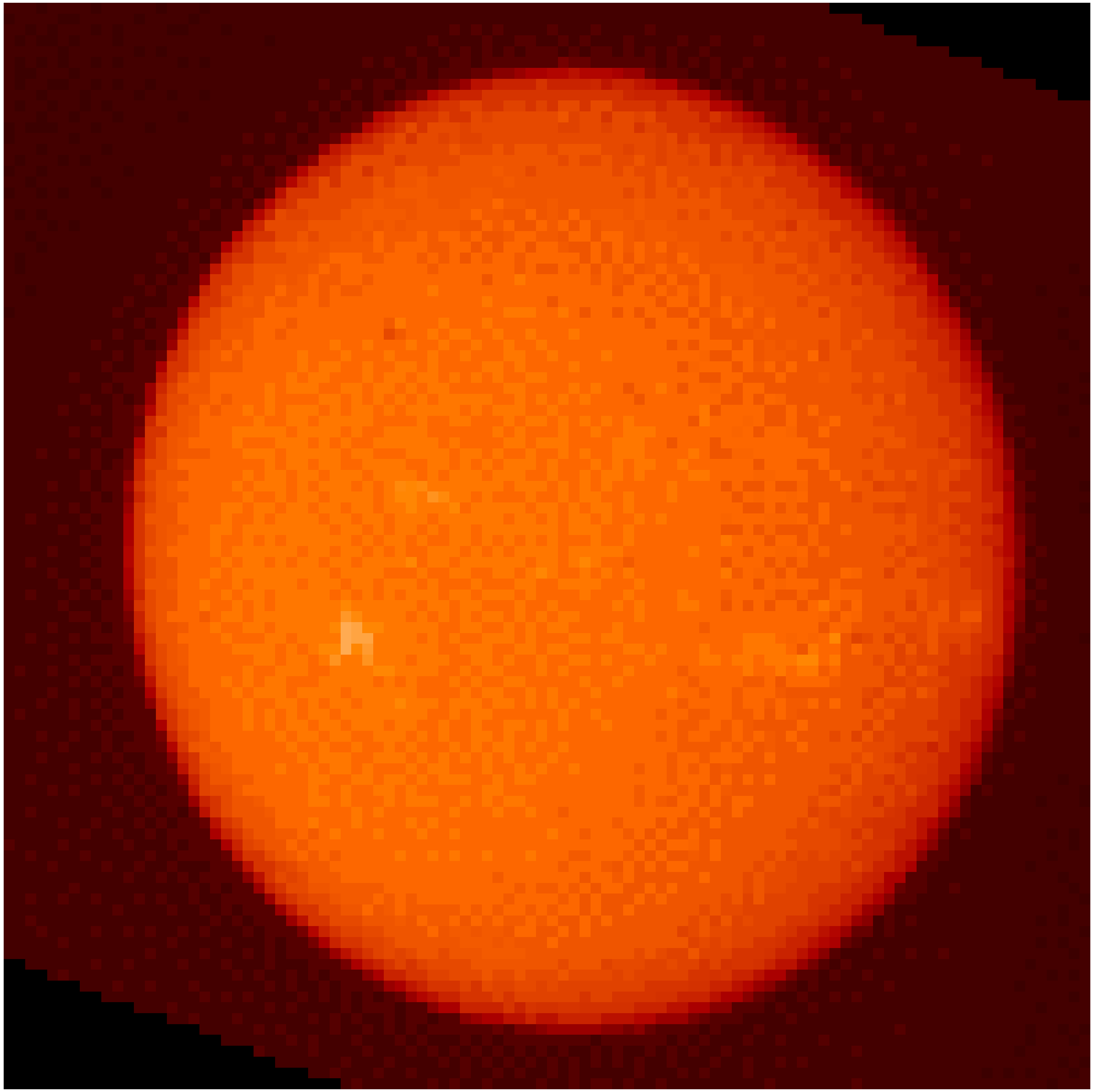
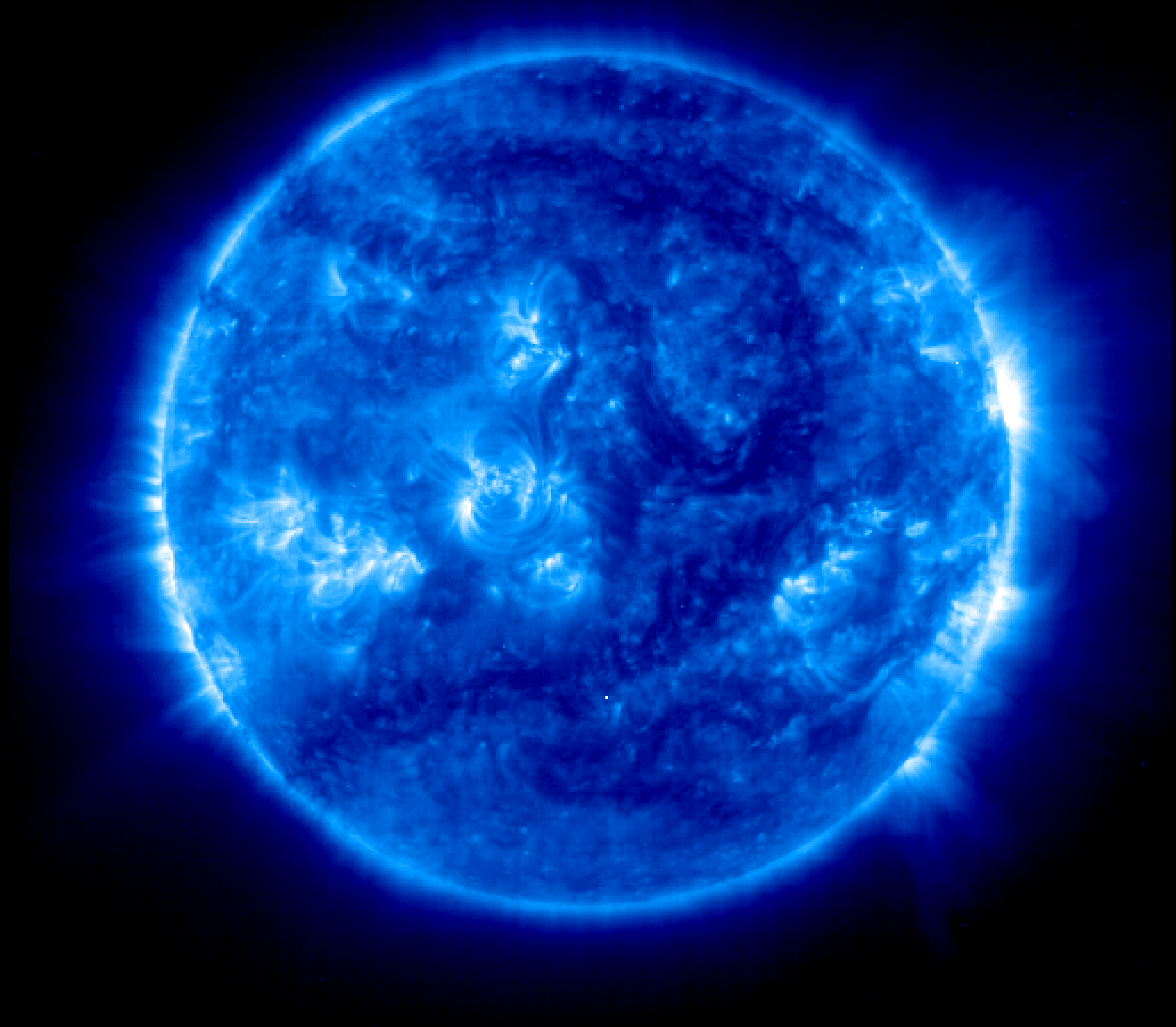
The sun shines 3.85e33 erg/s = 3.85e26 Watts for at least ~4.5 bio years





SOHO, 171A Fe emission line

… and its all nuclear physics:

- 1905 Einstein finds $E=mc^2$
- 1920 Aston measures mass defect of helium (!= 4p's)
- 1920 Nuclear Astrophysics is born with Sir Arthur Eddington remarks in his presidential address to the British Association for the Advancement of Science:

> "Certain physical investigations in the past year make it probable to my mind that some portion of sub-atomic energy is actually set free in the stars … If only five percent of a star's mass consists initially of hydrogen atoms which are gradually being combined to form more complex elements, the total heat liberated will more than suffice for our demands, and we need look no further for the source of a star's energy"

> "If, indeed, the sub-atomic energy in the stars is being freely used to maintain their great furnaces, it seems to bring a little nearer to fulllment our dream of controlling this latent power for the well-being of the human race|or for its suicide."

The p-p chains - ppI

As a star forms density and temperature (heat source ?) increase in its center

Fusion of hydrogen (^{1}H) is the first long term nuclear energy source that can ignite. Why ?

With only hydrogen available (for example in a first generation star right after it's formation) the ppI chain is the only possible sequence of reactions.
(all other reaction sequences require the presence of catalyst nuclei)

3- or 4-body reactions are too unlikely – chain has to proceed by steps of 2-body reactions or decays.

The ppI chain

Step 1: $\cancel{p + p \longrightarrow {}^2He}$ — ^{2}He is unstable

$$\boxed{p + p \longrightarrow d + e^+ + \nu_e}$$

Step 2: $$\boxed{d + p \longrightarrow {}^3He}$$

$\cancel{d + d \longrightarrow {}^4He}$ — d abundance is too low

Step 3: $\cancel{{}^3He + p \longrightarrow {}^4Li}$ — ^{4}Li is unstable

$\cancel{{}^3He + d \longrightarrow {}^4He + n}$ — d abundance is too low

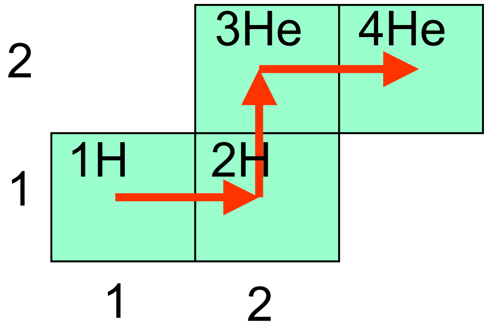
$$\boxed{{}^3He + {}^3He \longrightarrow {}^4He + 2p}$$

d+d not going because Y_d is small as d+p leads to rapid destruction

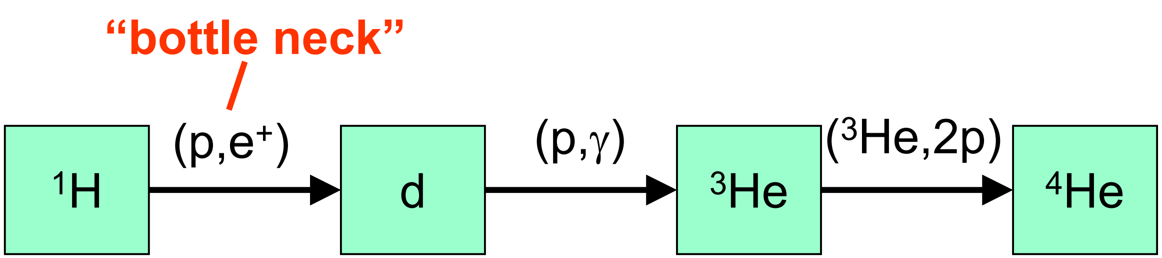
^{3}He+^{3}He goes because Y_{3He} gets large as there is no other rapid destruction

To summarize the ppI chain:

On chart of nuclides:



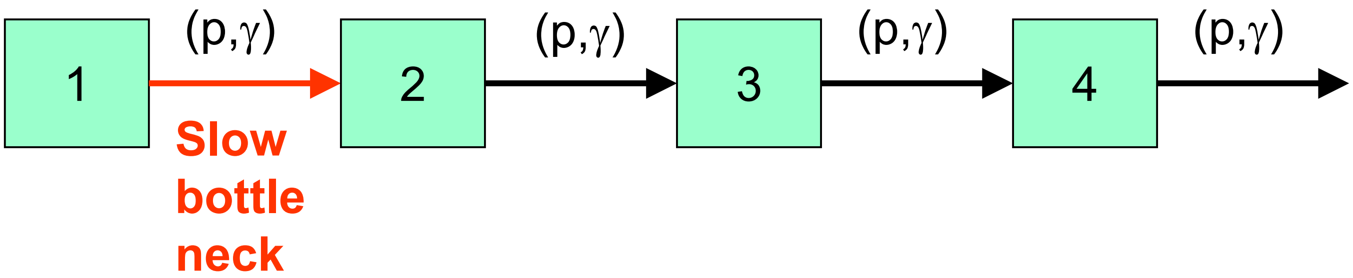
Or as a chain of reactions:



Sidebar: A chain of reactions after a "bottle neck"

Steady Flow

For simplicity consider chain of proton captures:



Assumptions:

- Y_1 ~ const as depletion is very slow because of "bottle neck"
- Capture rates constant (Y_p ~ const because of large "reservoir", conditions constant as well)

Abundance of nucleus 2 evolves according to:

$$\frac{dY_2}{dt} = \underbrace{Y_1\lambda_{12}}_{\text{production}} - \underbrace{Y_2\lambda_{23}}_{\text{destruction}}$$

$$\lambda_{12} = \frac{1}{1+\delta_{p1}} Y_p \rho \mathrm{N_A} < \sigma v >_{1 \longrightarrow 2}$$

For our assumptions Y_1~const and Y_p~ const, Y_2 will then, after some time reach an equilibrium value regardless of its initial abundance:

In equilibrium:

$$\frac{dY_2}{dt} = Y_1\lambda_{12} - Y_2\lambda_{23} = 0 \qquad \text{and} \qquad Y_2\lambda_{23} = Y_1\lambda_{12}$$

(this is equilibrium is called steady flow)

Same for Y_3 (after some longer time)

$$\frac{dY_3}{dt} = Y_2\lambda_{23} - Y_3\lambda_{34} = 0$$

and $Y_3\lambda_{34} = Y_2\lambda_{23}$ with result for Y_2: $Y_3\lambda_{34} = Y_1\lambda_{12}$

and so on …

So in steady flow: $Y_i\lambda_{i\,i+1} = \mathrm{const} = Y_1\lambda_{12}$ or $Y_i \propto \tau_i$

steady flow abundance destruction rate

Timescale to achieve steady flow equilibrium

for λ~const

$$\frac{dY_2}{dt} = Y_1\lambda_{12} - Y_2\lambda_{23}$$

has the solution:

$$Y_2(t) = \overline{Y}_2 - (\overline{Y}_2 - Y_{2\,\mathrm{initial}})\,\mathrm{e}^{-t/\tau_2}$$

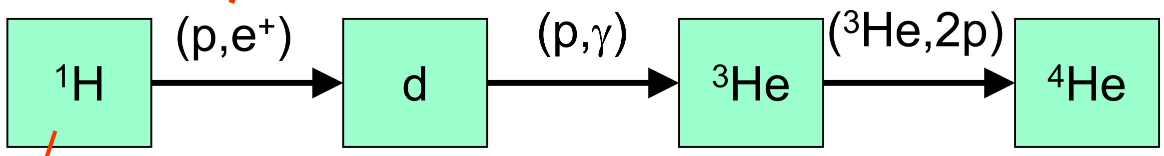
with $\overline{Y}_2$ equilibrium abundance

$Y_{2\,\mathrm{initial}}$ initial abundance

so independently of the initial abundance, the equilibrium is approached on a exponential timescale equal to the lifetime of the nucleus.

