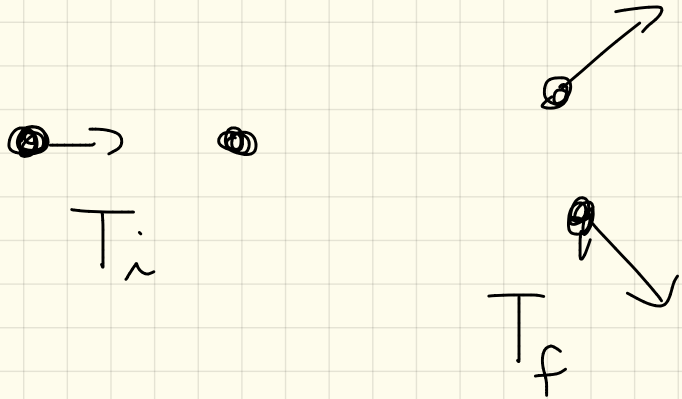
* Review for Exam 2

1.) Inelastic Collisions



$$T_i \neq T_f$$

$$Q = T_f - T_i \qquad \text{"Q-value"}$$

$$T = \frac{MV^2}{2} + \frac{\mu v^2}{2}$$

COM motion does NOT change

motion relative to COM can change

$$\varepsilon \equiv \frac{|\vec{v}_f|}{|\vec{v}_i|} = \frac{|\vec{v}_{1f} - \vec{v}_{2f}|}{|\vec{v}_{1i} - \vec{v}_{2i}|}$$

"Coeff. of Restitution" $\varepsilon = 1$ elastic
$= 0$ totally inelastic

② Scattering Cross Section

* Describes capability of a target to subject the incident particles to some fate (e.g., scatter into some direction. In QM, more exotic "fates" possible, e.g., incident particles can be absorbed & "new" particles emerge.)

Incident flux I

$$I = \frac{\#\text{ of incident particle}}{\text{time}\cdot\text{transverse area}}$$

* Expect $dN \propto I$

$$dN = I\, d\sigma$$

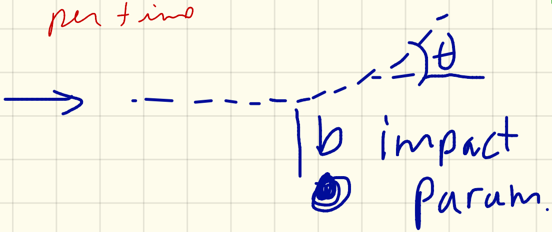
# of particles/time meeting some fate (e.g., getting deflected in some range of directions)

coefficient of proportionality "cross section" ($[d\sigma]$ = area)

$$\Rightarrow \frac{dN}{d\Omega} = I \frac{d\sigma}{d\Omega}$$

\# deflected into $d\Omega$ about some direction per time

"Differential cross section"



$$\boxed{\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|}$$

(valid for central forces)

* Total scattering cross section

$$\sigma_t = \int d\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{1}{I}\int dN = \frac{N}{I}$$

$\Rightarrow N = I\,\sigma_t$ = # incident particles per unit time affected (scattered) by the target

3. Rocket Motion

$$m\dot{v} = -mg + \alpha u$$

u = exhaust velocity wrt rocket

α = "burn rate" $= -\frac{dm}{dt}$

$$\Downarrow$$

$$v = -gt + u \ln\left(\frac{m_0}{m}\right)$$

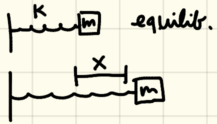
* be able to derive these ↑↑ eqns

"Thrust" $\tau \equiv \alpha u$

Need $\tau > m_0 g$ to lift off

* Understand why Multistage rockets Maximize v

④ Simple Harmonic Motion



$$\ddot{x} = -\omega_0^2 x$$

$\omega_0^2 = \frac{k}{m}$ $\quad \nu_0 = \frac{\omega_0}{2\pi}, \quad \tau_0 = \frac{1}{\nu_0}$ $\quad$ Energy $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

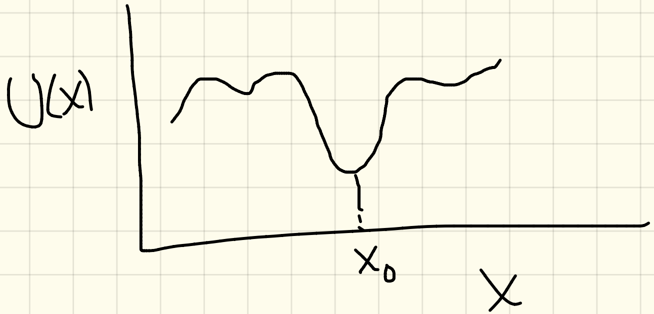
$x(t) = A_1 \cos(\omega_0 t) + A_2 \sin \omega_0 t$ or $a_1 e^{i\omega_0 t} + a_2 e^{-i\omega_0 t}$

or $A \cos(\omega_0 t - \delta)$

} 3 equiv. ways to write it

$A_1 + A_2$ (or $a_1 + a_2$ or A, δ) from IC's.

