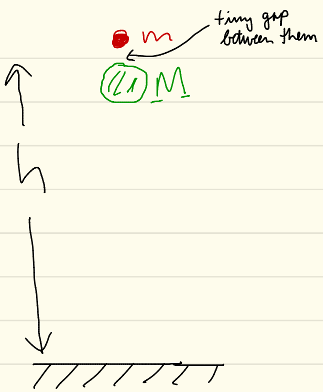
2.28

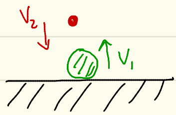


* ignore the finite spatial extent of both balls

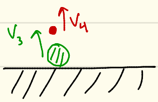
* assume elastic collision

* How high do both balls bounce?

Just before collision:



Just after collision:



Step 1: $|V_1| = |V_2|$ (but opposite direction. I.e, the superball has just hit the ground & rebounded, while the marble's still falling)

energy cons: $Mgh = \frac{1}{2}Mv^2 \Rightarrow \boxed{v = \sqrt{2gh}}$ ①

Step 2: energy cons: $mgh_{marble} = \frac{1}{2}mv_4^2 \Rightarrow \boxed{h_{marble} = \frac{v_4^2}{2g}}$ ②

$Mgh_{ball} = \frac{1}{2}Mv_3^2 \Rightarrow \boxed{h_{ball} = \frac{v_3^2}{2g}}$ ③

Step 3: Express V_3, V_4 in terms of v, m, M (i.e, known qty's)

Cons. of Momentum: $$Mv - mv = MV_3 + mV_4 \qquad (a)$$

Cons. of K.E.: $$\frac{1}{2}Mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}MV_3^2 + \frac{1}{2}mV_4^2 \qquad (b)$$

OK. These are 2 eqns. for 2 unknowns ($V_3 + V_4$). We can turn the crank + do some algebra. This simplifies a bit if we recall the result derived for elastic collisions

$$\left(\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}\right)$$

$$T_f = T_i \Rightarrow \cancel{\frac{P^2}{2(m+M)}} + \frac{p_i'^2}{2\mu} = \cancel{\frac{P^2}{2(m+M)}} + \frac{p_f'^2}{2\mu}$$

$$\Rightarrow p_i' = p_f' \qquad (\text{where relative momentum } \vec{p}' \equiv \mu \vec{v}_{rel})$$

$$\Rightarrow v_i = v_f$$

$$\therefore v_i = v_f \Rightarrow |v - (-v)| = |V_3 - V_4| \Rightarrow 2v = V_4 - V_3$$

$$\Rightarrow \boxed{V_4 = 2v + V_3}$$

plug in (a)

$$(M - m)v = MV_3 + 2mv + mV_3 \Rightarrow V_3(M+m) = (M - 3m)V$$

$$\boxed{\begin{aligned} &\Rightarrow V_3 = \left(\frac{1-3\alpha}{1+\alpha}\right)V \\ &\Rightarrow V_4 = \left(\frac{3-\alpha}{1+\alpha}\right)V \end{aligned}} \qquad \left(\alpha \equiv \frac{m}{M}\right)$$

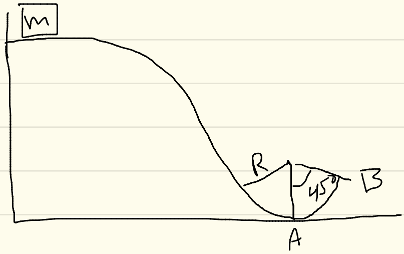
$\Rightarrow \boxed{h_{marble} = \frac{v_4^2}{2g}}$ (2)

$\Rightarrow \boxed{h_{ball} = \frac{v_3^2}{2g}}$ (3)

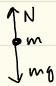
$$\Longrightarrow \boxed{\begin{gathered} h_{marble} = \left(\frac{3-\alpha}{1+\alpha}\right)^2 \frac{v^2}{2g} = \left(\frac{3-\alpha}{1+\alpha}\right)^2 h \\ h_{ball} = \left(\frac{1-3\alpha}{1+\alpha}\right)^2 h \end{gathered}}$$

eg: $\alpha \ll 1 \Rightarrow h^{marble} \approx 9h$!

2.25



a) Force on block @ A from track:



track counters gravity + provides centripetal acc.

$$\boxed{N - mg = \frac{mv^2}{R}}$$

* Energy cons to get v:

$$E_{top} = \cancel{T_{top}}^{\,0} + U_{top} = mgh$$

$$E_A = T_A + \cancel{U_A}^{\,0} = \frac{1}{2}mv^2$$

$E_{top} = E_A \Rightarrow \boxed{v = \sqrt{2gh}}$

$$\Rightarrow \boxed{N_A = mg + \frac{m}{R}(2gh) = mg\left(1+\frac{2h}{R}\right)}$$

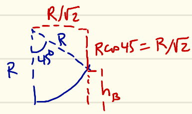
b) Find N_B:

$$N_B - mg\cos 45^\circ = \frac{m v_B^2}{R}$$

$$\Rightarrow N_B = \frac{m v_B^2}{R} + \frac{mg}{\sqrt{2}} \quad //$$

* Get v_B by cons. of E:

$$E_{top} = E_B$$



$$mgh = \frac{1}{2} m v_B^2 + mgh_B$$

$$R = \frac{R}{\sqrt{2}} + h_B$$

$$\Rightarrow h_B = R\left(1-\frac{1}{\sqrt{2}}\right)$$

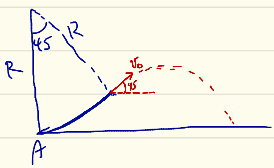
$$v_B^2 = 2\left[gh - gR\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

