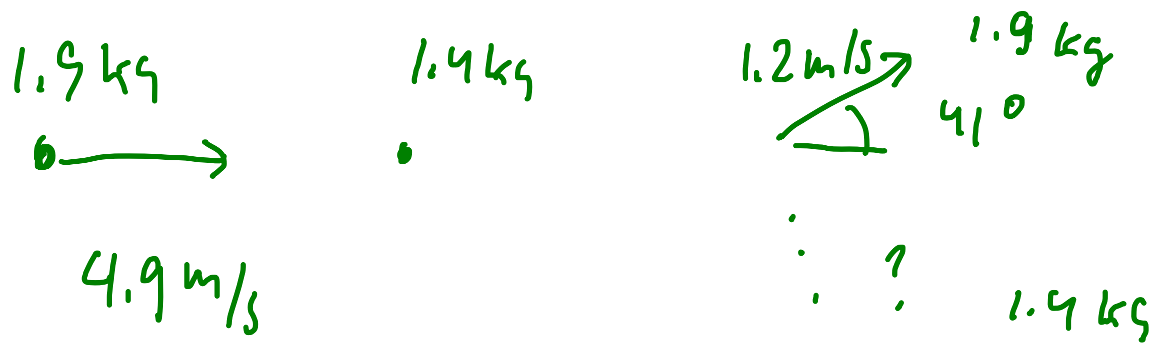
EXAMPLE: A 1.9 kg OBJECT MOVING AT A SPEED OF 4.9 m/s STRIKES A 1.4 kg OBJECT INITIALLY AT REST. IMMEDIATELY AFTER THE COLLISION, THE 1.9 kg OBJECT HAS A VELOCITY OF 1.2 m/s DIRECTED AT 41° FROM ITS INITIAL LINE OF MOTION. WHAT IS THE SPEED OF 1.4 kg OBJECT IMMEDIATELY AFTER THE COLLISION?



$\bar{P} = \text{const}$

$x: \quad 1{,}9 \times 4{,}9 = 1.9 \times 1{,}2 \times \cos 41^\circ + 1{,}4 \cdot v_x$

$y: \quad 0 = 1{,}9 \times 1{,}2 \times \sin 41^\circ + 1{,}4\, v_y$

$$v_y = -\frac{1.9 \times 1{,}2 \times \sin 41^\circ}{1{,}4} = -1{,}06 \ \frac{m}{s}$$

$$v_x = \frac{1.9}{1.4}(4.9 - 1.2 \times \cos 41^\circ) = 5.42\ \frac{m}{s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{1.06^2 + 5.42^2} = 5.52\ \frac{m}{s}$$

TAYLOR EXPANSION

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!} f''(x_0)(x - x_0)^2 + \frac{1}{3!} f'''(x_0)(x - x_0)^3 + \dots$$

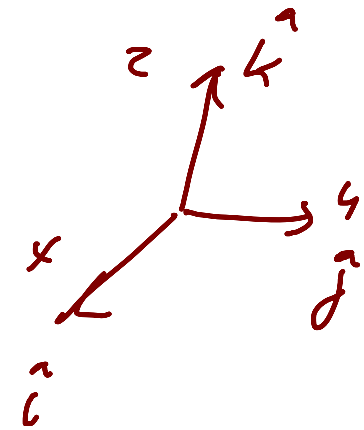
$$e^x = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \dots$$

$$\sin(x) = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 + \dots$$

$$\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$$

$$\vec{A}\,\vec{B} = AB\cos\Theta_{AB} = A_x B_x + A_y B_y + A_z A_z$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



$$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + \vec{A}_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\hat{i}\times\hat{i} = 0 \qquad \hat{i}\times\hat{j} = \hat{k}$$

$$|\vec{A}\times\vec{B}| = AB\cdot\sin\Theta_{AB} \qquad \vec{A}\times\vec{B} \perp \vec{A},\vec{B}$$

RH RULE

ϕ - SCALAR FUNCTION

$$\bar{\nabla}\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

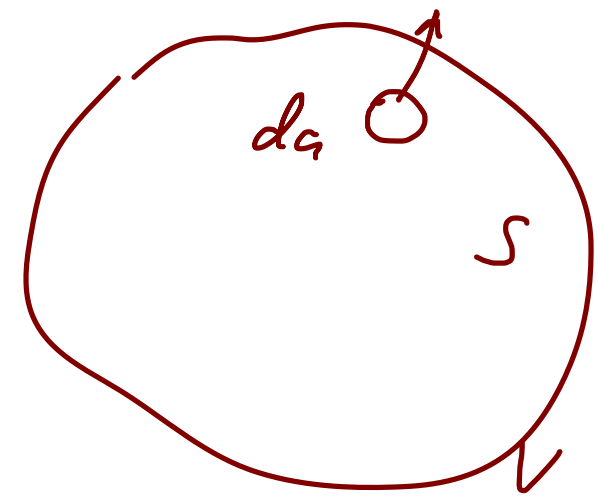
DIRECTION & RATE OF THE FASTEST CHANGE FOR THE FUNCTION

$$\operatorname{div}\vec{A} = \bar{\nabla}\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\operatorname{curl}\bar{A} = \bar{\nabla}\times\vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

