Determining Properties of the Quark-Gluon Plasma from Experiment

I. QGP Properties
II. Experiments
III. Models
IV. Phenomenology
V. Bayesian Analysis
VI. Results

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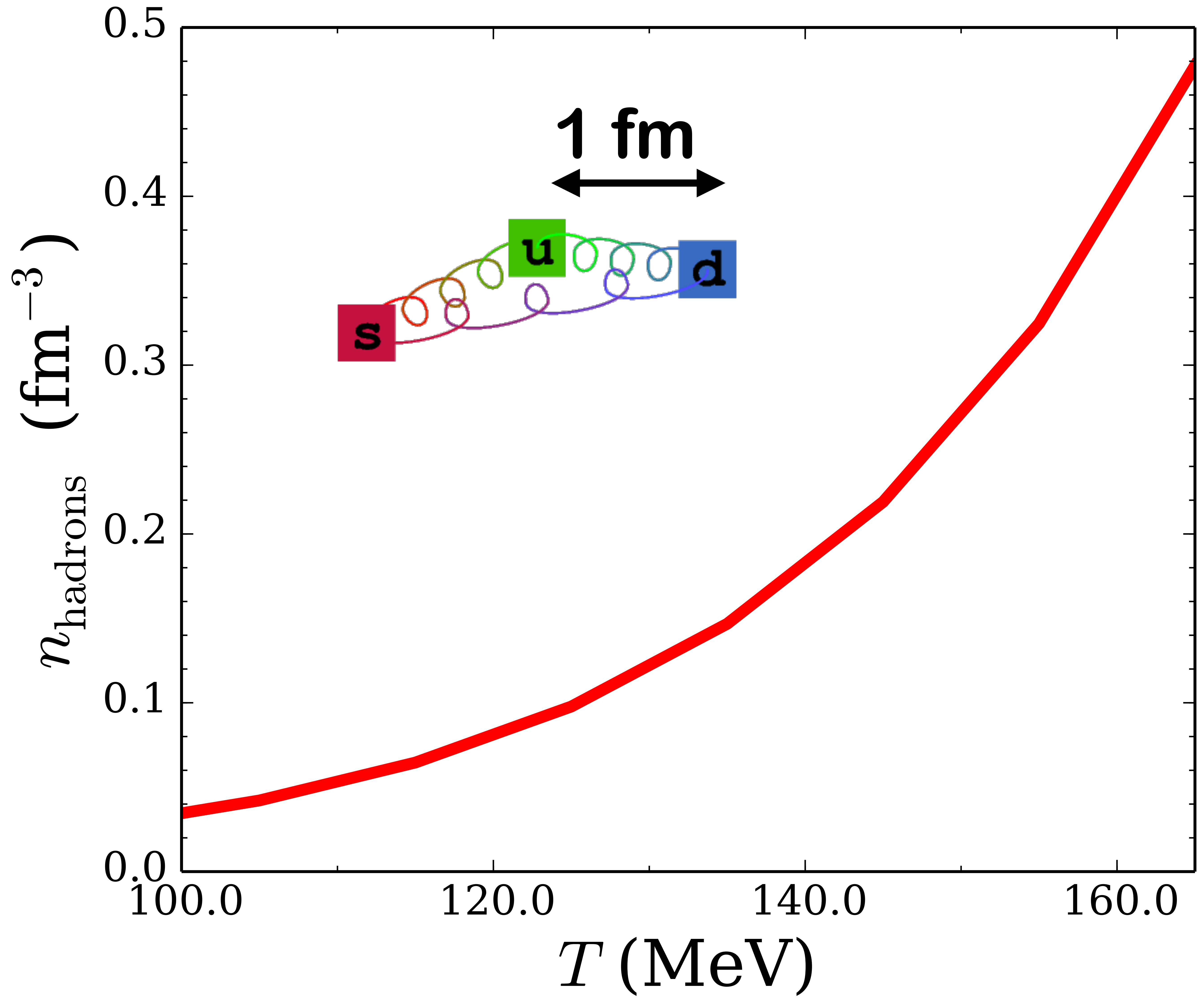
Michigan State University

