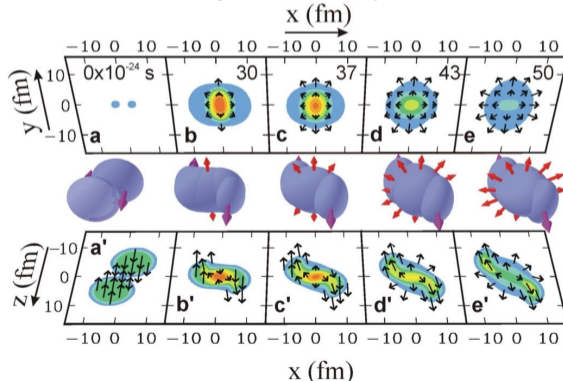
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## Sideward Flow Systematics

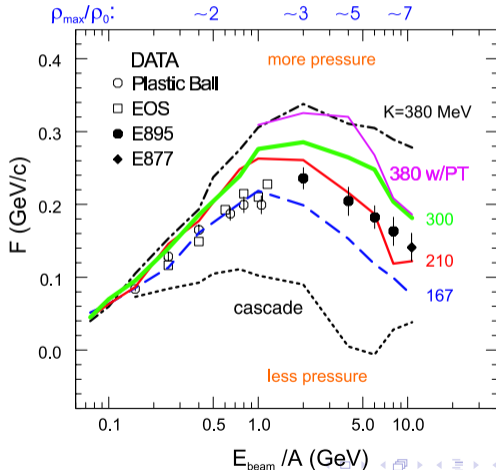
Deflection of forwards and backwards moving particles away from the beam axis, within the reaction plane

Au + Au Flow Excitation Function

Note:  $K$  used as a label

PD, Lacey & Lynch  
Science298(02)1592

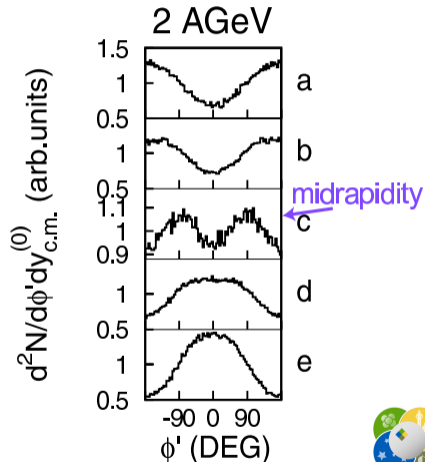
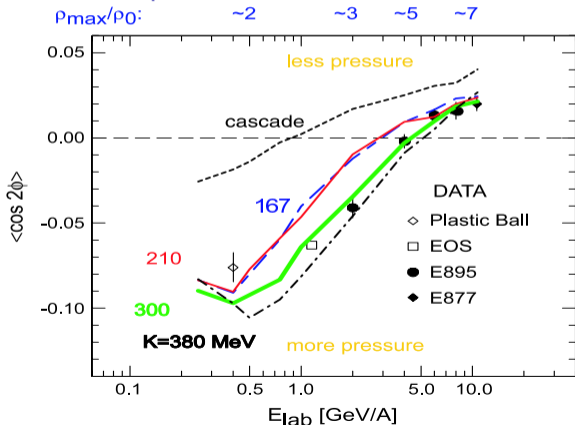
The sideward-flow observable results from dynamics that spans a density range varying with the incident energy



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## 2<sup>nd</sup>-Order or Elliptic Flow

Another anisotropy, studied at midrapidity:  
 $v_2 = \langle \cos 2\phi \rangle$ , where  $\phi$  is azimuthal angle relative  
 to reaction plane