Back to the ppI chain:

"bottle neck"



large reservoir (Y_p~const ok for some time)

d steady flow abundance ?

$$Y_d \lambda_{d+p} = \text{const} = Y_p \lambda_{p+p}$$

$$\frac{Y_d}{Y_p} = \frac{\lambda_{p+p}}{\lambda_{d+p}} = \frac{Y_p \frac{1}{2} \rho N_A < \sigma v >_{p+p}}{Y_p \rho N_A < \sigma v >_{d+p}}$$

$$\frac{Y_d}{Y_p} = \frac{< \sigma v >_{p+p}}{2 < \sigma v >_{d+p}}$$

S=3.8e-22 keV barn

S=2.5e-4 keV barn

therefore, equilibrium d-abundance extremely small (of the order of 4e-18 in the sun)

equilibrium reached within lifetime of d in the sun:

$N_A<\sigma v>_{pd}$=1e-2 cm^3/s/mole $\quad \tau_d = 1/(Y_p \rho N_A < \sigma v >_{p+d}) = 2\mathrm{s}$

^{3}He equilibrium abundance

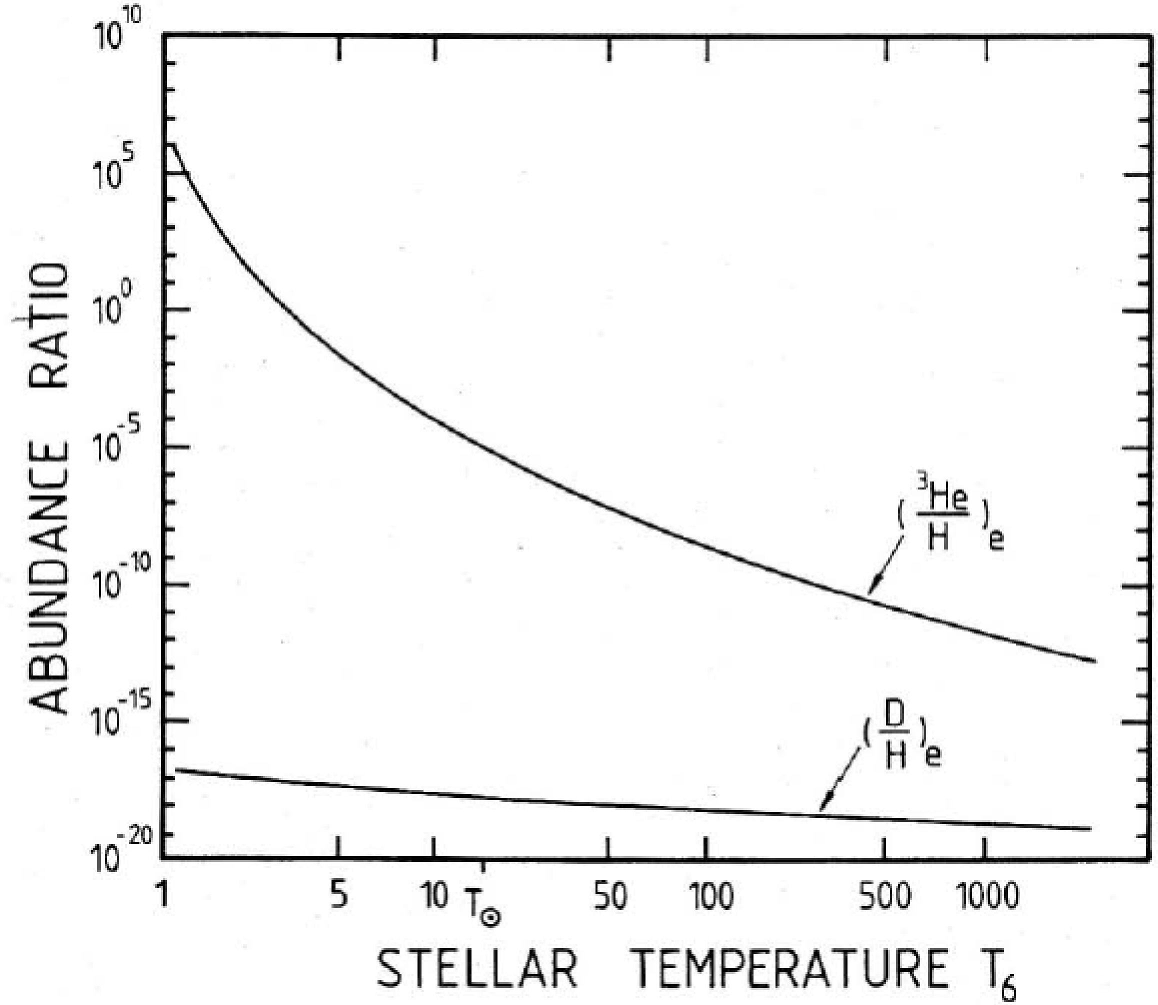
$$ {}^{1}H \xrightarrow{(p,e^+)} d \xrightarrow{(p,\gamma)} {}^{3}He \xrightarrow{({}^{3}He,2p)} {}^{4}He $$

different because two identical particles fuse
therefore destruction rate $\lambda_{3He+3He}$ obviously NOT constant:

$$\lambda_{3He+3He} = \frac{1}{2} Y_{3He} \rho N_A < \sigma v >_{3He+3He}$$

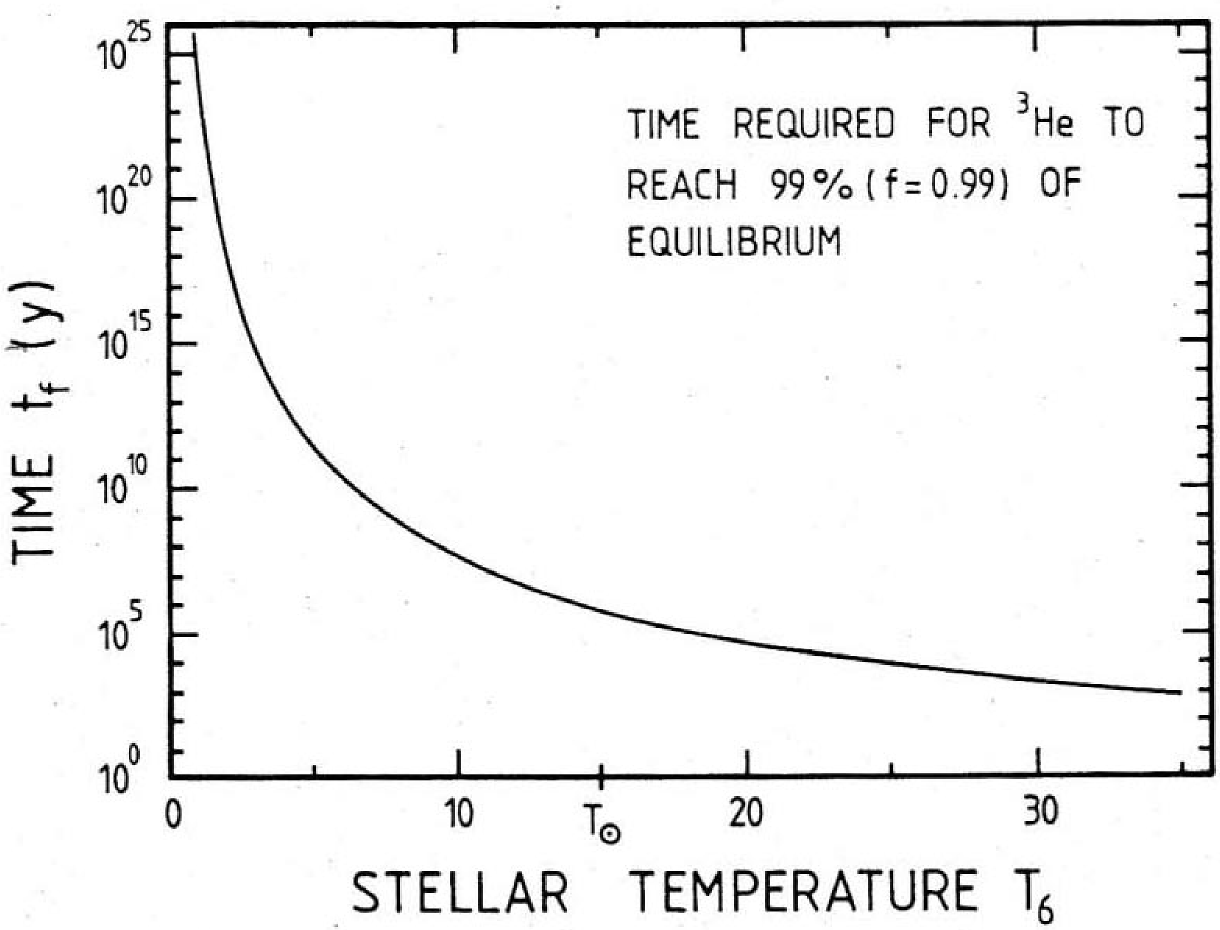
but depends strongly on Y_{3He} itself

But equations can be solved again (see Clayton)



^{3}He has a much higher equilibrium abundance than d

- therefore ^{3}He+^{3}He possible …

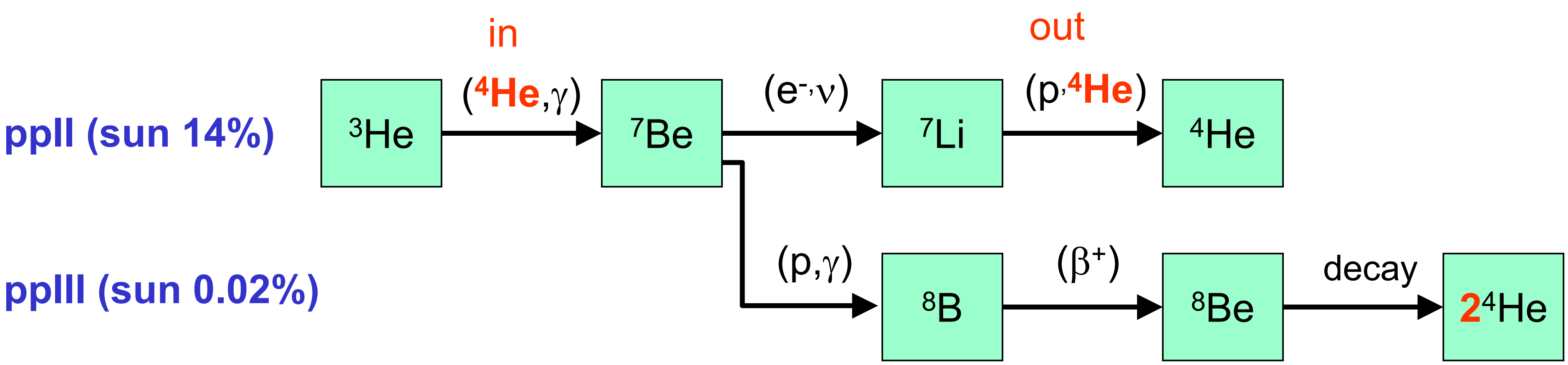

TIME REQUIRED FOR ^{3}He TO REACH 99% (f = 0.99) OF EQUILIBRIUM
TIME t_f (y)
STELLAR TEMPERATURE T_6
10^{25}
10^{20}
10^{15}
10^{10}
10^5
10^0
0
10
$T_\odot$
20
30

Hydrogen burning with catalysts

1. **ppII chain**
2. **ppIII chain**
3. **CNO cycle**

1. ppII and ppIII:

once ^{4}He has been produced it can serve as catalyst of the ppII and ppIII chains to synthesize more ^{4}He:



(Rolfs and Rodney)