I. QGP Properties

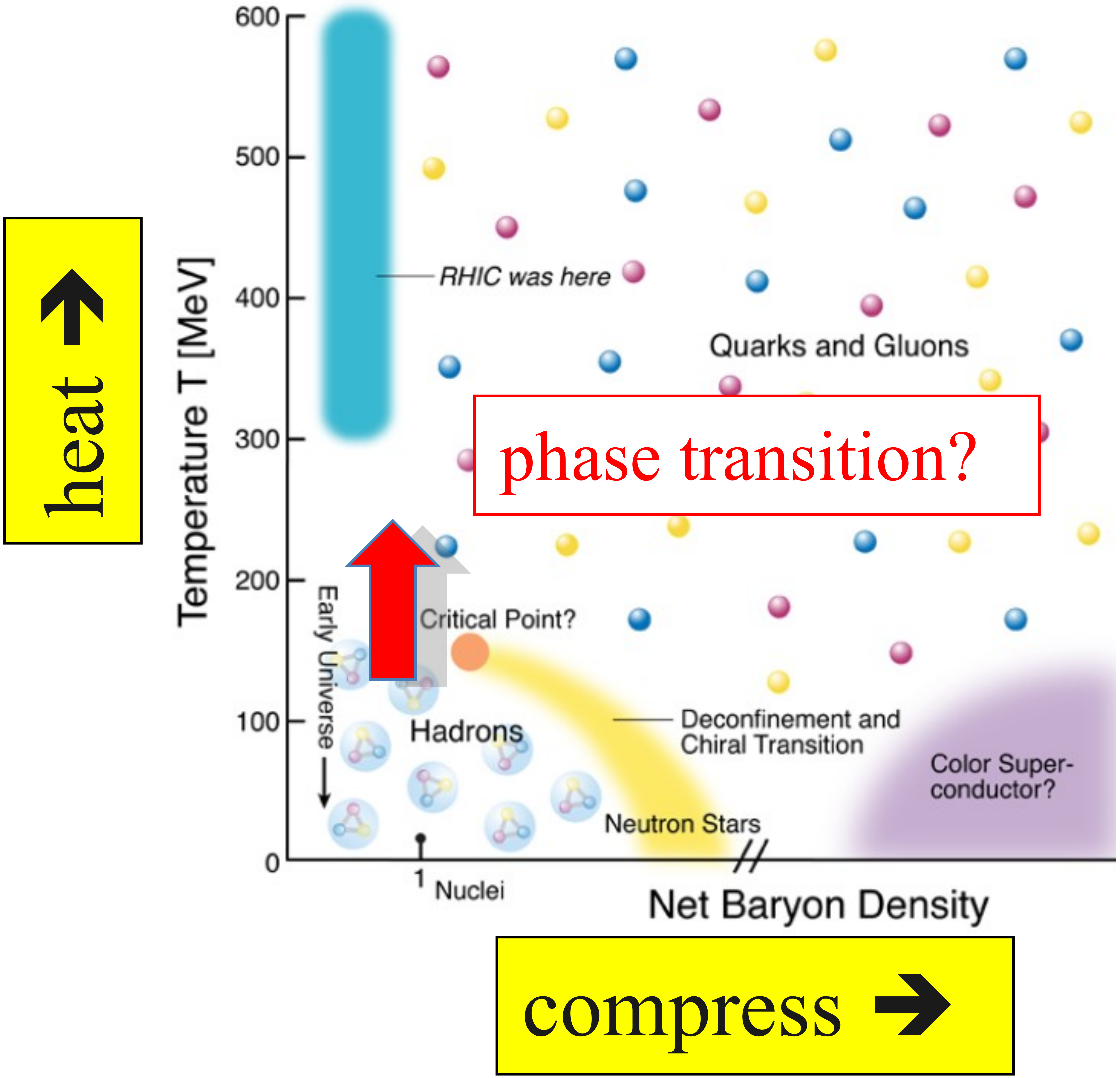


$T_c \approx$ 160 MeV
Hadrons → QGP

I. QGP Properties

QGP is Charge Rich!!!
52 colored degrees of freedom
16 gluons
36 quarks:
 up, down strange,
 anti-up, anti-down, anti-strange
 spin ↑, spin↓
 red,green,blue