Au+Au  $v_2$  Excitation Function FRIB

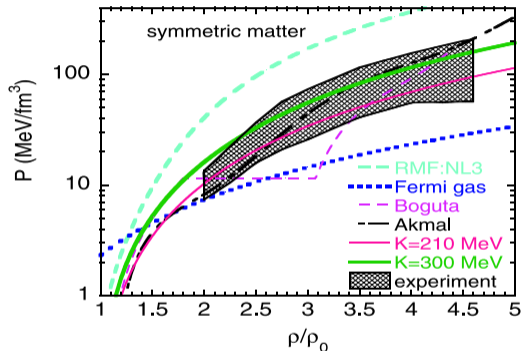
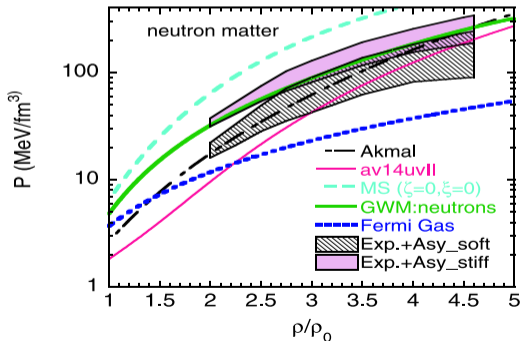
# Constraints on EOS

Au+Au flow anisotropies:  $\rho \simeq (2 - 4.6)\rho_0$

No single EOS yields both flows right

Discrepancies: inaccuracy of theory

Most extreme models for EOS can be eliminated



PD, Lacey&Lynch Science298(02)1592

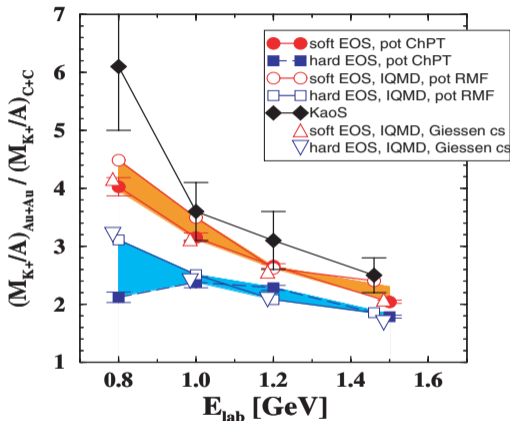
Neutron Matter:

Uncertainty in symmetry energy



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## Subthreshold Kaon Production



Ratio of kaons per participant nucleon in Au+Au collisions to kaons in C+C collisions vs beam energy

filled diamonds: KaoS data

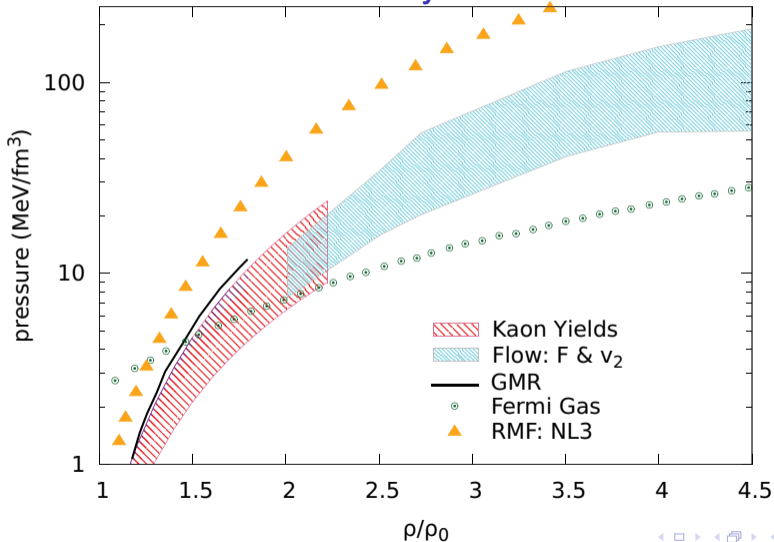
open symbols: theory

Fuchs *et al.* ProgPartNuclPhys 53(04)113

Kaon yield sensitive to EOS as multiple interactions needed for production, testing density

The data suggest a relatively soft EOS  
In-medium threshold effects?? (Dan Cozma)

