Useful definitions and Formulas

NOTE: This set of equations is not necessarily complete, but it can serve as a basis for your own equation sheets.

$\sin\theta$=opposite/hypotenuse; $\cos\theta$=adjacent/hypotenuse; $\tan\theta$=opposite/adjacent

Phytagorean theorem: $c^2=a^2+b^2$

Solution for t to a quadratic equation $at^2+bt+c=0$

$$t_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Average velocity $v_{average}=\Delta x/\Delta t$;

Average acceleration $a_{average}=\Delta v/\Delta t$; Instantaneous acceleration: $a=\lim_{\Delta t\to 0}(\Delta v/\Delta t)$

Displacement x as a function of time t with constant acceleration a: $x(t)=x(0)+v(0)t+½at^2$

Velocity v as a function of time t with constant acceleration a: $v(t)=v(0)+at$

Gravitational acceleration on earth surface: $g=9.81\ m/s^2$

Horizontal component of velocity of an object moving at velocity v, at an angle θ with the horizontal: $v_x=v\cos\theta$

Vertical component of velocity of an object moving at velocity v, at an angle θ with the horizontal: $v_y=v\sin\theta$

Newton's first law: if $\Sigma F=0$ the object continues in the original state of motion

Newton's second law: $\Sigma F=ma$

Newton's third law: $F_{1\to 2}=-F_{2\to 1}$

Newton's law of universal gravitation: $F_g=Gm_1m_2/r^2$ $G=6.67\times10^{-11}\ Nm^2/kg^2$

Weight: $w=mg$; Gravitational constant of planet with mass M and radius r: $g=GM/r^2$

Normal force n: elastic force acting perpendicular to the supporting surface

Force of static friction: $F_{static}\le\mu_s n$; μ_s: coefficient of static friction

Force of kinetic friction: $F_{kinetic}=\mu_k n$; μ_k: coefficient of kinetic friction

Work $W=F\cos\theta\ \Delta x$

Kinetic energy $KE=1/2\ mv^2$

Gravitational potential energy $PE=mgy$; Elastic potential energy $PE=1/2\ kx^2$

Principle of conservation of mechanical energy $KE_i+PE_i= KE_f+PE_f$

Work done by non conservative forces $W_{nc}= (KE_i+PE_i) - (KE_f+PE_f)$

Average Power $P=W/\Delta t$; Average Power for a constant force $P=Fv$

Linear momentum $p=mv$

Impulse = $F\ \Delta t$

Impulse-momentum theorem $F\ \Delta t=\Delta p$

Conservation of momentum (no net external force) $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

Conservation of kinetic energy (in elastic collisions only) $½ m_1 v_{1i}^2+ ½ m_2 v_{2i}^2= ½ m_1 v_{1f}^2+ ½ m_2 v_{2f}^2$

Average angular speed $\omega=\Delta\theta/\Delta t$; Average angular acceleration $\alpha=\Delta\omega/\Delta t$
Displacement θ as a function of time t with constant acceleration α: $\theta(t)=\theta(0)+\omega(0)t+½\alpha t^2$
Angular speed as a function of time t with constant acceleration α: $\omega(t)=\omega(0)+\alpha t$
Relations between tangential (linear) and angular speed and acceleration: $v_t=\omega r$ and $a_t=\alpha r$
Centripetal acceleration $a_c=v_t^{\,2}/r$
Kepler's third law $T^2=(4\pi^2/GM_s)\,r^3$

Angular momentum: for point-like object $L = mr^2\omega = mrv$; for rigid body: $L \equiv I\omega$;
Rotational K.E. of a rigid body = $½ I\omega^2$
Conservation of Total Mechanical Energy: $E = ½ I\omega^2 + ½ m v^2 + mgh$ (*if there is no friction*)
Angular momentum of a system is conserved (*provided there is no net torque on the system*).
Angular acceleration due to a net Torque: $\Sigma\tau=I\alpha$
Moments of inertia I: Cylindrical shell: $I=MR^2$; Solid Cylinder: $I=½MR^2$
Solid Sphere: $I=2/5MR^2$; Thin spherical shell: $I=2/3MR^2$
Thin Rod with axis through center: $I=1/12ML^2$
Thin rod with axis through end: $1/3ML^2$

Density: $\rho = M / V$ (mass / volume) ; Pressure : $P = F / A$ (force / area)
Specific Gravity= ρ/ρ_{water}, with $\rho_{water}=1.0\text{x}10^3\ kg/m^3$
Young's modulus : $Y = \frac{F}{A}\frac{L}{\Delta L}$
Pressure vs. Depth in Fluids:
$P = P_0 + \rho g h$ (P_0 : pressure at surface)
$P_1 + \rho g h_1 = P_2 + \rho g h_2$ (comparing two depths)
Buoyancy and Archimedes' principle:
$B = \rho_f g V = M_f g$ (*f* denotes the displaced fluid)
Fluid (nonviscous, incompressible) Flow: $A v = constant$ (constant flow rate)
Bernoulli's Equation $P + ½\rho v^2 + \rho g h = constant$
Poiseuille's Law for viscous fluid: $Rate\ of\ flow = \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}$

