<u>Reminders</u>: 1) HW2 due Friday 1/25 due to MLK holiday on Monday 1/21
2) Read Ch. 2 (Newton's Laws)
3) Midterm #1 tentatively Friday 2/8

* Newton's Laws + object trajectories

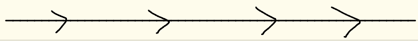
In an <u>inertial</u> reference frame (non-accelerating)

1.) Object stays at rest or moves uniformly if there's no force applied.

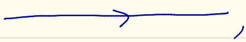
2.) $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \equiv \frac{d}{dt}\vec{p}$ $\quad (\vec{F} \propto \vec{a} \propto \dot{\vec{p}})$

3.)* For 2 objects that exert forces on each other, the forces are equal magnitude + opposite direction. * Only true for special class of forces, see later discussion on momentum conservation.

<u>Example</u>: If $\vec{F}_{Net} = 0$, then 1+2 imply $\vec{r}(t)$ either

• (static) OR 

Straight line

<u>NOTE</u>: If $\vec{r}(t)$: , $\vec{F}_{Net}$ not necessarily 0 (i.e. speed changes, but not direction.)

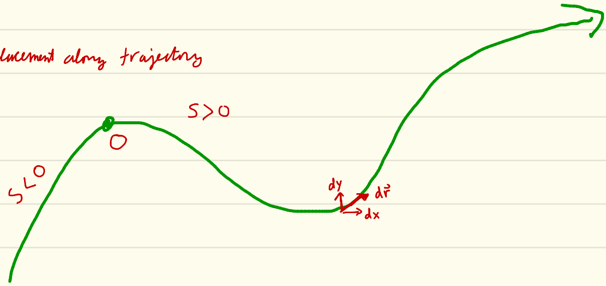
If $\vec{r}(t)$: 

$\vec{a} \neq 0$ necessarily $(\vec{F}_{Net} \neq 0)$

* General observations about trajectories

Consider a particle's trajectory:

$s = \int_0^{\vec{r}} ds$ = displacement along trajectory



* Let $\hat{e}_{\parallel} \equiv \dfrac{d\vec{r}}{ds}$ = Unit vector tangent (i.e., parallel) to trajectory

$$\hat{e}_{\parallel} \cdot \hat{e}_{\parallel} = \frac{d\vec{r}}{ds} \cdot \frac{d\vec{r}}{ds} \stackrel{?}{=} 1$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$ds = \pm |d\vec{r}| = \pm\sqrt{dx^2 + dy^2 + dz^2}$$

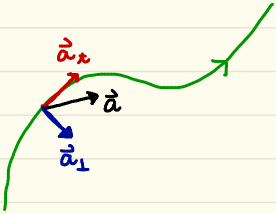
$$\therefore \hat{e}_{\parallel} \cdot \hat{e}_{\parallel} = \frac{ds^2}{ds^2} = 1$$

Yup. $\hat{e}_{\parallel} = \dfrac{d\vec{r}}{ds}$ is a unit vector

Velocity: $\vec{v} \equiv \dfrac{d\vec{r}}{dt} = \dfrac{d\vec{r}}{ds}\dfrac{ds}{dt} = \hat{e}_{\parallel} v$ speed $v = \dfrac{ds}{dt}$ $\left(\begin{array}{l} >0 \text{ if in + direction} \\ <0 \text{ if in − dir.} \end{array} \right)$

$\therefore$ **$\vec{v}$ tangent to $\vec{r}(t)$ at all times**

* Acceleration:



Useful breakdown of $\vec{a}$

$$\vec{a} = \vec{a}_\parallel + \vec{a}_\perp$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v\,\hat{e}_\parallel)$$

$$\therefore\ \vec{a} = \frac{dv}{dt}\hat{e}_\parallel + v\frac{d}{dt}\hat{e}_\parallel$$

tangent to $\vec{r}$

Is this purely $\perp$ to $\vec{r}$?

Is $\hat{e}_\parallel \cdot \frac{d}{dt}\hat{e}_\parallel = 0$?