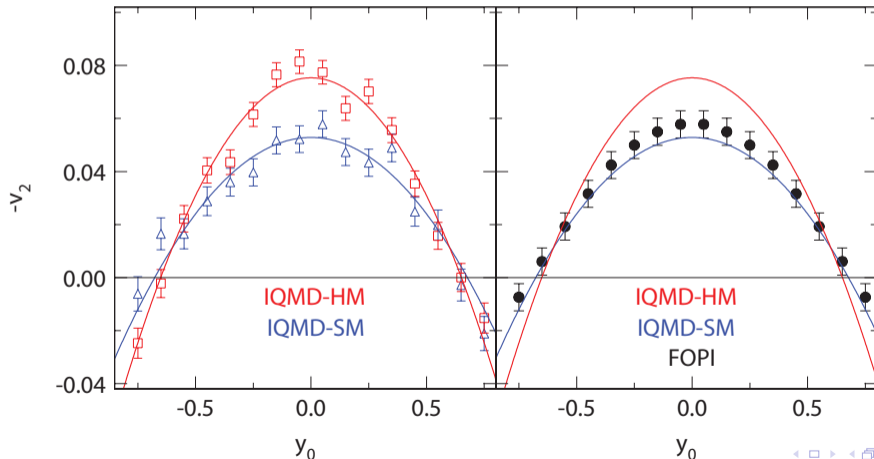
# Combination/Consistency of the Constraints?



# Flow Probing Vicinity of $\rho_0$

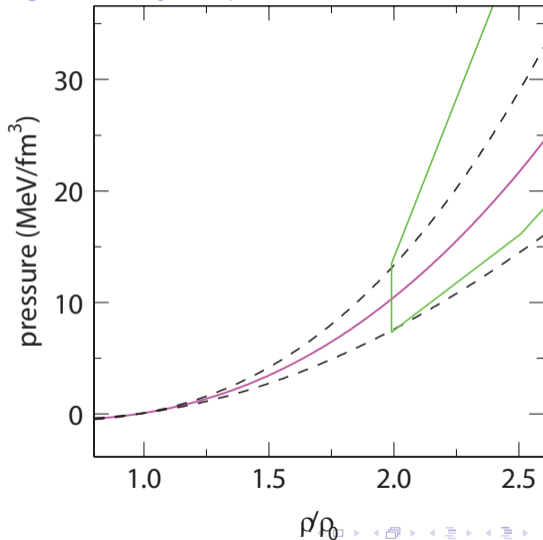
Le Fevre *et al.* NPA945(16)112 Elliptic flow in Au + Au between 0.4 and 1.5 GeV/u

Au+Au 1.2A GeV  $0.25 < b_0 < 0.45$  protons



## Flow Testing Vicinity of $\rho_0$

Le Fevre *et al.* NPA945(16)112  
Comparison to higher- $\rho$  inference

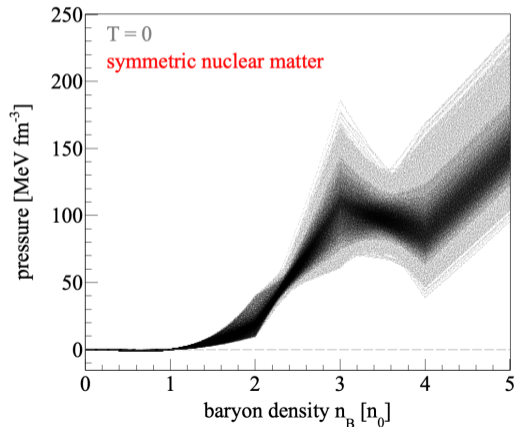
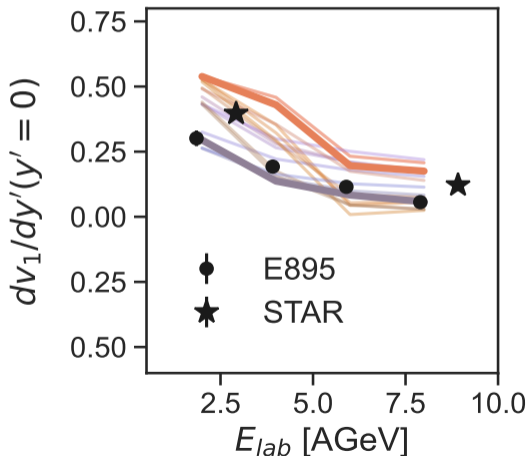