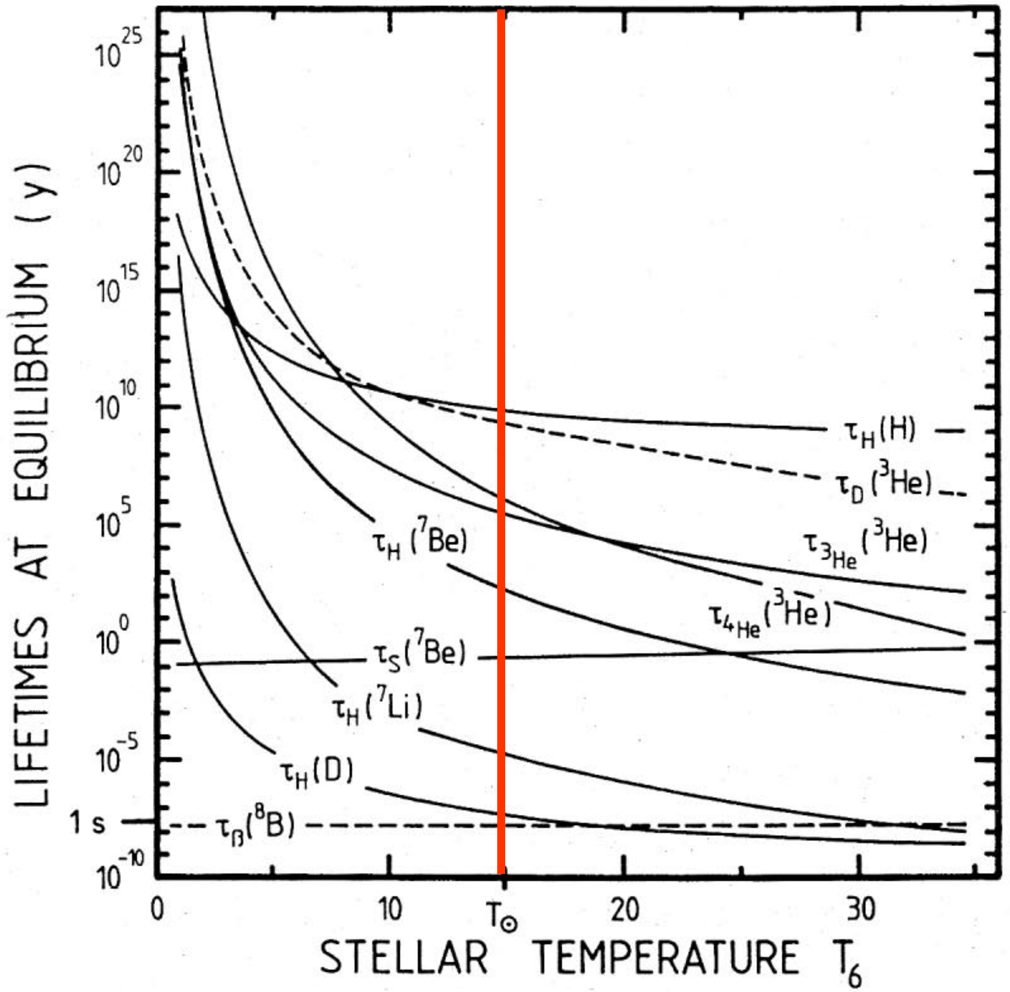


FIGURE 6.7. Plotted are the equilibrium lifetimes of ^{3}He resulting from different burning processes (Table 6.2). The $^3He(\alpha, \gamma)^7Be$ reaction leading to the $\tau_{^4He}(^3He)$–curve is important only in stars which have an appreciable amount of ^{4}He. Shown for comparison is the lifetime of hydrogen against destruction via the $p + p$ reaction and those of D, ^{7}Li, and ^{7}Be against destruction via hydrogen-burning interactions. The electron-capture lifetime of ^{7}Be in stars, $\tau_s(^7Be)$, and the laboratory lifetime of the positron decay for ^{8}B are also shown. All curves assume conditions of $\rho = 100$ g cm^{-3}, $X_H = X_{He} = 0.5$.

Electron capture decay of ^{7}Be

Why electron capture:

Q_{EC}=862 keV

$Q_{\beta+}=Q_{EC}-1022 = -160$ keV

only possible decay mode

Earth: Capture of bound K-electron

$T_{1/2}$=77 days

Sun: Ionized fraction: Capture of continuum electrons

$\longrightarrow$ depends on density and temperature

$$\tau_{^7Be} = 4.72\mathrm{e}8 \frac{T_6^{1/2}}{\rho(1+X_H)} s$$

Not completely ionized fraction: capture of bound K-electron

(21% correction in sun)

$\Big\}$ $T_{1/2}$=120 days

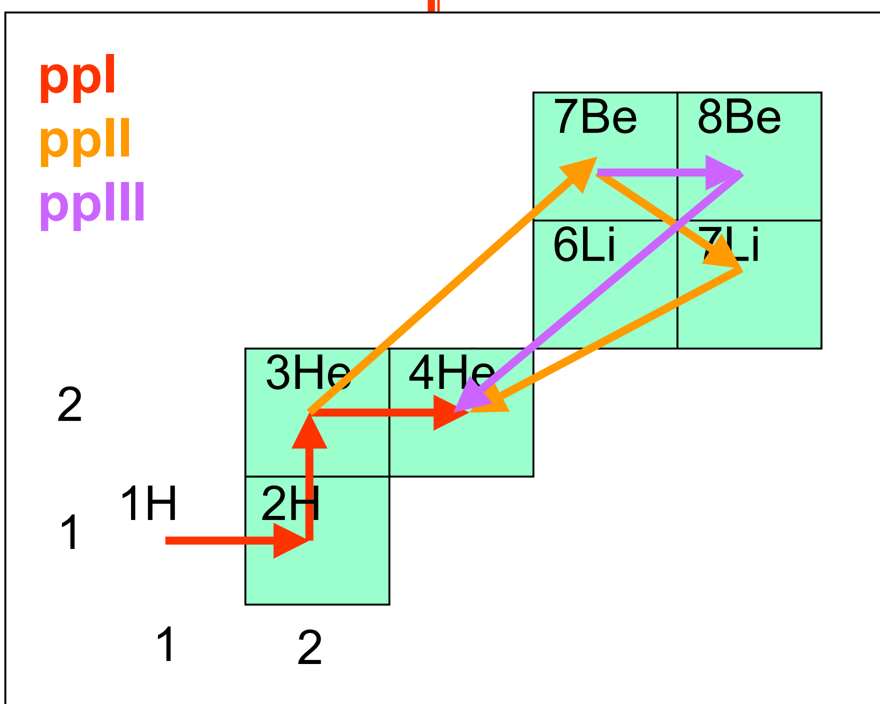
Summary pp-chains:

$p(p,e^+\nu)d$
$d(p,\gamma)^3He$

86% → $^3He(^3He,2p)^4He$

14% → $^3He(\alpha,\gamma)^7Be$

- 14% → $Be(e^-,\nu)^7Li$, $^7Li(p,\alpha)^4He$
- 0.02% → $^7Be(p,\gamma)^8B$, $^8B(e^+,\nu)^8Be$, $^8Be(\alpha)^4He$



Why do additional pp chains matter ?

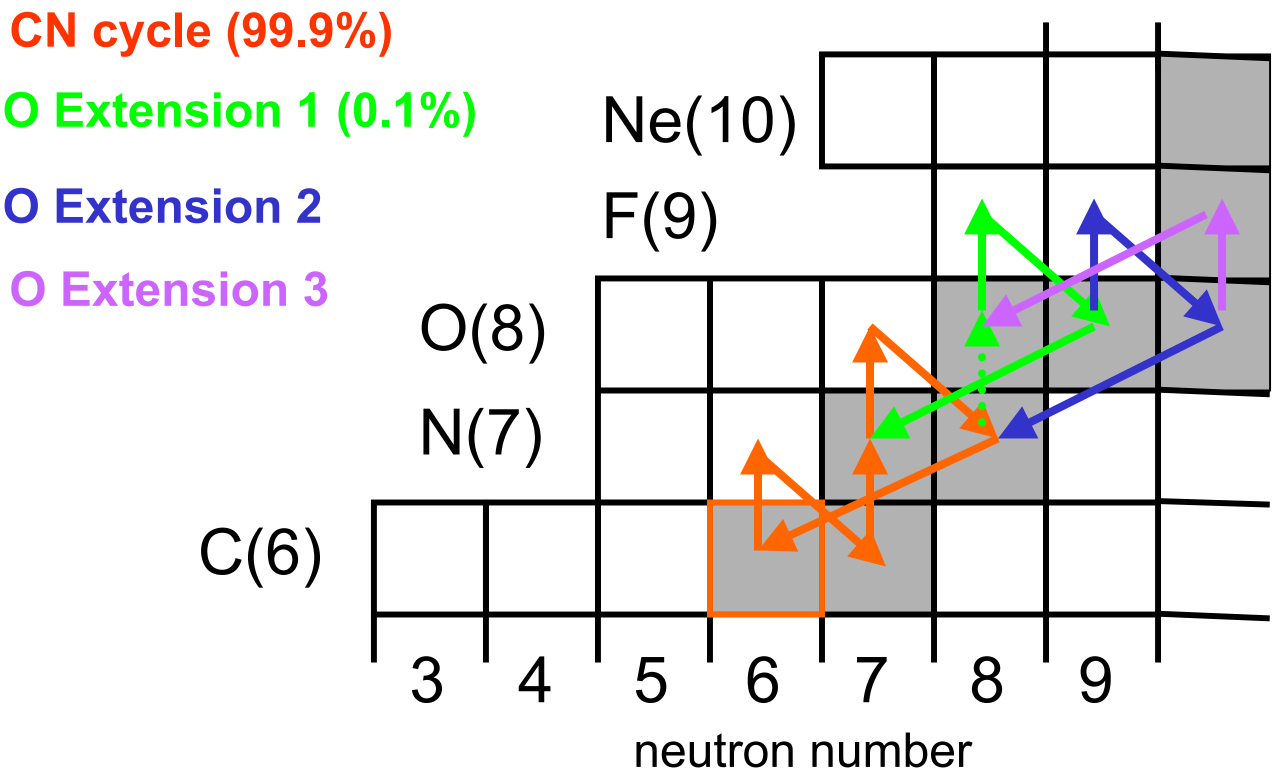
p+p dominates timescale BUT

ppI produces 1/2 ^4He per p+p reaction

ppI+II+III produces 1 ^4He per p+p reaction

→ **double burning rate**

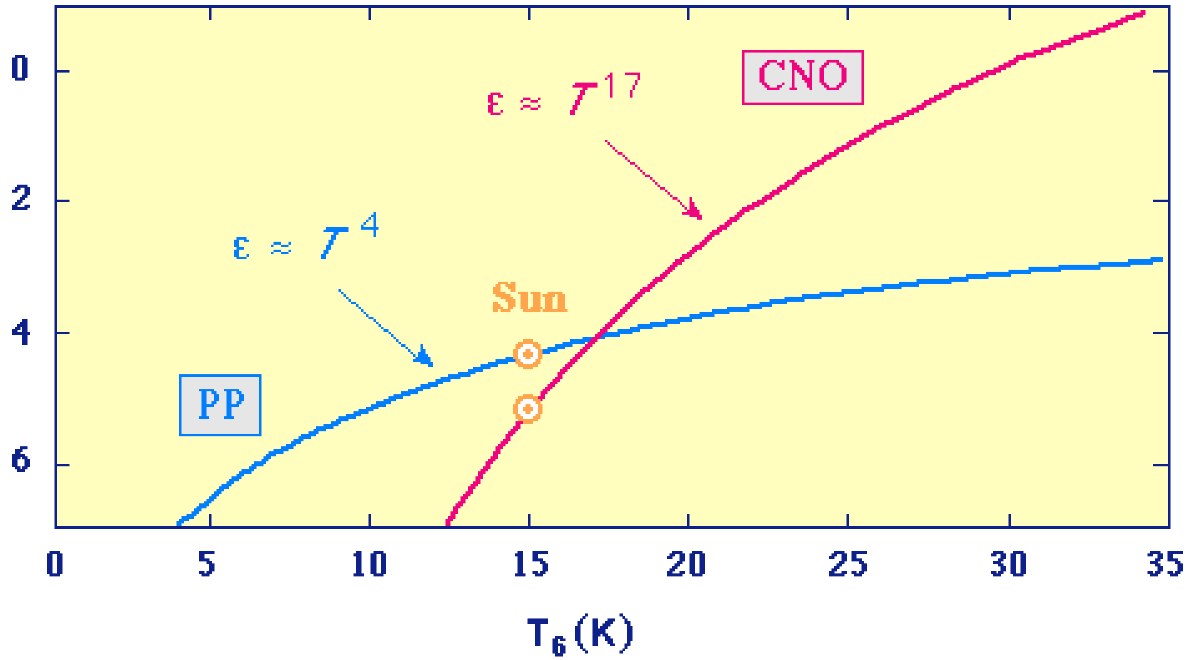
CNO cycle



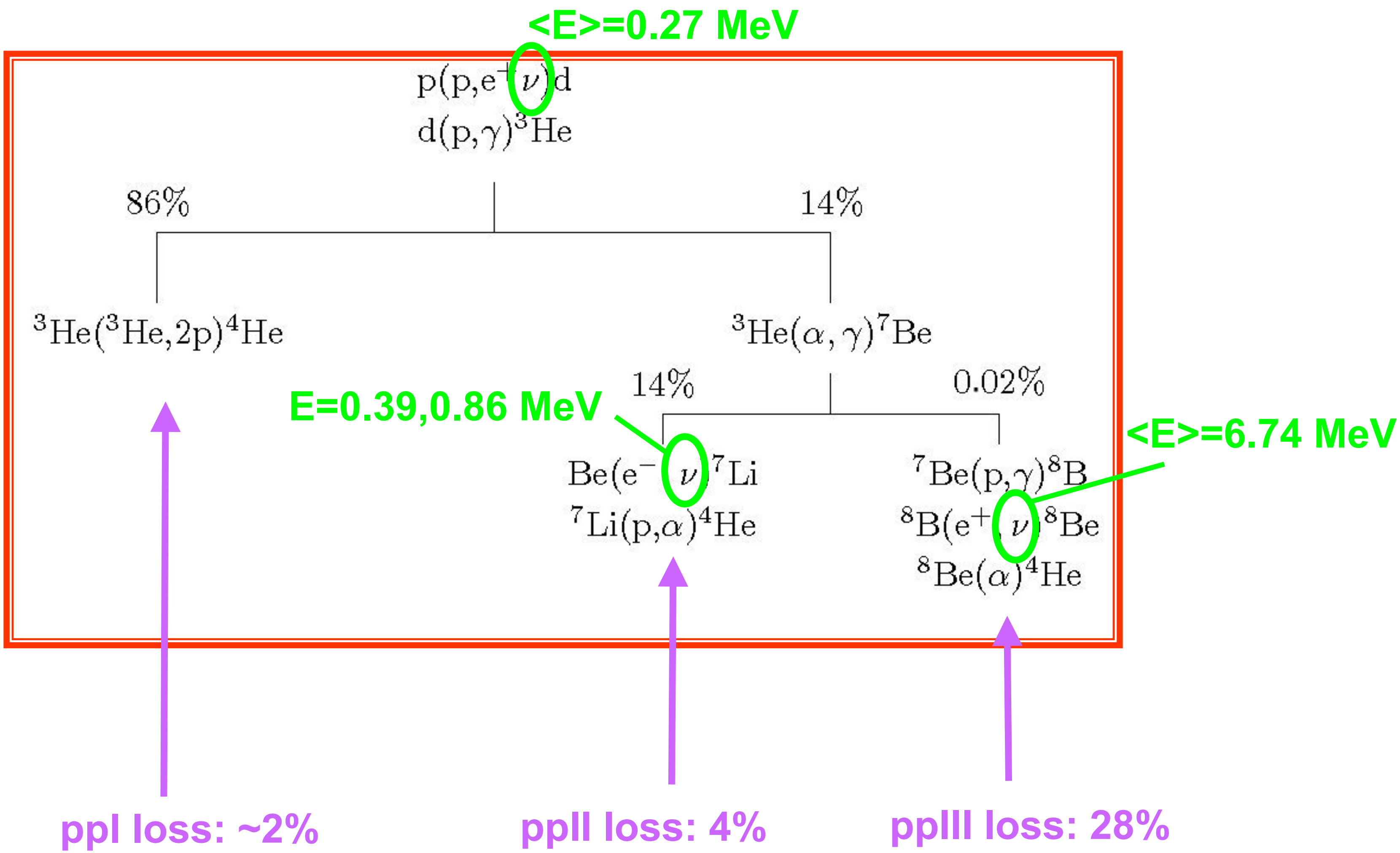
All initial abundances within a cycle serve as catalysts and accumulate at largest τ