GAUSS' THEOREM

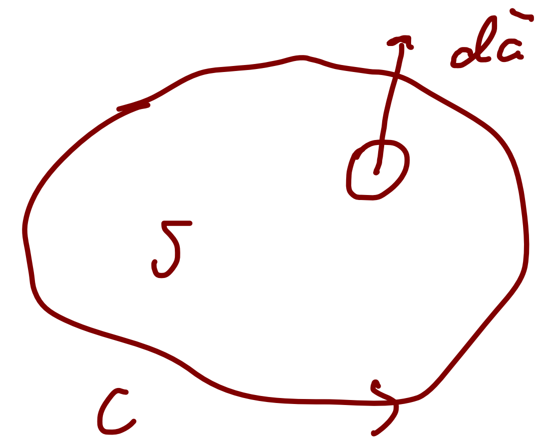
$$\int_V \operatorname{div} \bar{A}\, dV = \oint \vec{A}\, d\vec{a}$$



$$d\vec{a} = da \cdot \vec{n}$$

STOKES' THEOREM

$$\int \operatorname{curl} \bar{A}\, dS = \oint_C \bar{A}\, d\bar{\ell}$$



$\operatorname{curl} \bar{F} = 0 \Leftrightarrow \bar{F}$ IS CONSERVATIVE

$$\bar{F} = -\vec{\nabla} U$$

← POT ENERGY

PROJECTILE MOTION

$$v_x = v_0 \cos\theta$$

$$v_y = v_0 \sin\theta - gt$$

$$x = x_0 + v_0 \cos\theta \, t$$

$$y = y_0 + v_0 \sin\theta - \frac{1}{2} g t^2$$

WITH AIR RESISTANCE

$$\vec{F} = m\vec{g} - km\vec{v}$$

$$\vec{g} = (0, -g)$$

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

$$\frac{d\vec{v}}{dt} = \vec{g} - k\vec{v}$$

$x:$ $\dfrac{dv_x}{dt} = -kv_x$ $\qquad v_x = v_x^0 e^{-kt}$

$v_x = v_0 \sin\theta \, e^{-kt}$

$y:$ $m\dfrac{dv_y}{dt} = -mg - kmv_y$

$$\frac{dv_y}{dt} = -g - kv_y = -k\left(v_y + \frac{g}{k}\right)$$

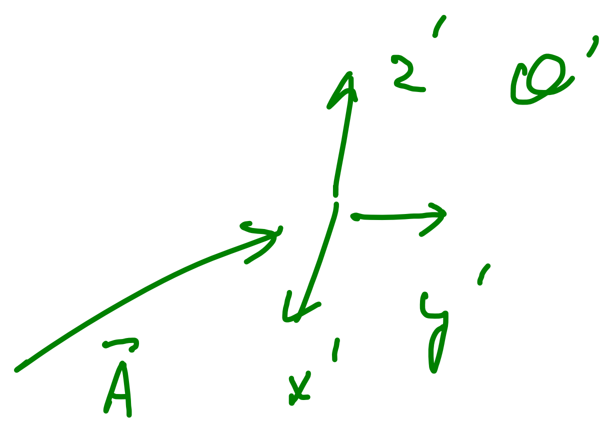
$$\frac{d}{dt}\left(v_y + \frac{g}{k}\right) = -k\left(v_y + \frac{g}{k}\right)$$

$\dfrac{dF}{dt} = -kF$

$$v_y + \frac{g}{k} = \left(v_y^0 + \frac{g}{k}\right) e^{-kt} = \left(v^0 \sin\theta + \frac{g}{k}\right) e^{-kt}$$

$$v_y = \left(v^0 \sin\theta + \frac{g}{k}\right) e^{-kt} - \frac{g}{k}$$

NONINERTIAL FRAMES



EFFECTIVE FORCE

$$m\vec{a}' = \vec{F} + \vec{F}_{ef} = \vec{F} - m\vec{A}$$

NEWTON'S FORCE

SYSTEMS OF PARTICLES

$$\vec{R} = \frac{\sum_\alpha m_\alpha \vec{r}_\alpha}{\sum m_\alpha} \qquad \vec{R} = \frac{\int \vec{r}\, dm}{M}$$

$$\vec{V} = \dot{\vec{R}} = \frac{\sum \vec{p}_\alpha}{\sum m_\alpha} = \frac{\vec{P}}{M} \qquad \vec{P} = M\vec{V}$$

$$\frac{d\vec{P}}{dt} = \vec{F}_{ext}$$

$$T = \frac{MV^2}{2} + T' \qquad \vec{L} = \vec{R} \times \vec{P} + \vec{L}'$$

$\vec{R}$ - CM POSITION

$$\frac{d\vec{L}}{dt} = \vec{\tau}^{ext}$$

FOR CENTRAL INTRINSIC FORCES

$$\frac{d}{dt} \vec{R} \times \vec{P} = \vec{R} \times \vec{F}^{ex}$$

$$\frac{d\vec{L}'}{dt} = \vec{\tau}^{ext\,\prime}$$

2-BODY COLLISIONS

$\bar{P}$ - CONSERVED $\quad \vec{L}$ - CONSERVED

ELASTIC COLLISIONS : T CONSERVED AS WELL

$\vec{V}$ - DOES NOT CHANGE

$\bar{v}_{rel} = \bar{v}_1 - \bar{v}_2$ DOES NOT DEPEND ON REF FRAME & v_{rel} DOES NOT CHANGE IN ELASTIC

$$T = \frac{MV^2}{2} + \underbrace{\frac{\mu v_{rel}^2}{2}}_{T'}$$

$$\cdots$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$