# Flow in SMASH

Oliinychenko, Sorensen *et al.* arXiv:2208.11996

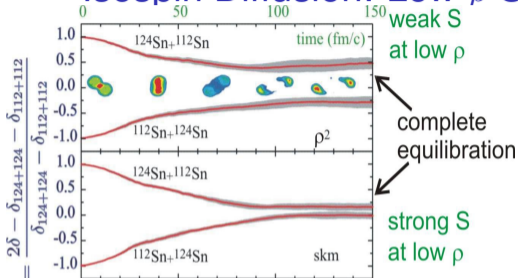
Changing speed of sound in density intervals and comparing to flow data



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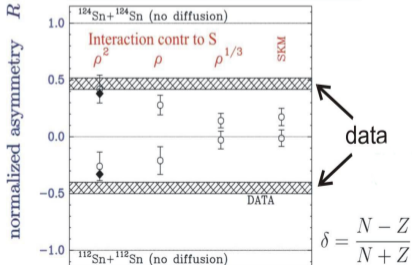


# Isospin Diffusion: Low- $\rho$ Symmetry Energy



Linear theory:  

$$\vec{j}_{np} \simeq -4\nu \vec{\nabla} [S \delta]$$

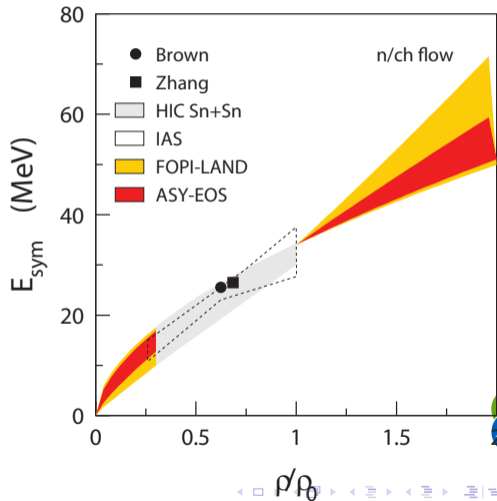
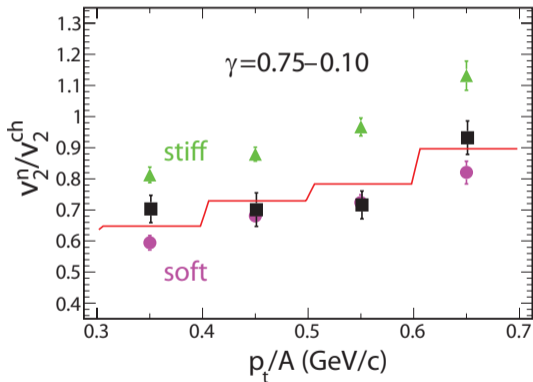


Tsang *et al.*  
 PRL92(04)062701  
 Shi&PD  
 PRC68(03)064604



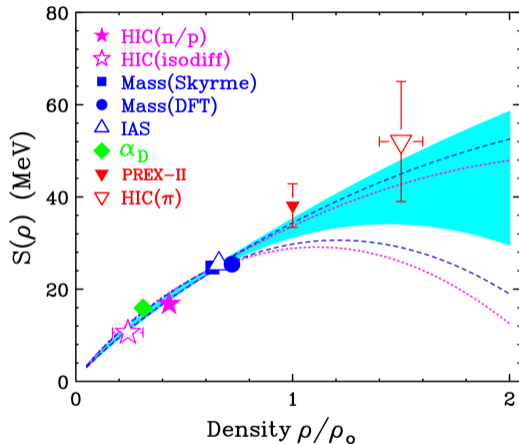
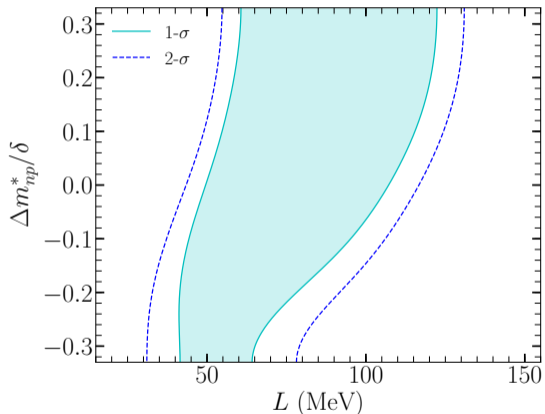
# Supranormal Densities: Baryon Differential Flow