Extended cycles introduce outside material into CN cycle (Oxygen, …)

Competition between the p-p chain and the CNO Cycle



Neutrino emission

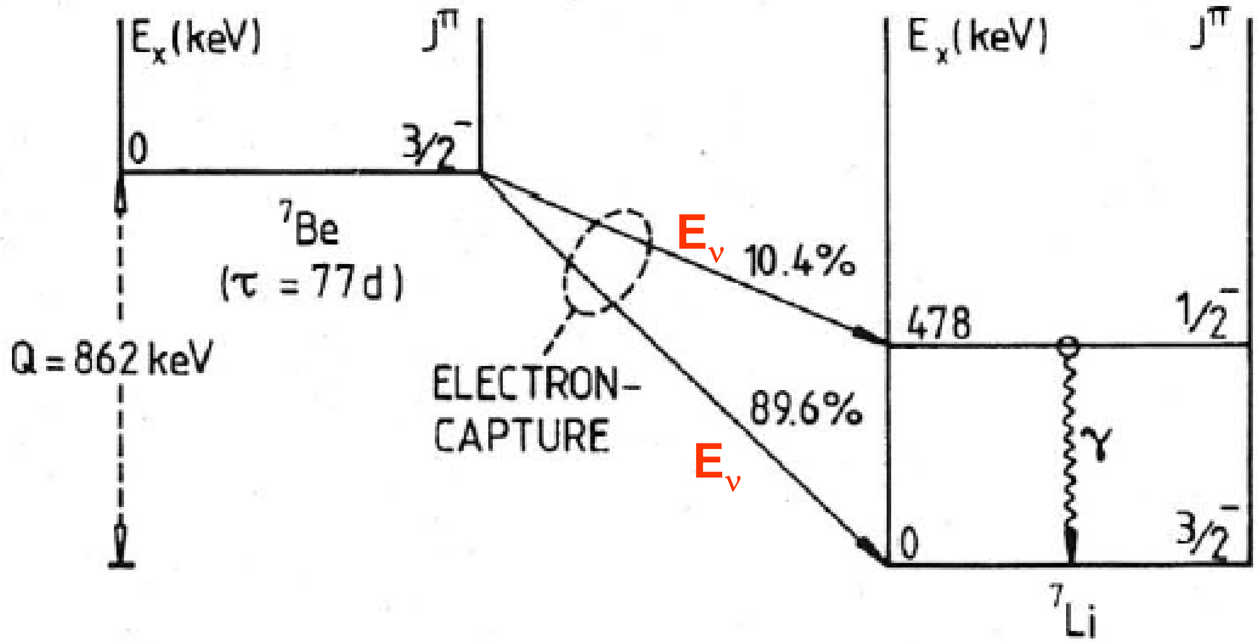


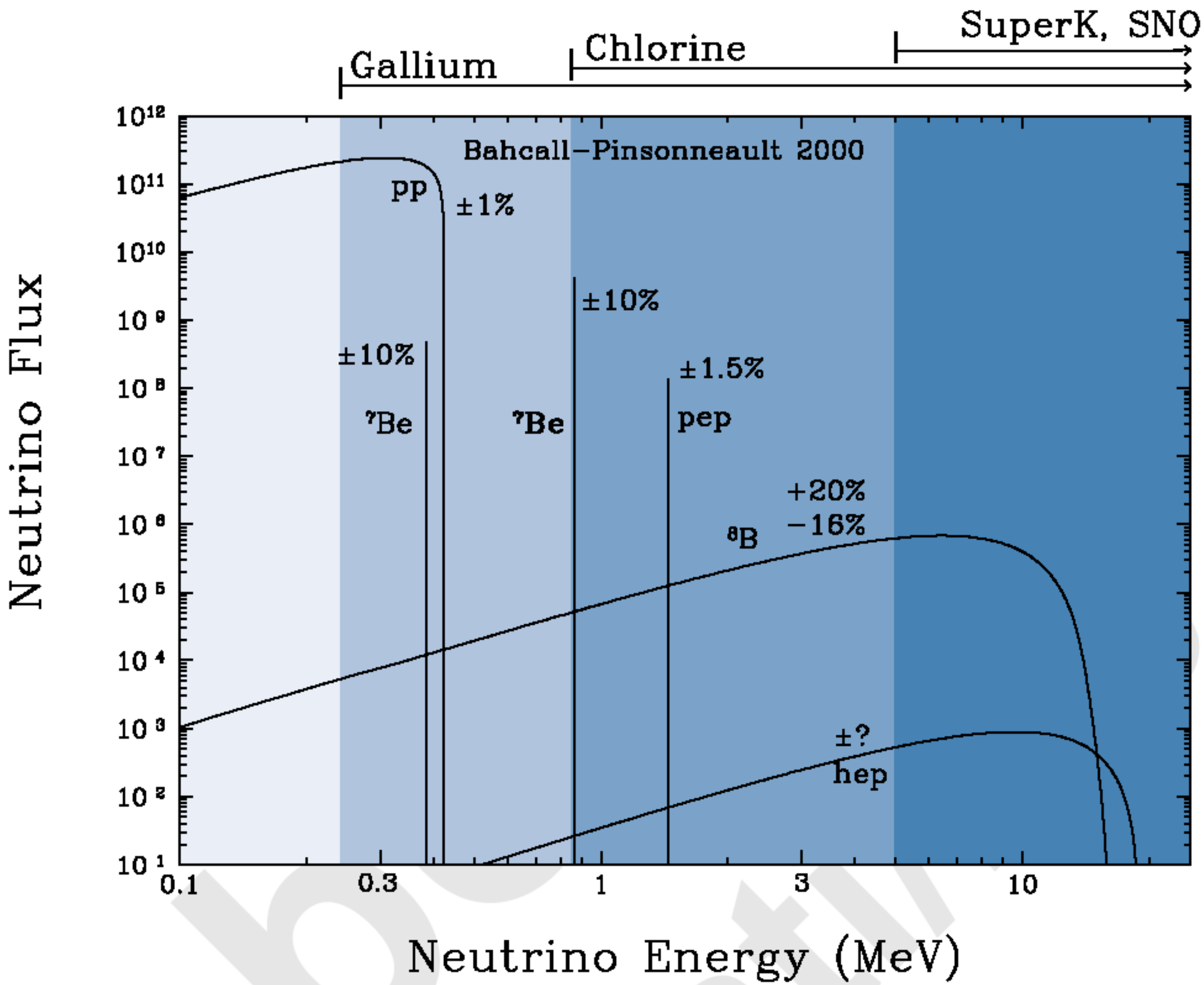
note: <E>/Q=
0.27/26.73 = 1%

Total loss: 2.3%

2 neutrino energies from 7Be electron capture ?

$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_{\varepsilon}$





Continuous fluxes in /cm^2/s/MeV
Discrete fluxes in /cm^2/s

Neutrino Astronomy

Photons emitted from sun are not the photons created by nuclear reactions
(heat is transported by absorption and emission of photons plus convection
to the surface over timescales of 10 Mio years)

But neutrinos escape !

Every second, 10 Bio solar neutrinos pass through your thumbnail !

But hard to detect (they pass through 1e33 g solar material largely undisturbed !)

First experimental detection of solar neutrinos:

- **1964** John Bahcall and Ray Davis have the idea to detect solar neutrinos using the reaction:

$$^{37}Cl + \nu_e \longrightarrow {}^{37}Ar + e^-$$

- **1967** **Homestake experiment starts taking data**
  - 100,000 Gallons of cleaning fluid in a tank 4850 feet underground
  - ^{37}Ar extracted chemically every few months (single atoms !) and decay counted in counting station (35 days half-life)
  - event rate: ~1 neutrino capture per day !
- **1968** **First results: only 34% of predicted neutrino flux !**

solar neutrino problem is born - for next 20 years no other detector !

Neutrino production in solar core ~ T^{25}

$\longrightarrow$ **nuclear energy source of sun directly and unambiguously confirmed**

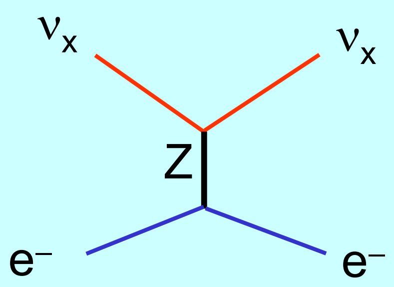
$\longrightarrow$ **solar models precise enough so that deficit points to serious problem**

Are the neutrinos really coming from the sun ?

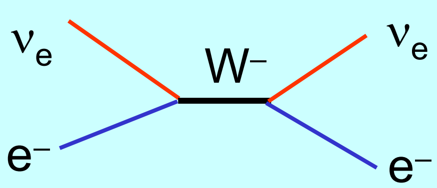
<u>Water Cerenkov detector:</u>

$$\upsilon + e^- \longrightarrow \upsilon + e^-$$

high energy (compared to rest mass)
- produces cerenkov radiation when traveling in water **(can get direction)**



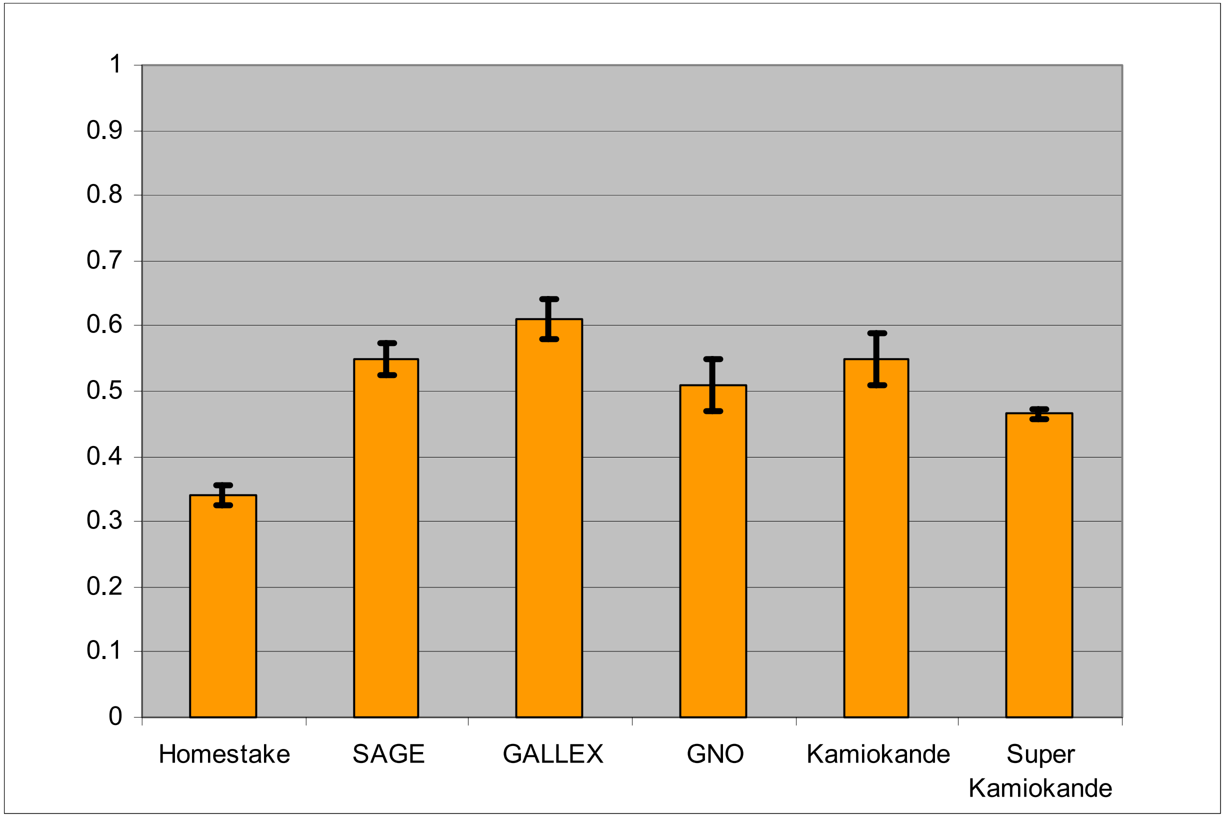
neutral current (NC)



charged current (CC)

Super-Kamiokande Detector

many more experiments over the years with very different energy thresholds:

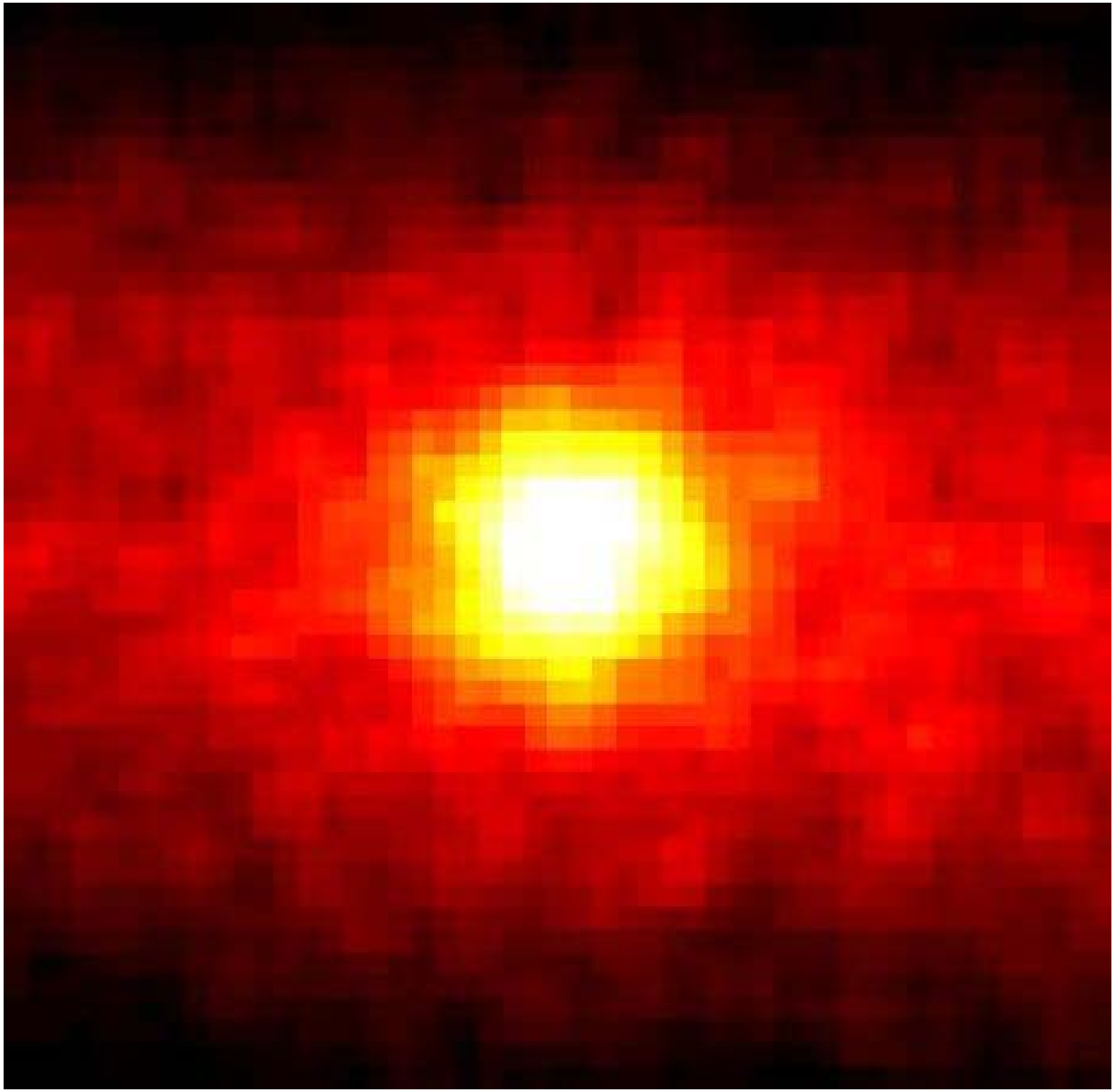


all show deficit to standard solar model

ν_e only

all flavors, but ν_τ,ν_μ only 16% of ν_e cross section because no CC, only NC

Astronomy Picture of the Day June 5, 1998



Neutrino image of the sun by Super-Kamiokande – next step in neutrino astronomy

The solution: neutrino oscillations

Neutrinos can change flavor while travelling from sun to earth

The arguments:

1. SNO solar neutrino experiment

uses three reactions in heavy water:

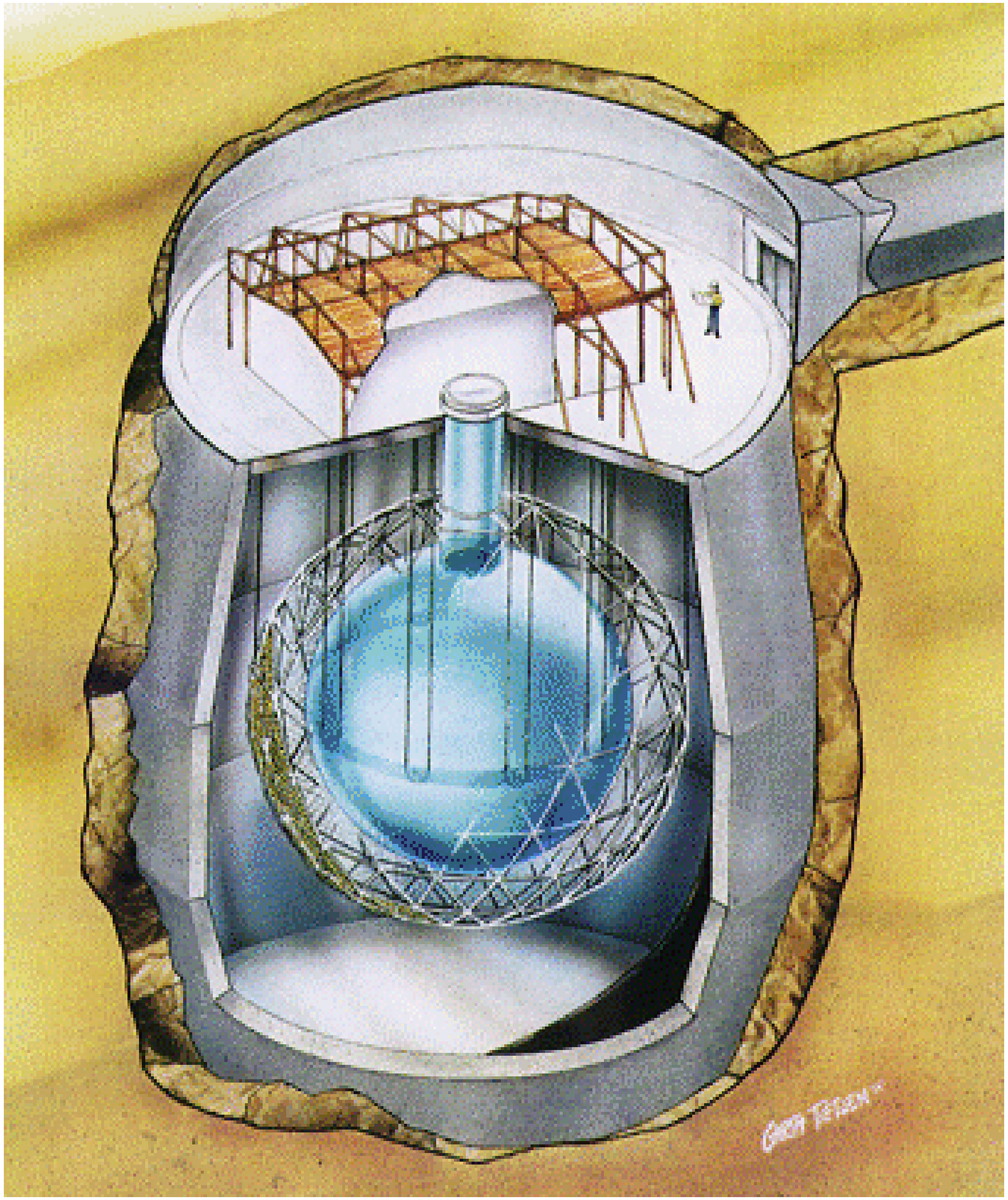
CC $\upsilon_e + d \longrightarrow p + p + e^-$ (Cerenkov)

ES $\upsilon + e^- \longrightarrow \upsilon + e^-$ (Cerenkov)

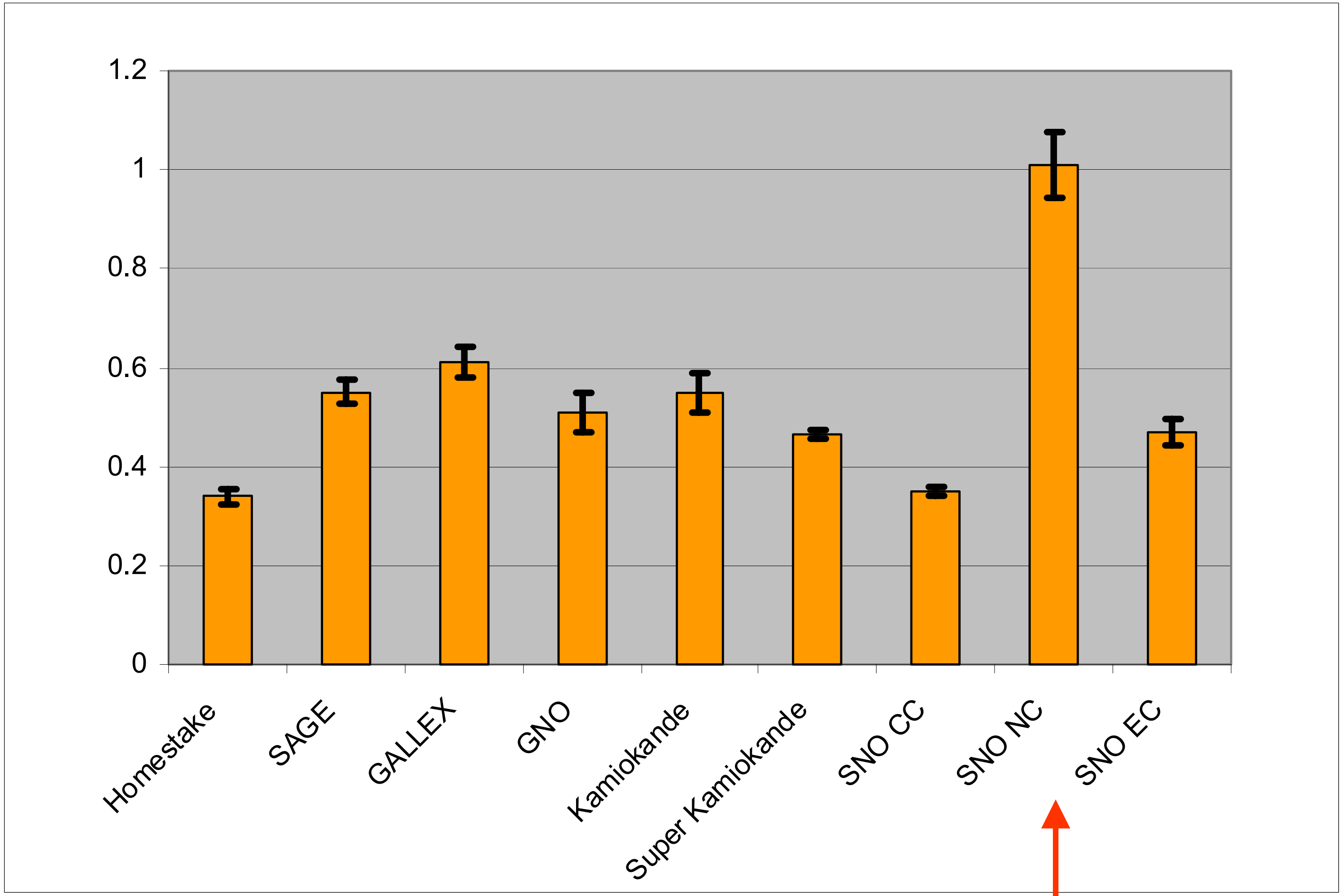
NC $\upsilon + d \longrightarrow p + n + \upsilon$ (n-capture by ^{35}Cl - g scatter - Cerenkov)

key:
- NC independent of flavor - should always equal solar model prediction if oscillations explain the solar neutrino problem
- Difference between CC and ES indicates additional flavors present

Sudbury Neutrino Observatory



With SNO results:



Puzzle solved …

more arguments for neutrino oscillation solution:

2. Indication for neutrino oscillations in two other experiments:

- 1998 Super Kamiokande reports evidence for ν_μ --> ν_τ oscillations for neutrinos created by cosmic ray interaction with the atmosphere
- 2003 KamLAND reports evidence for disappearance of electron anti neutrinos from reactors

3. There is a (single) solution for oscillation parameters that is consistent with all solar neutrino experiments and the new KamLAND results