$\hat{e}_\parallel \cdot \hat{e}_\parallel = 1$ (at all times t)

$$\therefore\ \frac{d}{dt}(\hat{e}_\parallel \cdot \hat{e}_\parallel) = 2\,\hat{e}_\parallel \cdot \frac{d\hat{e}_\parallel}{dt} = \frac{d}{dt}(1) = 0$$

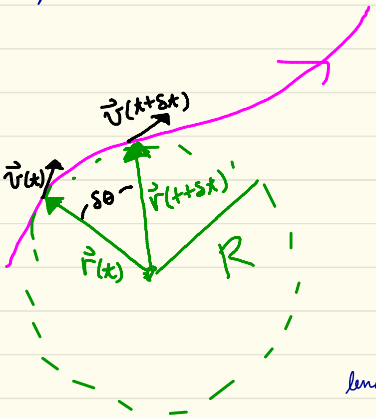
$\Rightarrow$ $\hat{e}_\parallel$ + $\frac{d}{dt}\hat{e}_\parallel$ are indeed perpendicular

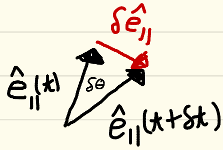
Therefore, we have

$$\boxed{\begin{aligned} \vec{a}_\parallel &= \frac{dv}{dt}\hat{e}_\parallel \quad &\text{(changing speed)} \\ \vec{a}_\perp &= v\frac{d}{dt}\hat{e}_\parallel \quad &\text{(changing direction)} \end{aligned}}$$

* Angular Velocity

Around any point of trajectory, we can treat instantaneous motion as circular motion for circle of radius R





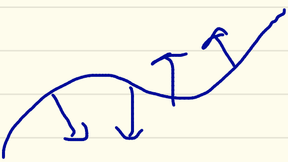
recall, $\vec{a}_{\perp} = v \frac{d\hat{e}_{\parallel}}{dt}$

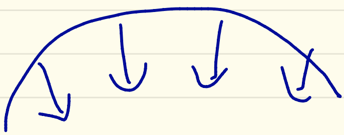
length $\delta e_{\parallel} = e_{\parallel}\,\delta\theta = \delta\theta \quad (e_{\parallel} \equiv |\hat{e}_{\parallel}| = 1)$

$$\therefore \quad \frac{\delta e_{\parallel}}{\delta t} = \frac{\delta\theta}{\delta t} = \omega = \frac{v}{R}$$

$\Rightarrow a_{\perp} = v\omega = \frac{v^2}{R}$ and $\vec{a}_{\perp}$ points towards the instantaneous center of curvature.