~50 particles in one thermal wavelength

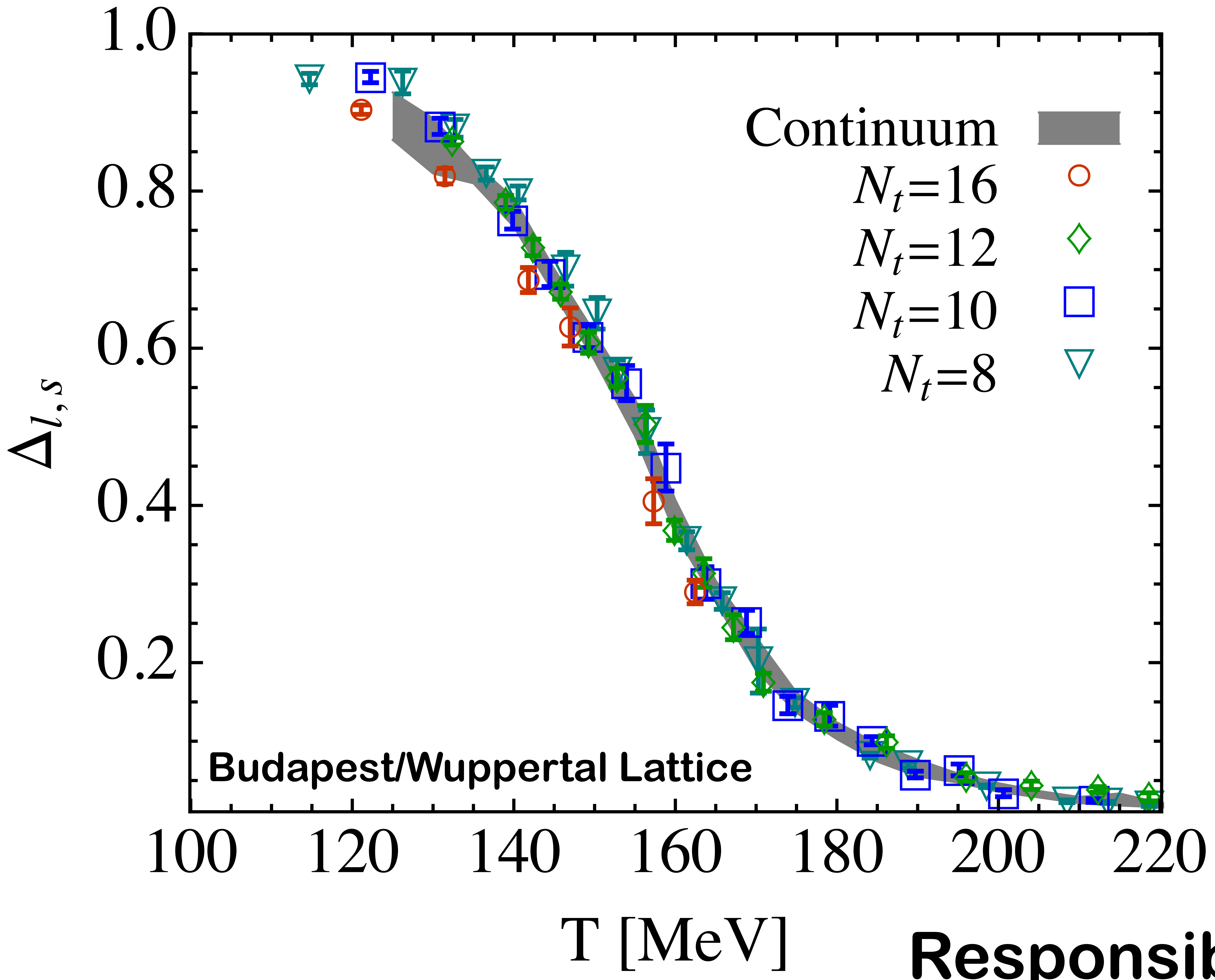


I. QGP Properties

Eq. of state

- possibly 1^{st} order ??
- phase separation & critical point ??