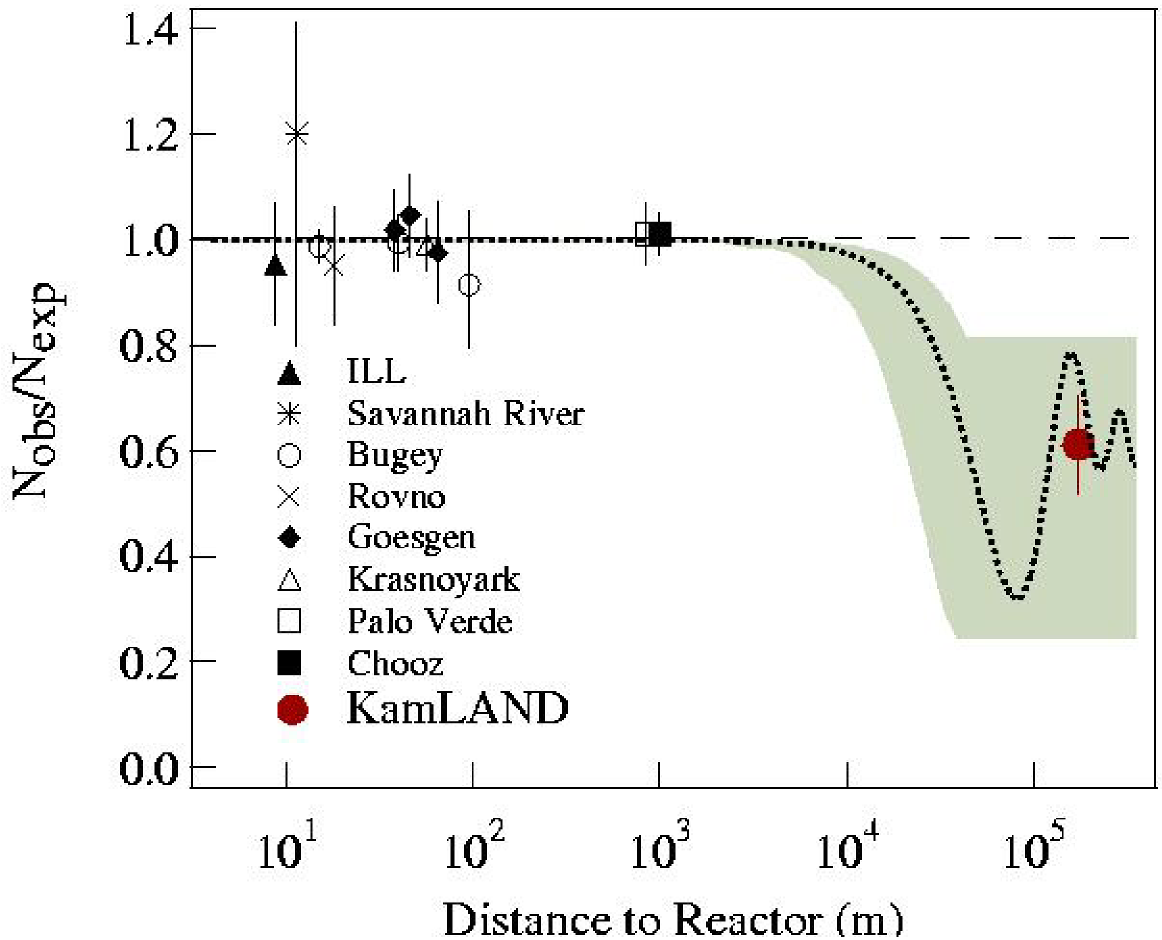
KamLAND:

Reactor proudUces $\overline{\upsilon}_e$ from beta decay of radioactive material in core:

Detection in liquid scintillator tank in Kamiokande mine ~180 km away

$\longrightarrow$ check whether neutrinos disappear

2003 Results:



dashed: Best fit: LMA $\sin^2 2\Theta=0.833$, $\Delta m^2=5.5\text{e-}5\ \text{eV}^2$
shaded: 95% CL LMA from solar neutrino data

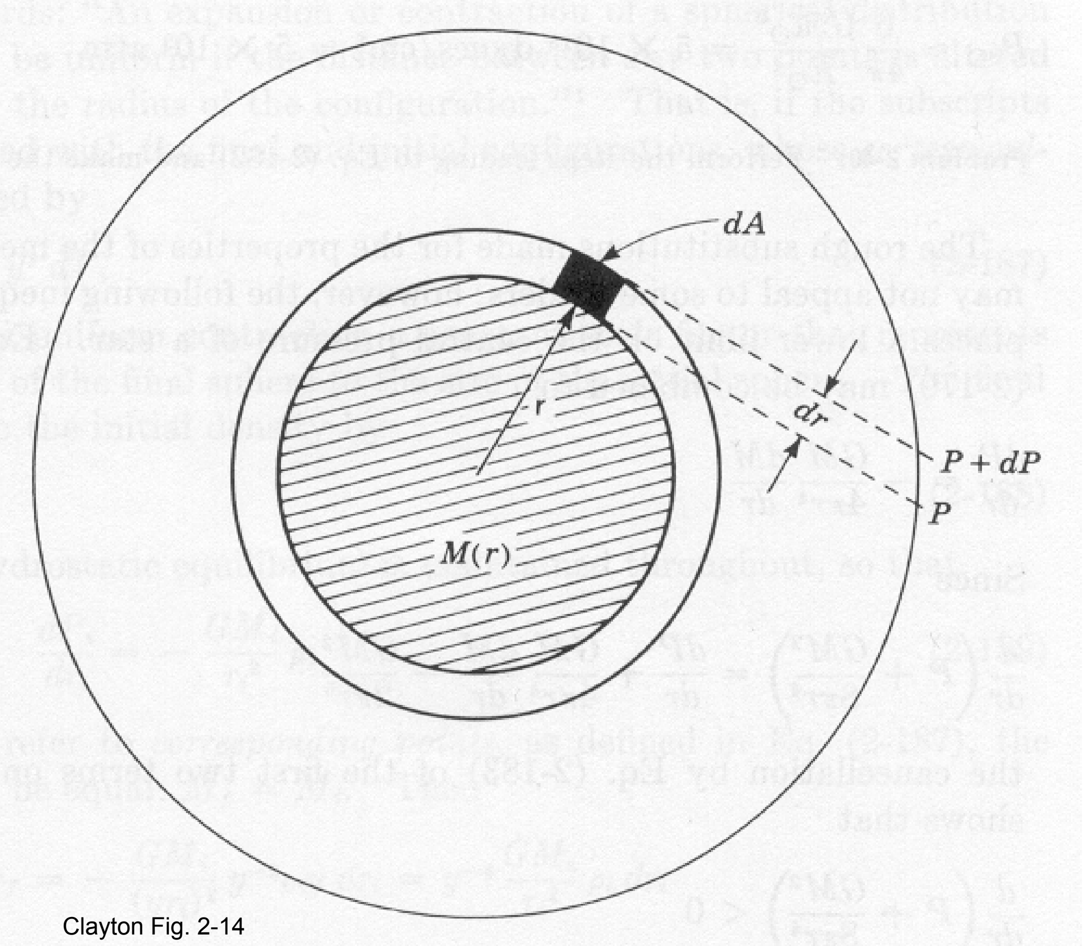
K. Eguchi, PRL 90 (2003) 021802

Properties of stars during hydrogen burning

Hydrogen burning is first major **hydrostatic burning phase** of a star:
Star is "stable" - radius and temperature everywhere do not change drastically with time

Hydrostatic equilibrium:

a fluid element is "held in place" by a pressure gradient that balances gravity



Clayton Fig. 2-14

Force from pressure:

$$F_p = PdA - (P + dP)dA$$
$$= -dPdA$$

Force from gravity:

$$F_G = -GM(r)\rho(r)\,dAdr / r^2$$

For balance: $F_G = F_P$ need:

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}}$$

The origin of pressure: equation of state

Under the simplest assumption of an ideal gas: $P = \rho N_A RT / \mu_I$

$\longrightarrow$ need high temperature !

Keeping the star hot:

The star cools at the surface - energy loss is luminosity L

To keep the temperature constant everywhere luminosity must be generated

In general, for the luminosity of a spherical shell at radius r in the star:

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon$$ (energy equation)

where ε is the energy generation rate (sum of all energy sources and losses) per g and s

Luminosity is generated in the center region of the star (L(r) rises) by nuclear reactions and then transported to the surface (L(r)=const)

Energy transport requires a temperature gradient ("heat flows from hot to cold")

For example for radiative transport ("photon diffusion" - mean free path ~1cm in sun):
to carry a luminosity of L, a temperature gradient dT/dr is needed:

$$L(r) = -4\pi r^2 \frac{4acT^3}{3\kappa\rho} \frac{dT}{dr}$$

a: radiation density constant =7.56591e-15 erg/cm^3/K^4

k is the "opacity", for example luminosity L in a layer r gets attenuated by photon absorption with a cross section σ:

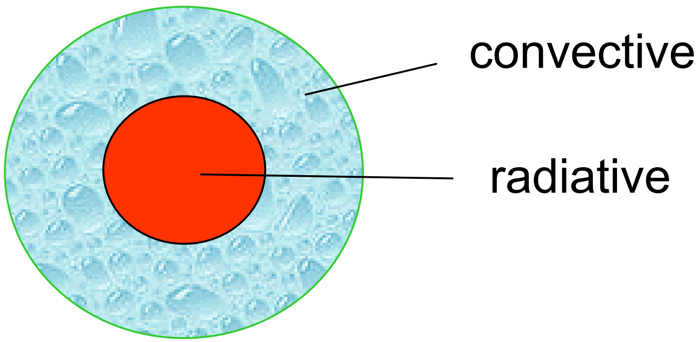
$$L = L_0 e^{-\sigma n r} = L_0 e^{-\kappa \rho r}$$

Therefore the star has a temperature gradient (hot in the core, cooler at the surface)
As pressure and temperature drop towards the surface, the density drops as well.

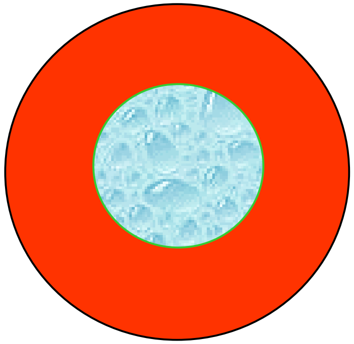
Convective energy transport: takes over when necessary temperature gradient too steep (hot gas moves up, cool gas moves down, within convective zone)

not discussed here, but needs a temperature gradient as well

Stars with $M<1.2\ M_0$ have radiative core and convective outer layer (like the sun):



Stars with $M>1.2\ M_0$ have convective core and radiative outer layer: why ?



(convective core about 50% of mass for $15M_0$ star)

The structure of a star in hydrostatic equilibrium:

Equations so far:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$ (hydrostatic equilibrium)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho\varepsilon$$ (energy)

$$L(r) = -4\pi r^2 \frac{4acT^3}{3\kappa\rho}\frac{dT}{dr}$$ (radiative energy transfer)

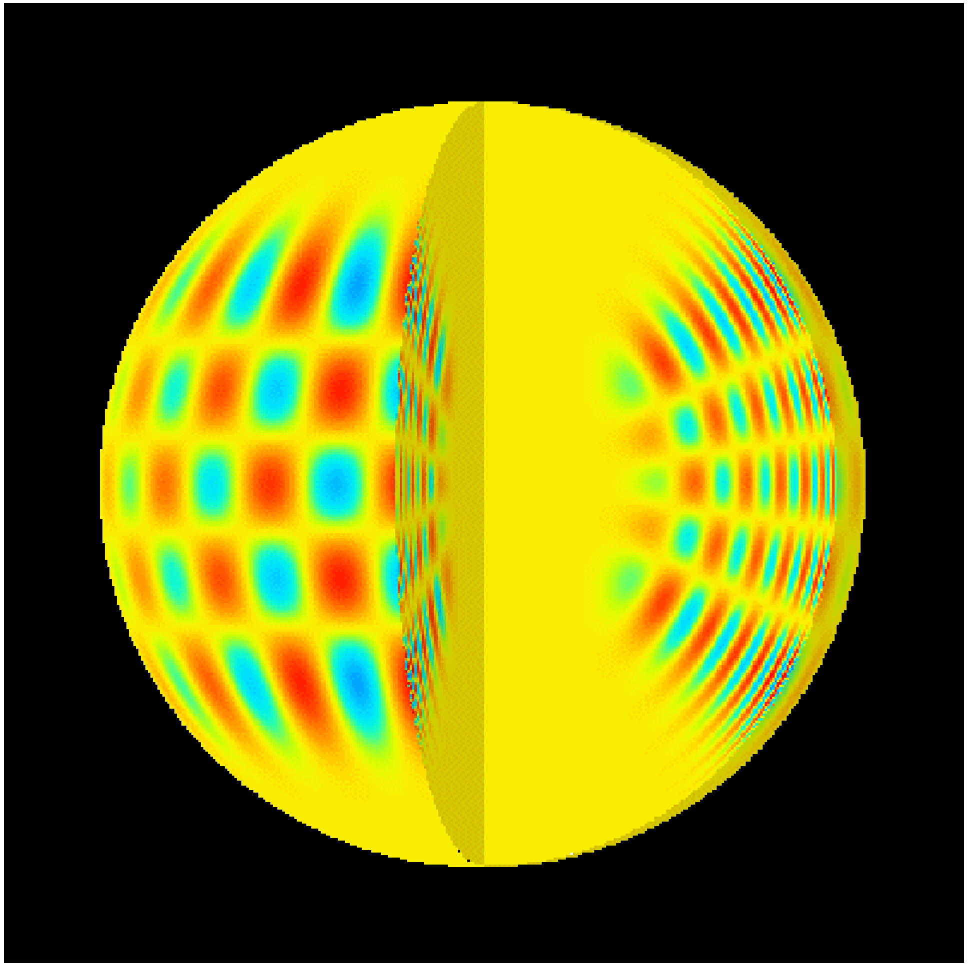
In addition of course: $$\frac{dM(r)}{dr} = 4\pi\rho r^2$$ (mass)

and an equation of state

BUT: solution not trivial, especially as ε,κ in general depend strongly on composition, temperature, and density