⇓ back into N_B eqn

$$\boxed{N_B = mg\left[\frac{2h}{R} + \left(\frac{3}{\sqrt{2}} - 2\right)\right]}$$

c) find v_B (see above)

d) Where does block land? (How far from A)



$$x = x_0 + v_{0,x} t \qquad x_0 = R\sin 45^\circ = \frac{R}{\sqrt{2}}$$

$$y = y_0 + v_{0,y} t - \frac{1}{2} g t^2 \qquad y_0 = h_B = R\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow X = \frac{R}{\sqrt{2}} + \frac{V_B}{\sqrt{2}} t$$

$$y = h_B + \frac{V_B}{\sqrt{2}} t - \frac{1}{2} g t^2$$

set $y=0$ + solve for t:

$$g t^2 - \sqrt{2} V_B t - 2 h_B = 0$$

$$t = \frac{\sqrt{2} V_B \pm \sqrt{2 V_B^2 + 8 g h_B}}{2g}$$

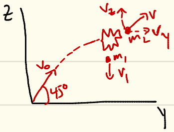
* ignore (-) root (require $t>0$)

$\Downarrow$

plug t into $X(t)$

$$\boxed{\begin{aligned} &\Rightarrow X = \frac{R}{\sqrt{2}} + \frac{V_B}{\sqrt{2}}\left[\frac{\sqrt{2} V_B + \sqrt{2 V_B^2 + 8 g h_B}}{2g}\right] \\ &= \text{(using } V_B + h_B \text{ expressions)} \\ &= (\sqrt{2}-1)R + h + \left[h^2 - \tfrac{3}{2} R^2 + \sqrt{2} R^2\right]^{1/2} \end{aligned}}$$

9.9



initial E_0

at explosion, added energy E_0 into 2 fragments $m_1 + m_2$

m_1 goes straight down

* Find $\vec{V}_1$ + $\vec{V}_2$.
* Find max. possible m_1

$$\frac{1}{2}(m_1+m_2)v_0^2 = E_0$$

* at explosion, $V_{0y} = V_0 \cos 45 = \frac{V_0}{\sqrt{2}} = \frac{\sqrt{\frac{2E_0}{m_1+m_2}}}{\sqrt{2}} = \sqrt{\frac{E_0}{m_1+m_2}}$

* initial mom. $\vec{P}_i = MV_{0,y} = (m_1+m_2)\sqrt{\frac{E_0}{m_1+m_2}} = \sqrt{E_0(m_1+m_2)}\,\hat{j}$

* Final mom: $\vec{P}_f = \vec{P}_1 + \vec{P}_2$ $\qquad \vec{P}_1 = -m_1 v_1 \hat{k}$ $\qquad \vec{P}_2 = m_2 \vec{v}$

$\vec{P}_i = \vec{P}_f \Rightarrow$ X: $0 = m_2 v_x \Rightarrow \boxed{v_x = 0}$

Y: $\sqrt{E_0(m_1+m_2)} = m_2 v_y \Rightarrow \boxed{V_y = \frac{1}{m_2}\sqrt{E_0(m_1+m_2)}}$ ①

Z: $0 = -m_1 v_1 + m_2 v_z \Rightarrow \boxed{v_z = \frac{m_1}{m_2} v_1}$ ②

Energy: $\frac{1}{2}(m_1+m_2)V_{0,y}^2 + E_0 = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2(V_y^2+V_z^2)$

$$\frac{1}{2}(m_1+m_2)\frac{E_0}{m_1+m_2} + E_0 = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2(V^2)$$

$$\boxed{\Rightarrow 3E_0 = m_1 V_1^2 + m_2(V_y^2+V_z^2)}$$ ③

Eqns ①,②,③ + algebra $\Rightarrow$ $V_z = \sqrt{\frac{E_0 m_1(2m_2-m_1)}{m_2^2(m_1+m_2)}}$, $V_y = \frac{1}{m_2}\sqrt{E_0(m_1+m_2)}$ + $V_1 = \sqrt{\frac{E_0(2m_2-m_1)}{m_1(m_1+m_2)}}$

$$V_z = \sqrt{\frac{E_0 m_1(2m_2-m_1)}{m_2^2(m_1+m_2)}}, \quad V_y = \frac{1}{m_2}\sqrt{E_0(m_1+m_2)} \quad \& \quad V_1 = \sqrt{\frac{E_0(2m_2-m_1)}{m_1(m_1+m_2)}}$$

$$\Rightarrow V_2 = \sqrt{V_y^2+V_z^2} = \sqrt{\frac{E_0(4m_1+m_2)}{m_2(m_1+m_2)}}$$

$$\theta = \tan^{-1}\left(\frac{V_z}{V_y}\right) = \tan^{-1}\sqrt{\frac{m_1(2m_2-m_1)}{(m_1+m_2)}}$$

Largest m_1 when $V_1 = 0$ $\Rightarrow \boxed{2m_2 = m_1}$