I. QGP Properties



Quark-antiquark condensate melts

Responsible for much of baryon mass

I. QGP Properties

QGP is strongly interacting

$$n_h(T_c) \approx 0.5 \quad \mathrm{fm}^3$$

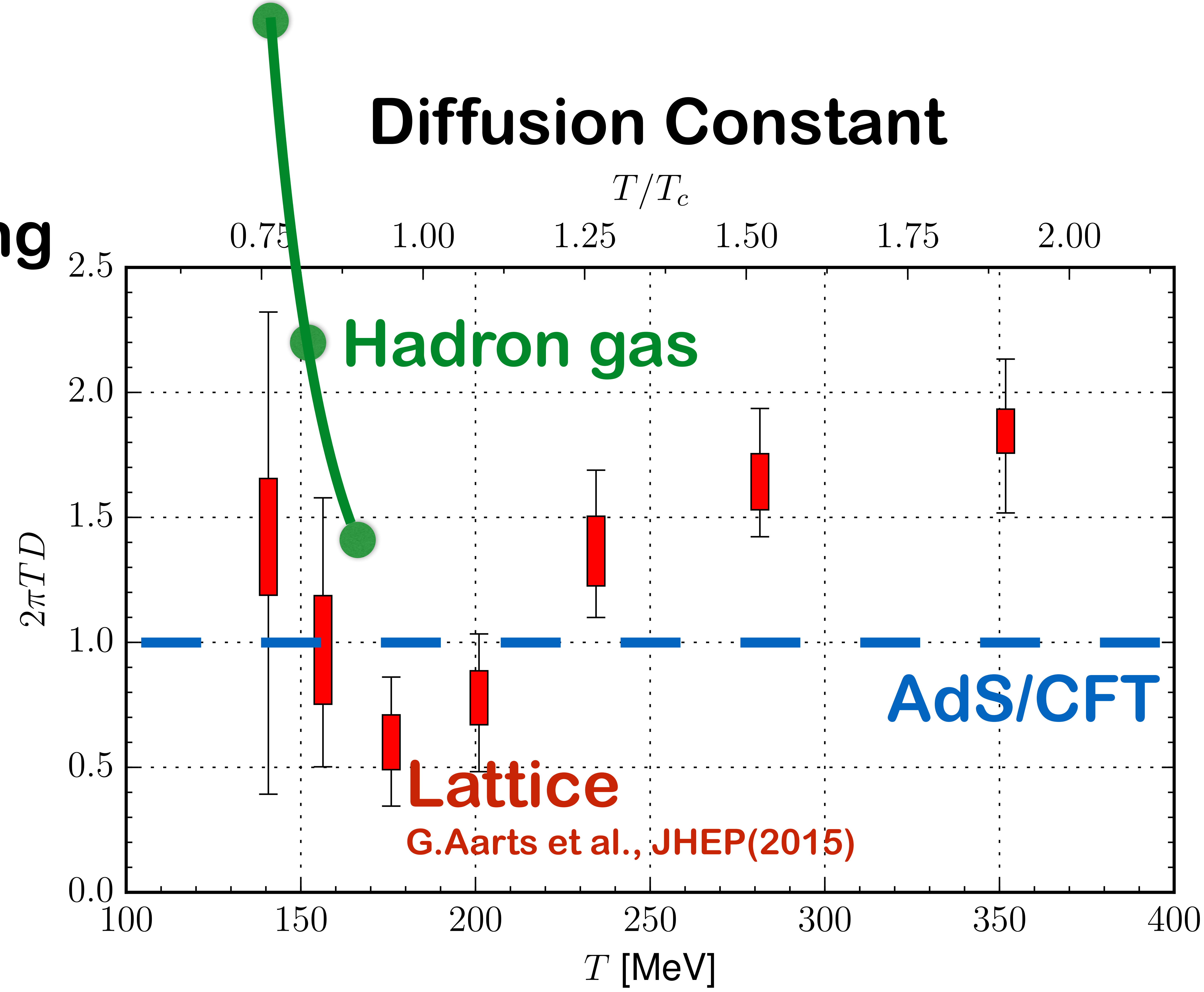
$$\sigma_{\mathrm{had}} \approx 2.5 \quad \mathrm{fm}^2$$

$$\text{Char. size} \approx 10 \quad \mathrm{fm}$$

- **Low viscosity**
  - "perfect liquid"
  - uncertainty limit

P.Danielewicz and M.Gyulassy, PRD(1985)