Russotto *et al.* PRC94(16)034608 Au+Au @ 400MeV/u, neutron measurements w/LAND



# Constraints from Charged Pion Yields

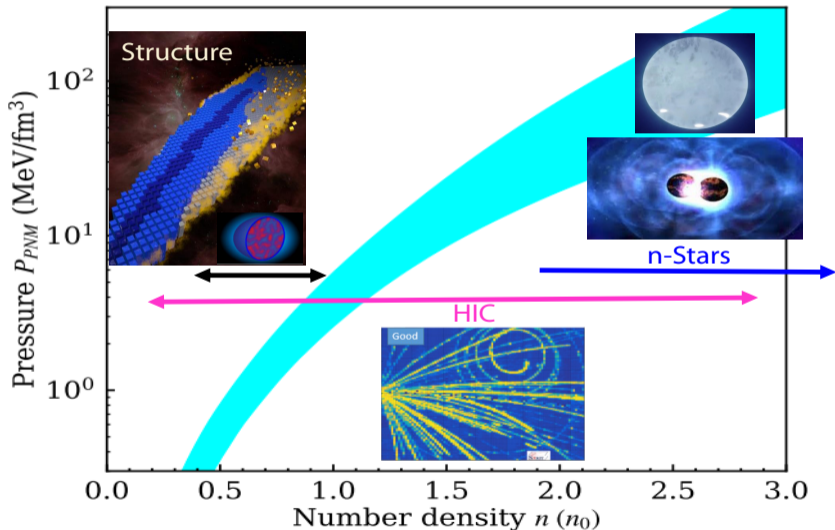
proposition: Bao-An Li PRL88(02)192701



results: Reisdorf *et al.* NPA781(07)459; Estee *et al.* PRL126(21)162701  
Liu *et al.* PRC103(21)014616; Lynch&Tsang PLB830(22)137098

# Combined Inferences: HIC + Structure + Astrophysics

Different  
probed  
densities



From Betty Tsang



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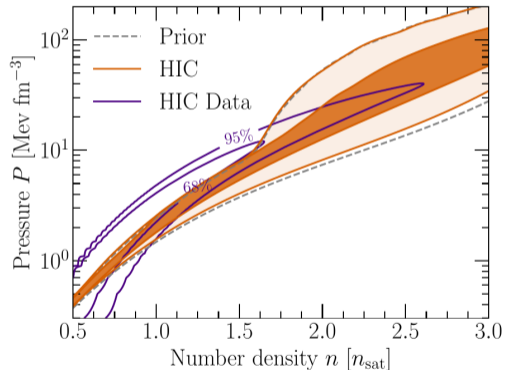
# Analysis by Huth et al.

Constraining neutron-star matter with microscopic and macroscopic collisions

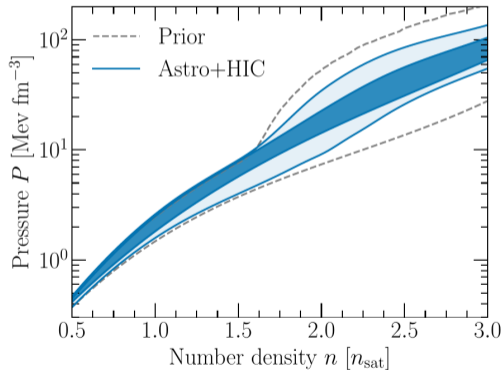
Bayesian combinations

Huth, Pang *et al.* Nature 606(22)276

**HIC experiments:**



**HIC and Astro combined:**



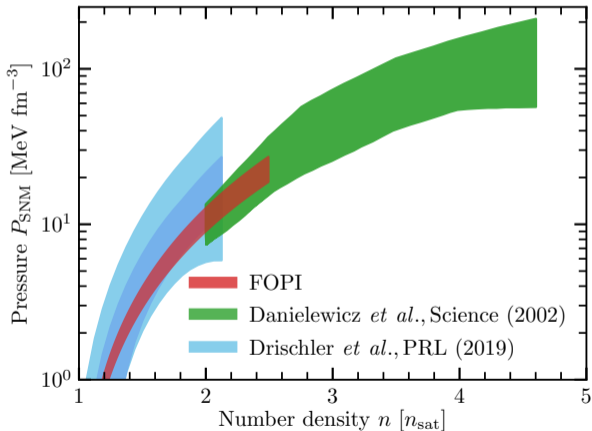
Astrophysical observations narrow constraints above  $2\rho_0$