SH motion near PE minimum



$$U(x) \simeq \underbrace{U(x_0)}_{\text{const}} + (x-x_0)\underbrace{U'(x_0)}_{0} + \frac{1}{2}(x-x_0)^2 U''(x_0).$$

$\therefore F(x \simeq x_0) = -\frac{dU}{dx} = -U''(x_0)(x-x_0)$ $\quad$ Hook's Law

$\Rightarrow k = U''(x_0)$ $\quad$ (>0, why?)

⑤ Damped HO

$$m\ddot{x} = -kx - b\dot{x}$$

$$\Downarrow$$

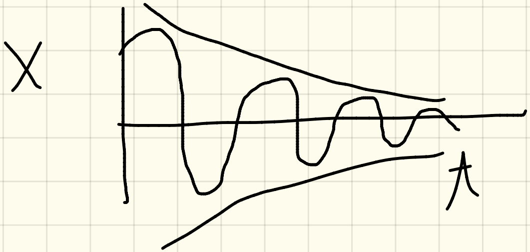
$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

$$\omega_0^2 = \frac{k}{m}$$
$$\beta = \frac{b}{2m}$$

Soln:
$$x(t) = e^{-\beta t}\left[A_1 e^{\sqrt{\beta^2-\omega_0^2}\,t} + A_2 e^{-\sqrt{\beta^2-\omega_0^2}\,t}\right]$$

(1.) $\omega_0 > \beta$ "Underdamped"

$$x(t) = e^{-\beta t} A\cos(\omega_1 t - \delta)$$
$$\omega_1 = \sqrt{\omega_0^2 - \beta^2}$$



② $\beta > \omega_0$: Overdamped

$$x = e^{-\beta t}\left[A_1 e^{\omega_2 t} + A_2 e^{-\omega_2 t}\right]$$

No oscillation

$$\omega_2 = \sqrt{\beta^2 - \omega_0^2}$$

SLOW for $\beta \gg \omega_0$

③ $\beta = \omega_0$: Critical Damping

$$x = e^{-\beta t}(A + Bt)$$

"best" for returning to equil. as fast as possible + w/out overshoot

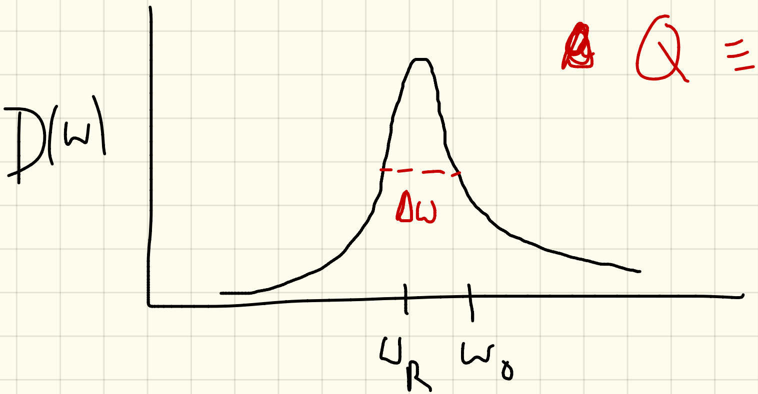
7 Driven HO

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$$

Steady sol'n: $x(t) = D(\omega)\cos(\omega t - \delta(\omega))$

$$D(\omega) = \frac{f}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2}} \qquad \delta = \tan^{-1}\left[\frac{2\beta\omega}{\omega_0^2-\omega^2}\right]$$

resonance: $\omega_R = \sqrt{\omega_0^2 - 2\beta}$ $\quad \mathrm{Max}[D(\omega)] = D(\omega_R)$



$$Q \equiv \frac{\omega_R}{\Delta\omega} = \frac{\omega_R}{2\beta}$$

"Quality factor"