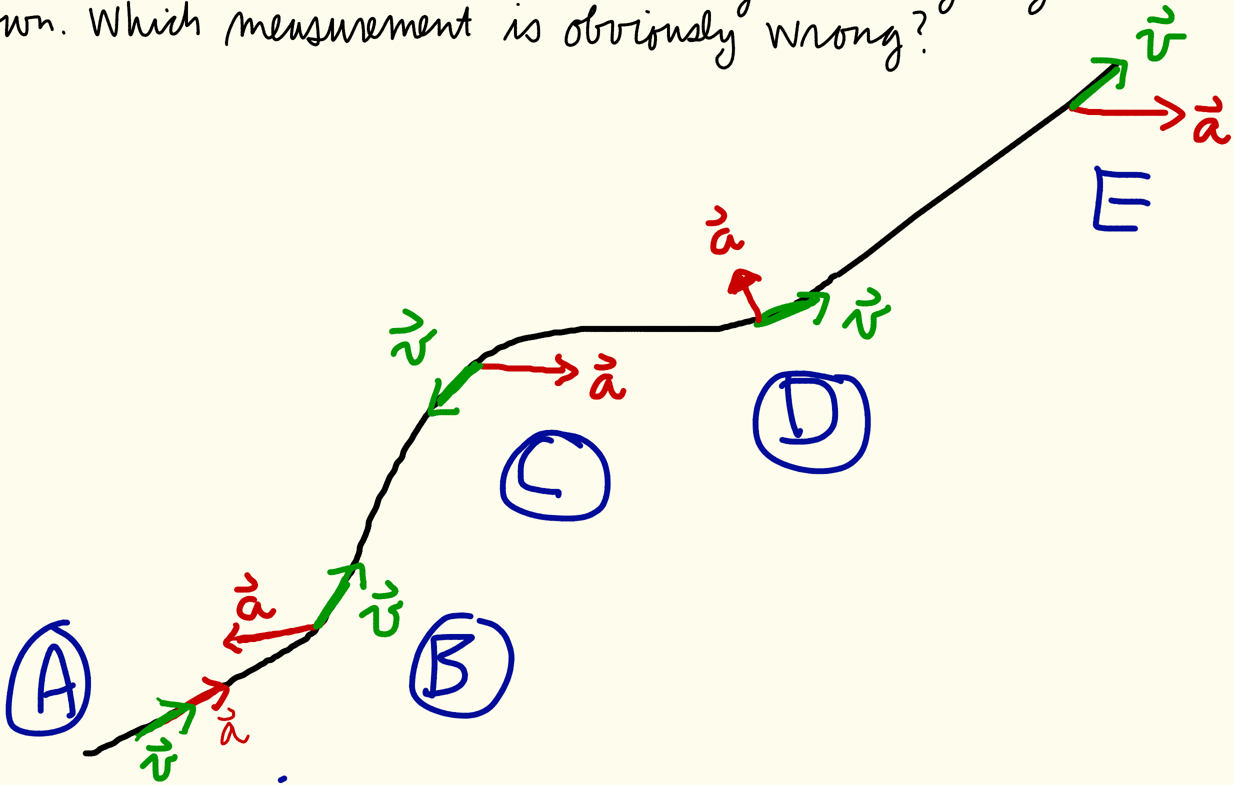
$\therefore$ $\vec{a}$ will always point to the side which trajectory curves



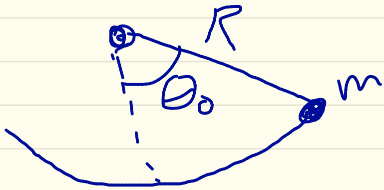


Clicker ?

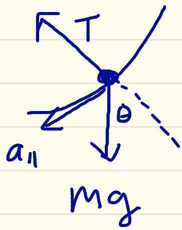
A student measures $\vec{a}(t)$ & $\vec{v}(t)$ along the trajectory shown. Which measurement is obviously wrong?



Example: Pendulum released from $\theta = \theta_0$ w/ mass m.



* Find tension T of string in terms of θ & θ_0



$$\vec{F} = m\vec{a}$$

$$m a_{\parallel} = -mg\sin\theta$$

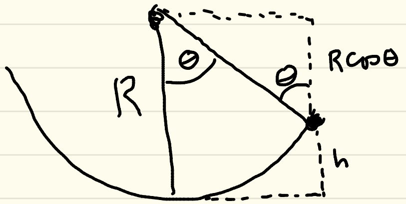
$$m a_{\perp} = T - mg\cos\theta$$

*here, the circular motion is exact (i.e., not just for small δt)

$$\therefore\ m a_{\perp} = m\frac{v^2}{R} = T - mg\cos\theta$$

$$\boxed{\Rightarrow T = \frac{mv^2}{R} + mg\cos\theta}$$

* How to eliminate v^2? Energy conservation!



<u>Energy Conservation:</u> $\underset{(KE)}{\frac{1}{2}mv^2} + \underset{(PE)}{mgh} = E \qquad (\dot{E} = 0)$

$$h + R\cos\theta = R \Rightarrow h = R(1-\cos\theta)$$

$$\therefore E(\theta) = \frac{1}{2}mv^2 + mgR(1-\cos\theta) = E(\theta_0) \underset{KE=0}{=} mgR(1-\cos\theta_0)$$

Solving: $mv^2 = 2mgR(\cos\theta - \cos\theta_0)$ + plug into $T = \frac{mv^2}{R} + mg\cos\theta$

$$\boxed{\Rightarrow T = \frac{2mg\cancel{R}(\cos\theta-\cos\theta_0)}{\cancel{R}} + mg\cos\theta = mg(3\cos\theta - 2\cos\theta_0)}$$