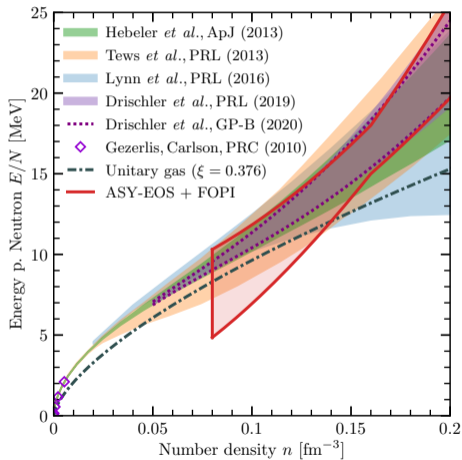
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# Huth et al: Theory vs Heavy Ions

## Symmetric Matter



## Neutron Matter



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# Conclusions

- Heavy-ion collisions allow to dial densities for studying EOS, by changing beam energy
- Window in the energy that addresses the densities around  $2\rho_0$ , where the collisions are particularly called for, is actually easier from the standpoint of transport than either significantly lower or significantly higher energies
- FRIB should deliver a wider range of exotic projectiles, at intensities needed for heavy-ion experiments focussed on EOS, than any other accelerator in the world in the foreseeable future
- More refined observables are needed for more stringent EOS constraints (PD 3D: need beam separation from products)

Supported by US Department of Energy under Grant US DE-SC0019209





# Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment



no control over plane



some control,  $v_n$



full control,  $\frac{d^3N}{dp^3}$

Claim: You can go from center to right panel through deblurring

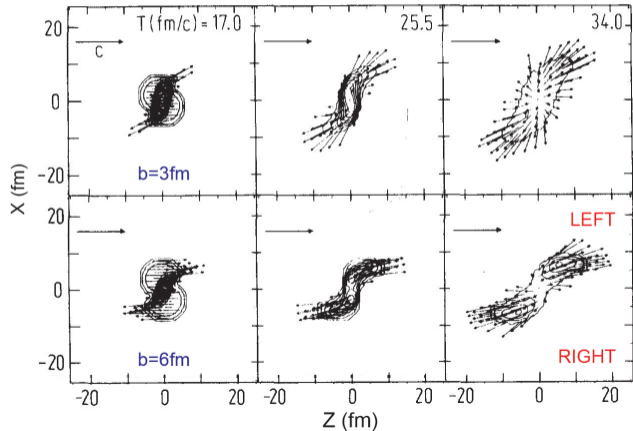


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# Side Splash in Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion **away from the beam**, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

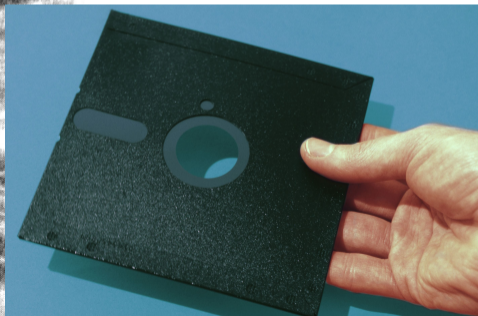
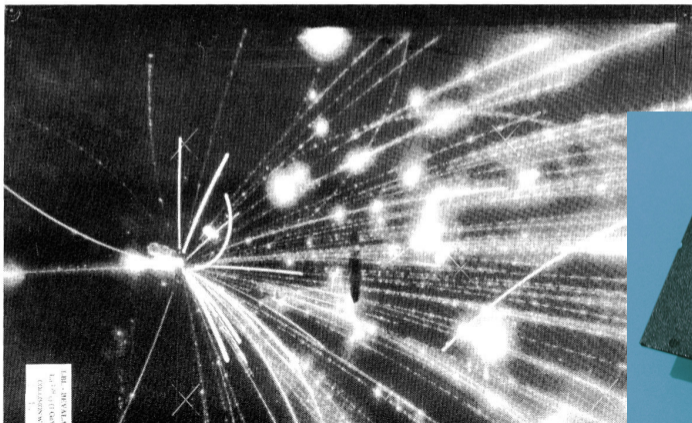