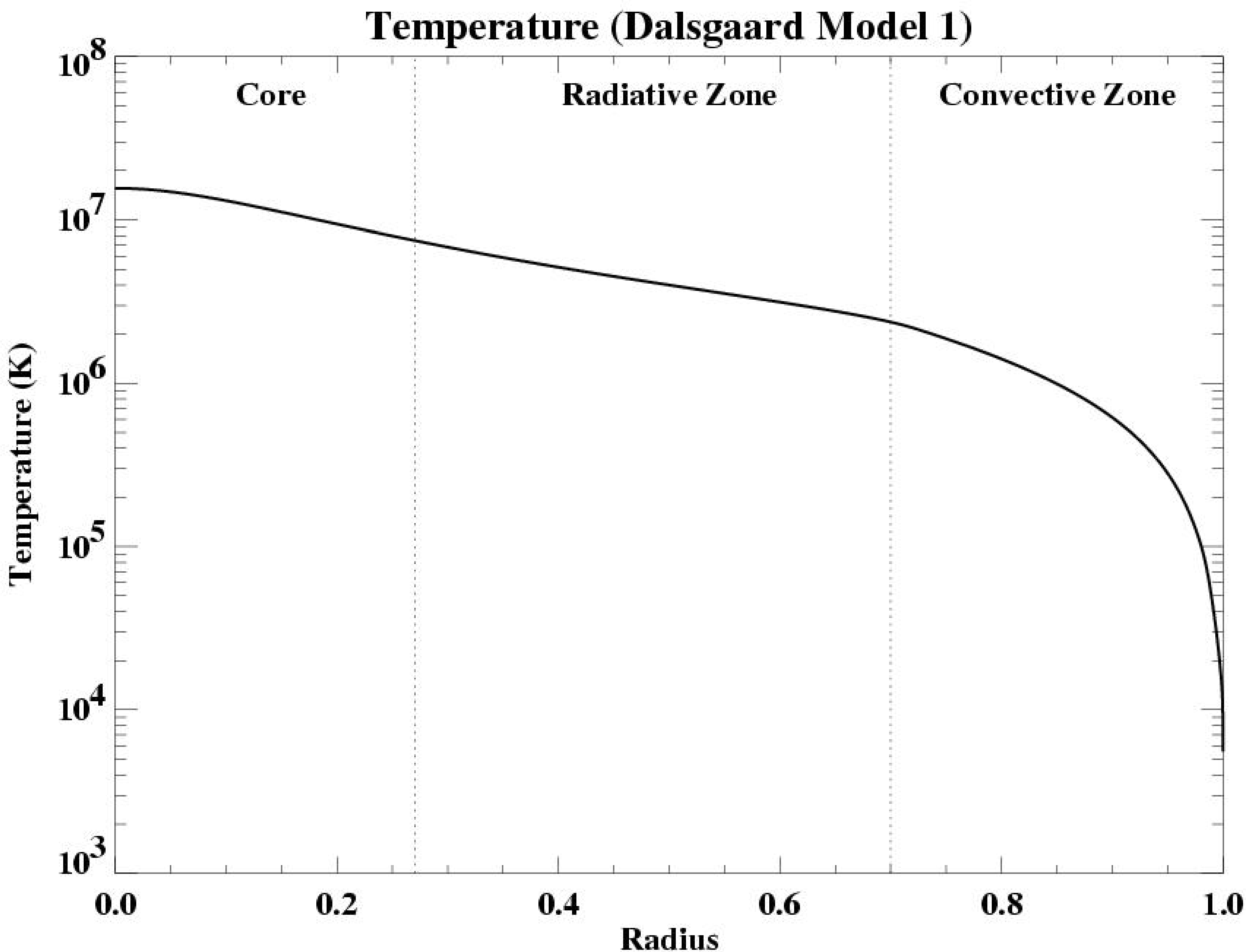
Example: The sun

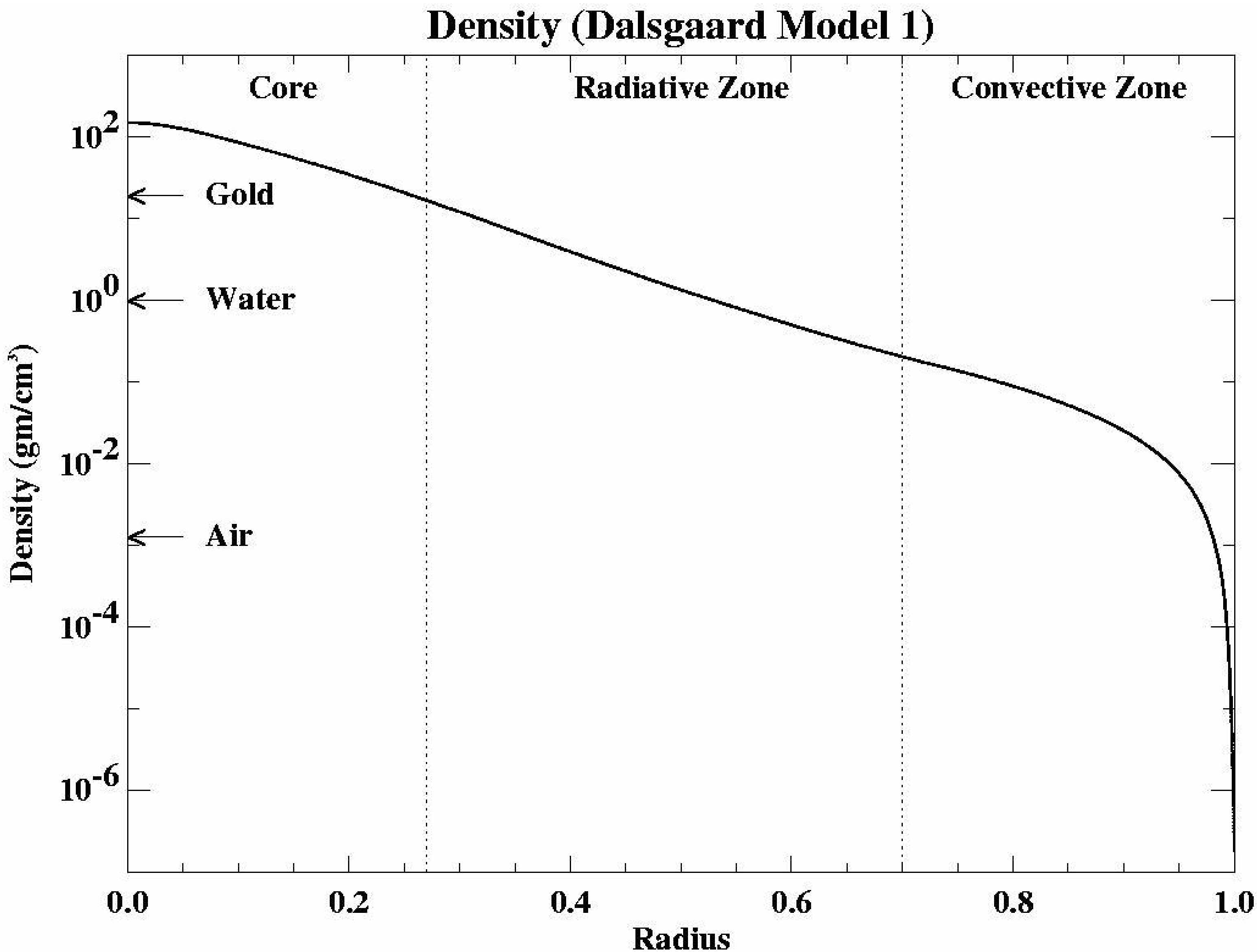
But - thanks to **helioseismology** one does not have to rely on theoretical calculations, but can directly measure the internal structure of the sun



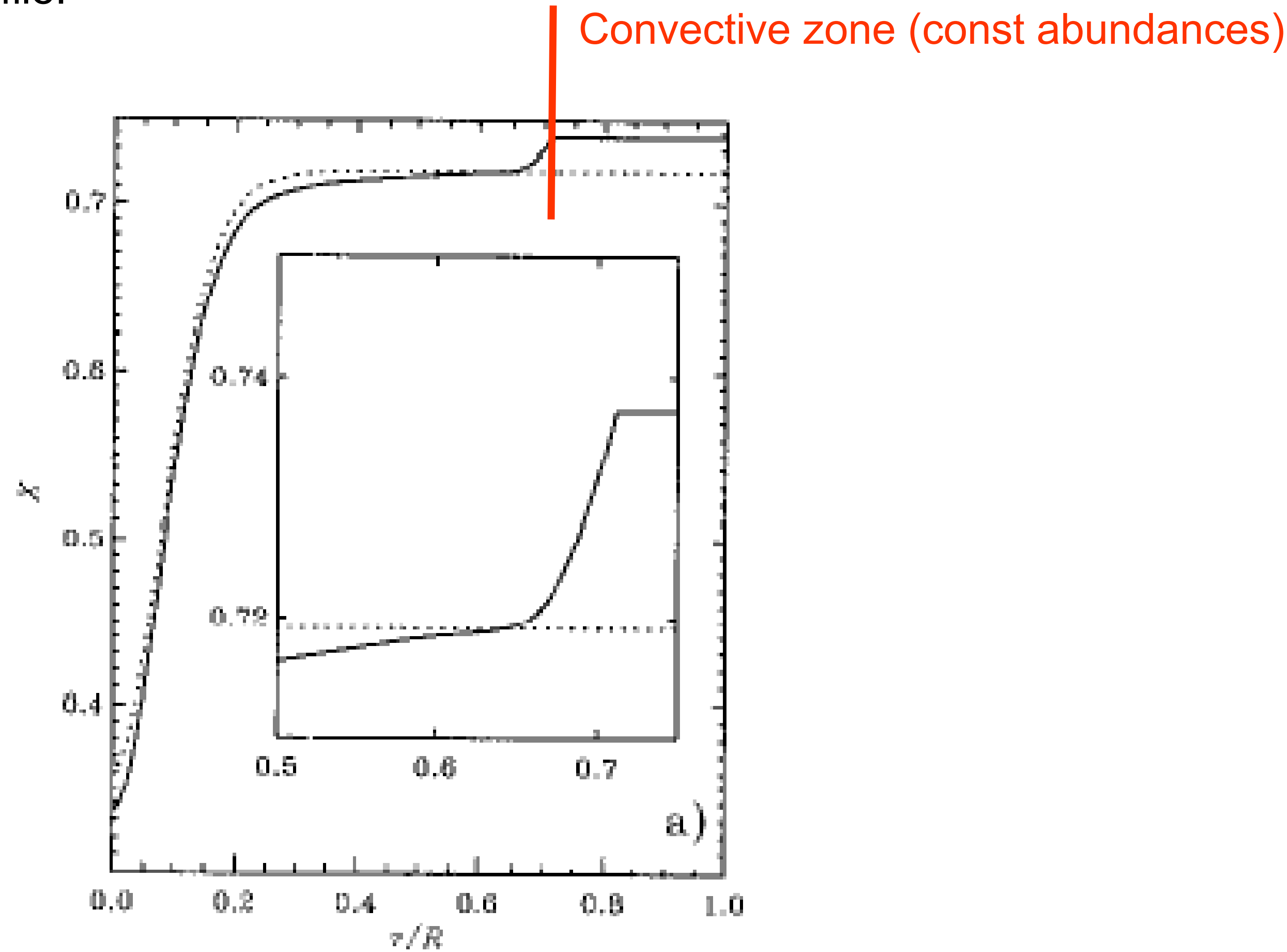
oscillations with periods of 1-20 minutes

max 0.1 m/s


Temperature (Dalsgaard Model 1)
Core
Radiative Zone
Convective Zone
Temperature (K)
10^8
10^7
10^6
10^5
10^4
10^3
0.0
0.2
0.4
0.6
0.8
1.0
Radius

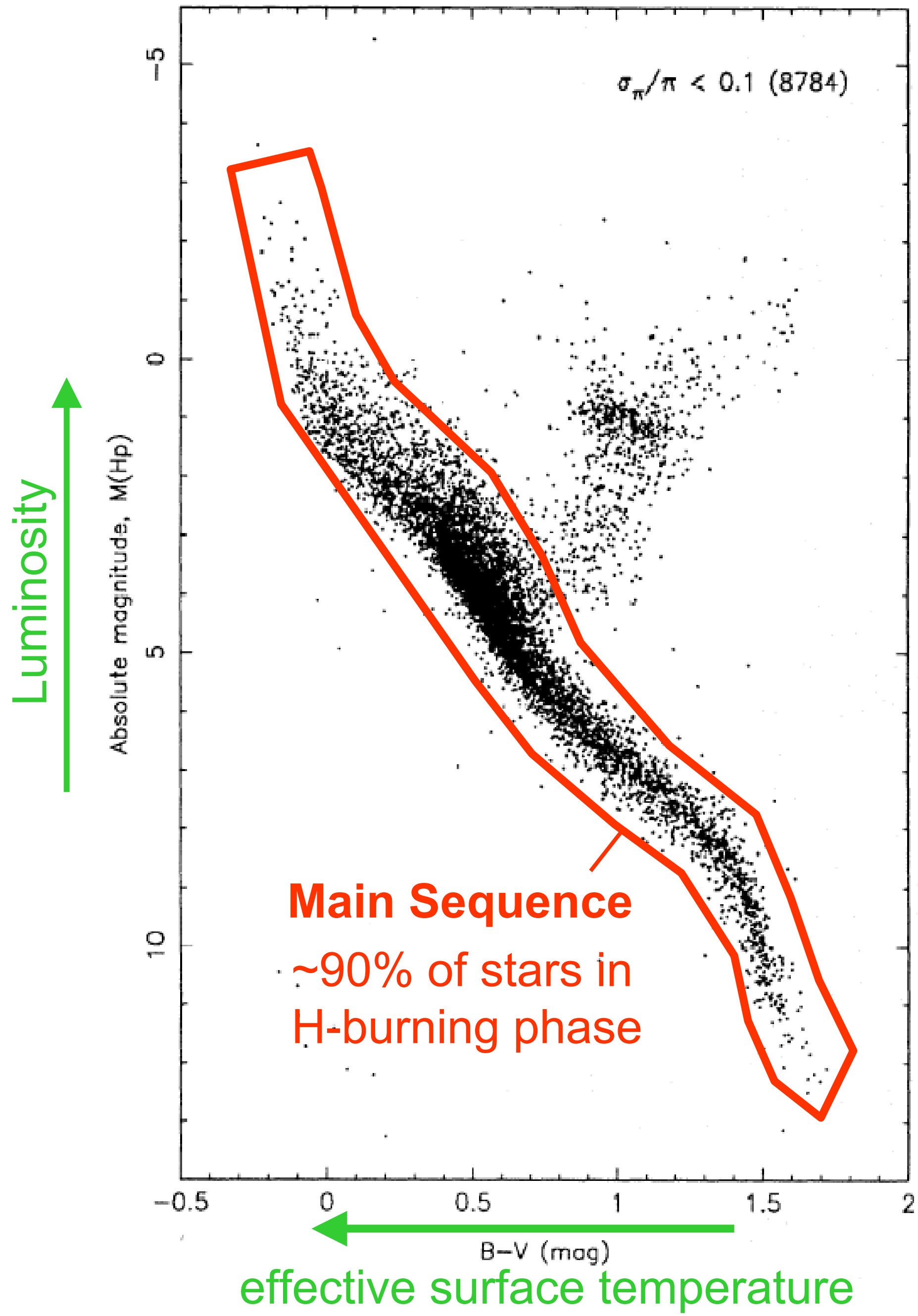

Density (Dalsgaard Model 1)
Core
Radiative Zone
Convective Zone
Gold
Water
Air
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
Density (gm/cm^3)
0.0
0.2
0.4
0.6
0.8
1.0
Radius