Temperature conversions: $T_C = T_K - 273.15°$ $T_F = 1.8\,T_C + 32°$
Thermal expansion: $\Delta L = \alpha L_0 \Delta T$; $\Delta A = \gamma A_0 \Delta T$; $\Delta V = \beta V_0 \Delta T$
($\gamma \cong 2\alpha$) ($\beta \cong 3\alpha$)
The Ideal Gas Law: $PV = nRT = Nk_BT$; R=8.31 J / mol °K
Avogadro's Number: $N_A = 6.02 \times 10^{23}$ molecules/mol.
Kinetic theory of gas : $(½ m v^2)_{average} = (3/2)(R/N_A)T$
Root-mean-square speed of molecule (mass m; molar mass M) in ideal gas: $v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$

Heat capacity c: $Q = mc\Delta T$ 1 calorie = 4.186 J
Specific heat of ice: 2090 J/kg^{0}C (0.5 cal/g^{o}C); Specific heat of water: 4186 J/kg^{0}C (1 cal/g^{o}C)
Specific heat of steam: 2010 J/kg^{0}C (0.48 cal/g^{o}C)

Latent heat *L*: $Q = mL$
Latent heat of fusion for water: $3.33\text{x}10^5$ J/kg (79.7 cal/g)
Latent heat of vaporization for water: $2.26\text{x}10^6$ J/kg (540 cal/g)
Heat conductivity *k* : rate of heat transfer = $P = \frac{\Delta Q}{\Delta t} = kA\left(\frac{\Delta T}{\Delta x}\right)$

Through a compound slab: $P = \frac{A\Delta T}{\sum_i (L_i / k_i)}$

Power of radiation energy (Stefan's Law): $P=\sigma AeT^4$

Rate of radiation energy transfer: $P=\sigma Ae(T^4 - T_0^4)$

Wien's Displacement Law: $\lambda_{max}T=0.2898\text{x}10^{-2}$ mK

Work done <u>on</u> a gas at constant pressure $W_{on\ gas}= -P\ \Delta V$; $\Delta V = V_{final}-V_{initial}$

(Note: work done <u>by</u> the gas is $W_{by\ gas} = P\ \Delta V$)

First law of thermodynamics: $\Delta U = Q - P\ \Delta V$ (true for both definition of work)

Work done by a Heat engine $W_{eng}=|Q_{hot}|-|Q_{cold}|$

Thermal efficiency $\varepsilon=W_{eng}/Q_{hot}=1-|Q_{cold}|/|Q_{hot}|$

Thermal efficiency for an ideal engine (the Carnot cycle): $\varepsilon=1-T_{cold}/T_{hot}$

Change in Entropy in a reversible process involving heat transfer of Q: $\Delta S=Q_r/T$

Elastic force (Hooke'e Law): F = -kx

Acceleration in the Simple Harmonic Oscillator (S.H.O.) a=-(k/m) x

Elasctic potential energy PE=1/2 kx^2

Velocity in a simple harmonic oscillator $v = \sqrt{\frac{k}{m}(A^2 - x^2)}$

Position, velocity and acceleration of a S.H.O (if x is at a maximum at t=0):

$x(t)=A\cos(\omega t)$; $v(t)=-\omega A\sin(\omega t)$; $a(t)=-\omega^2 A\cos(\omega t)$

Period and angular frequency of a simple harmonic oscillator $T = 2\pi\sqrt{\frac{m}{k}}$ and $\omega = 2\pi f = \sqrt{\frac{k}{m}}$

Total energy of a S.H.O.: $E = ½kx^2+½mv^2 = ½kA^2 = ½mv_{max}^2$

Period a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

Relation between the speed, the wavelength, and the frequency of a wave: $v=\lambda f$

Speed of a wave on a stretched string $v = \sqrt{\frac{F}{\mu}}$

Speed of sound wave in a medium $v = \sqrt{\frac{B}{\rho}}$

Speed of sound as a function of temperature: $v_{sound}=331\sqrt{(T/273)}$

Intensity of wave propagation: I = Energy transfer per unit area per unit time = Average Power / Area

Intensity of sound in decibel $\beta=10 \log (I/I_0)$ where $I_0=1.0\text{x}10^{-12}\text{W/m}^2$

Intensity of a spherical wave $I=P_{av}/(4\pi r^2)$

Doppler effect $f' = f\,\frac{v+v_O}{v-v_S}$

Standing waves on a stretched string $f = n\frac{v}{2L}$

Standing waves in a tube of air open in both sides $f = n\frac{v}{2L}$ with n=1,2,3,…

Standing waves in a tube of air closed in one side and open in the other $f = n\frac{v}{4L}$ with n=1,3,5,…