# Ar + KCl @ 1.8 GeV/nucI

Ströbele PRC 27(83)1349

495 events from Streamer Chamber,  $b \lesssim 2.4$  fm

PD&Odyniec PLB 157(85)146

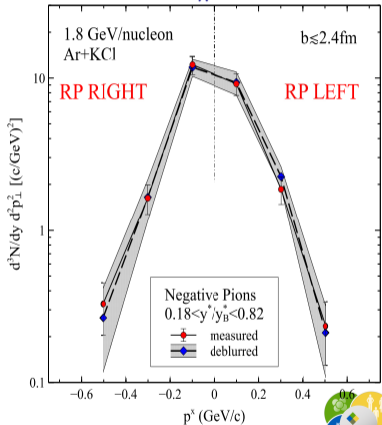
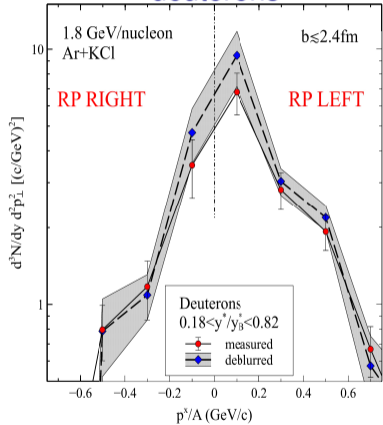
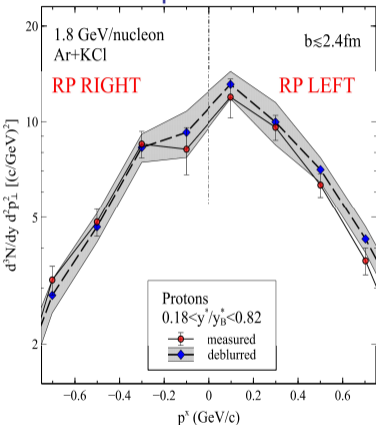


# Side-Splash in Ar + KCl 1.8 GeV/nuclel

protons

deuterons

$\pi^-$

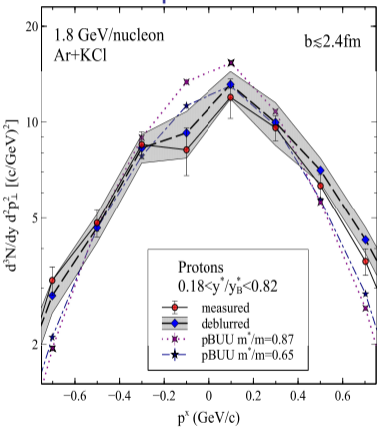


Particles in the forward hemisphere,  $y^* \sim 0.5y_B^*$

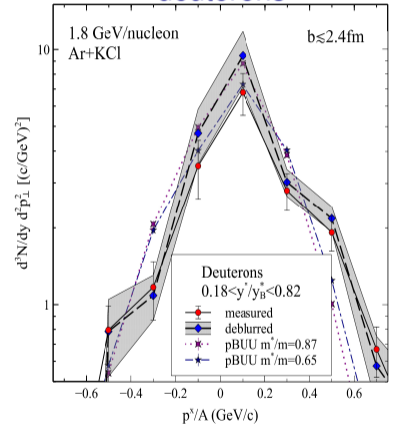


# Side-Splash: Experiment vs Theory

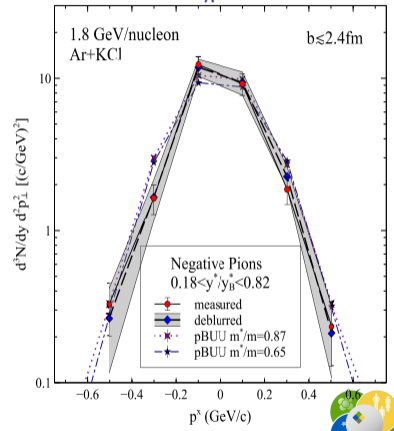
protons



deuterons



$\pi^-$



Calculations in pBUU: sensitivity to mean-field  $p$ -dependence in Ar+KCl