Hydrogen profile:



Christensen-Dalsgaard, Space Science Review 85 (1998) 19

Hertzsprung-Russell diagram



Perryman et al. A&A 304 (1995) 69

HIPPARCOS distance measurements

Magnitude:

Measure of stars brightness

Def: difference in magnitudes m from ratio of brightnesses b:

$$m_2 - m_1 = 2.5 \log \frac{b_1}{b_2}$$

(star that is x100 **brighter** has by 5 **lower** magnitude)

absolute scale historically defined
(Sirius: -1.5, Sun: -26.2
naked eye easy: <0, limit: <4)

absolute magnitude is measure of luminosity = magnitude that star would have at 10 pc distance

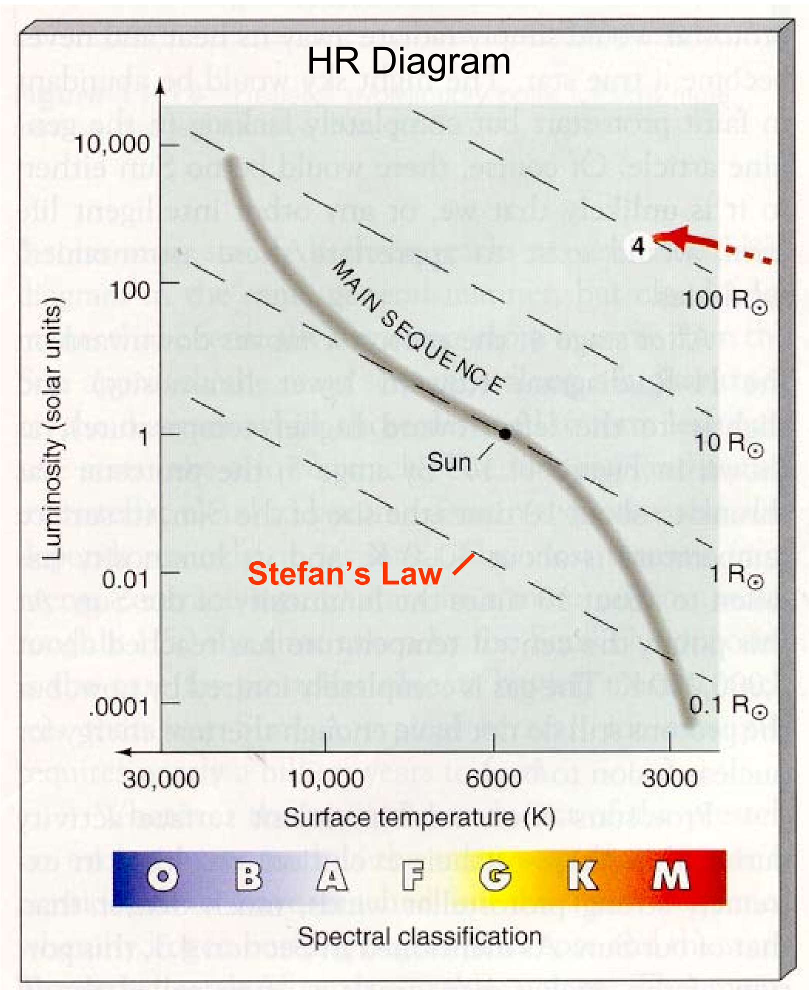
Sun: +4.77

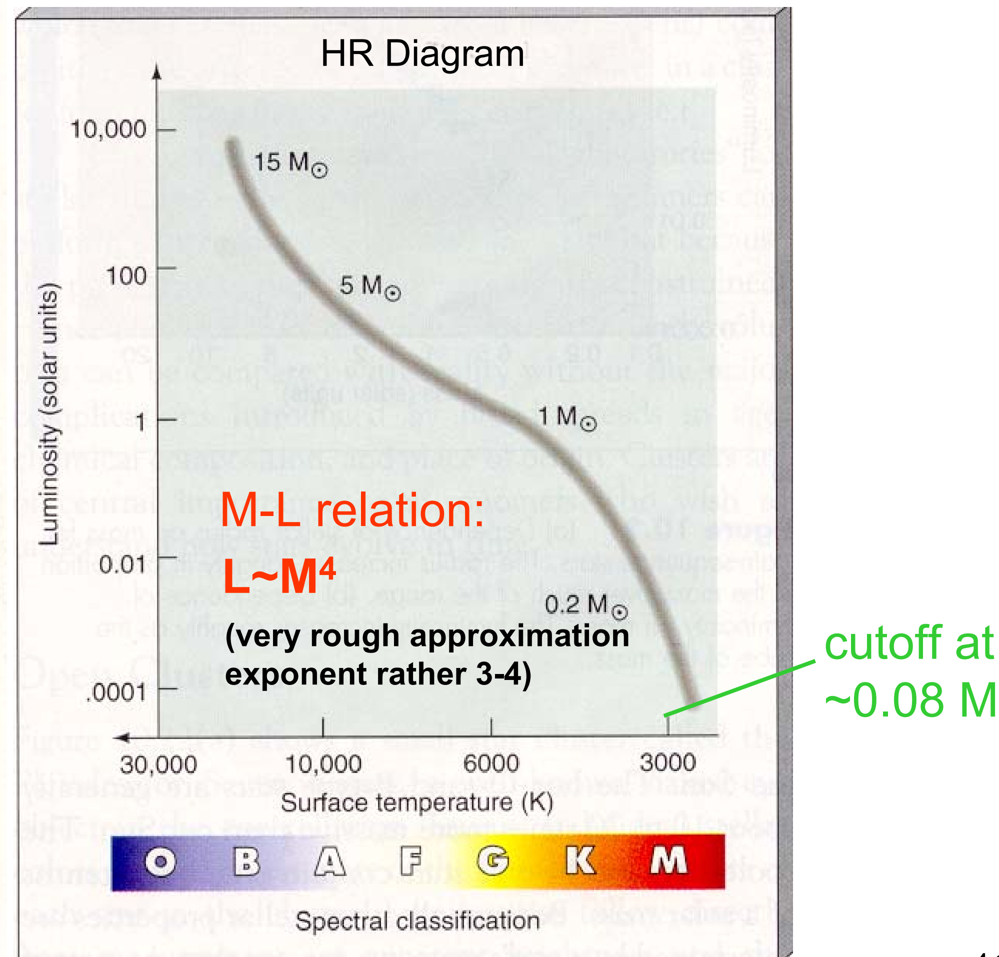
Temperature,Luminosity, Mass relation during H-burning:

It turns out that as a function of mass there is a rather unique relationship between

- surface temperature (can be measured from contineous spectrum)
- luminosity (can be measured if distance is known)

(recall Stefan's Law $L \sim R^2\,T^4$, so this rather a R-T relation)





(from Chaisson McMillan)

Main Sequence evolution:

Main sequence lifetime:

H Fuel reservoir F~M
Luminosity L~M^4 $\longrightarrow$ lifetime $\tau_{\mathrm{MS}} = \frac{F}{L} \propto M^{-3}$

Recall from Homework: H-burning lifetime of sun ~ 10^{10} years

$$\boxed{\tau_{\mathrm{MS}} \approx \left(\frac{M}{M_{\oplus}}\right)^{-3} 10^{10}\ \text{years}}$$

note: very approximate
exponent is really
between 2 and 3

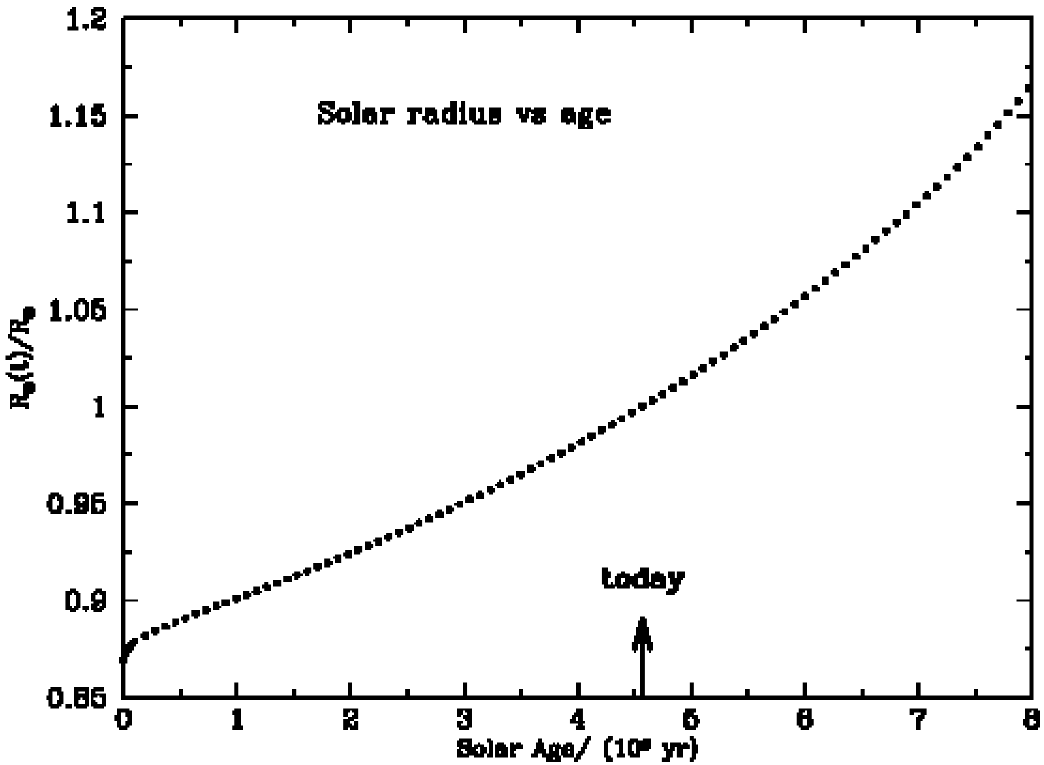
so a 10 solar mass star lives only for 10-100 Mio years
a 100 solar mass star only for 10-100 thousand years !

Changes during Main Sequence evolution:

With the growing He abundance in the center of the star slight changes occur (star gets somewhat cooler and bigger) and the stars moves in the HR diagram slightly

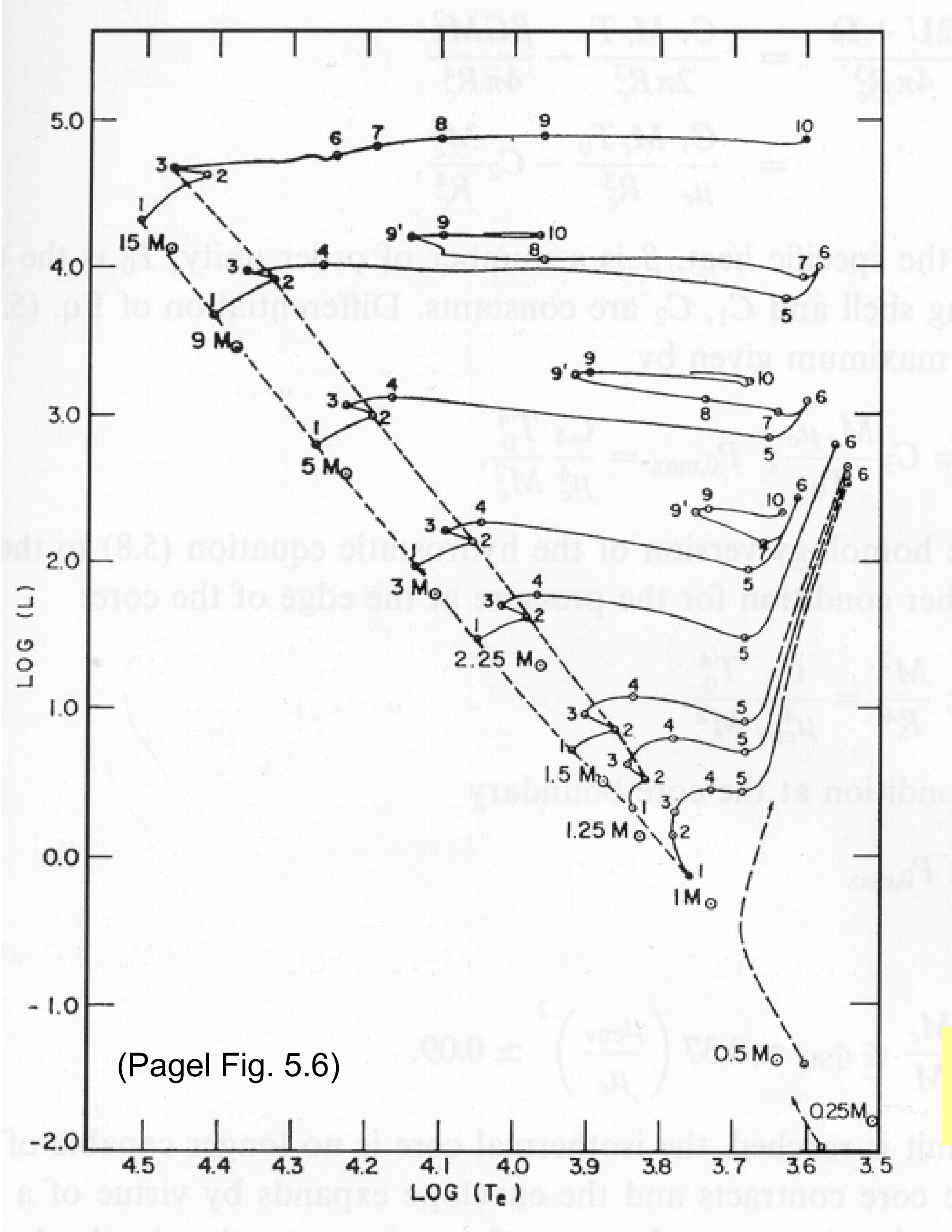
→ main sequence is a band with a certain width

For example, predicted radius change of the sun according to Bahcall et al. ApJ555(2001)990



Zero Age Main Sequence (ZAMS): "1"

End of Main Sequence: "2"



(Pagel Fig. 5.6)

Stellar masses are usually given in ZAMS mass !