- **Low diffusivity**



I. QGP Properties

1. Eq. of state (B=0 & B$\neq$0)
   P(n_B,ε) or P(μ,T) or c_s^2(n_B,ε)…
   Quasi-first-order
2. Charge susceptibility
   $\chi_{ab} = \langle \delta Q_a \delta Q_b \rangle / V$ - describes chemistry
3. Quark-antiquark condensate $\langle \bar{\psi}\psi \rangle$
   "Chiral symmetry" restoration
4. Viscosity — response to flow gradient
   $$\delta T_{ij} = -\eta \left[\partial_i v_j + \partial_j v_i - (2/3)\delta_{ij} \nabla \cdot \mathbf{v} \right] - \zeta \nabla \cdot \mathbf{v}$$
   η(shear) and ς(bulk) remarkably small
5. Diffusivity — response to density gradient
   $\mathbf{j}_a = -D_{ab} \nabla \rho_b$
   Poor conductor

I. QGP Properties

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Well-determined by lattice

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Not-so-well-determined by lattice

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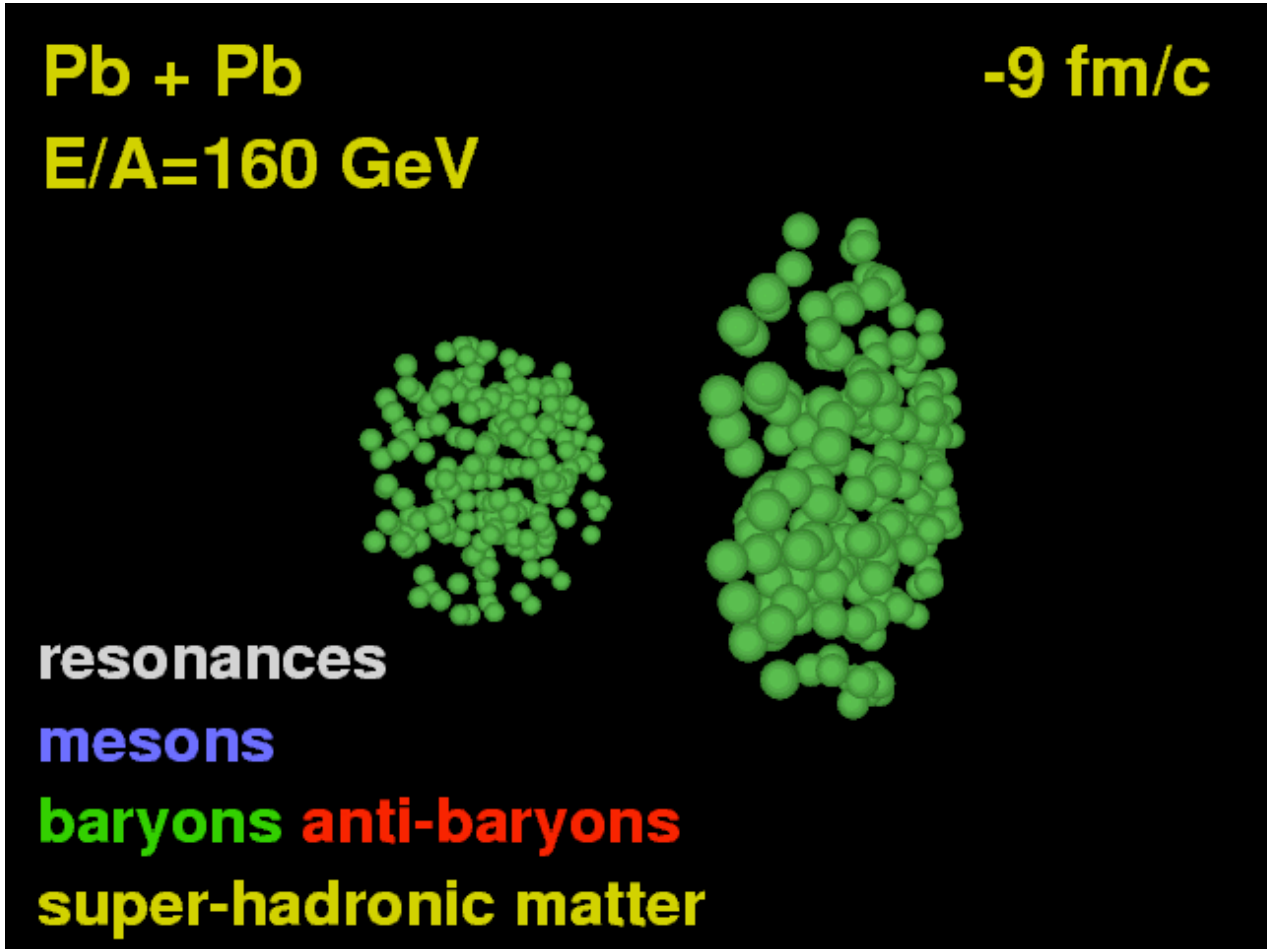
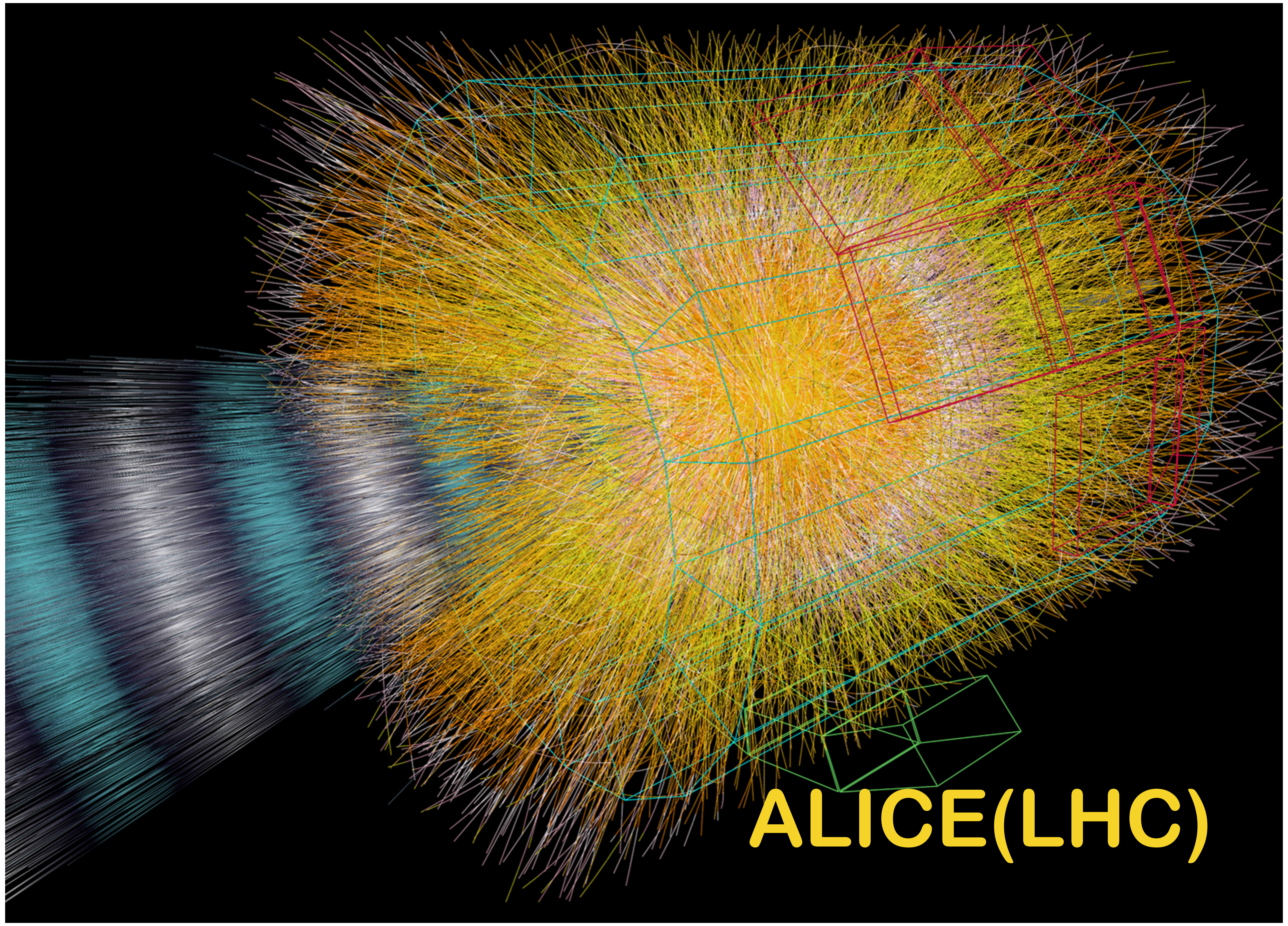
Experimentally accessible

II. Experiments





AGS(11A GeV), SPS(160A GeV), RHIC(100A+100A GeV), LHC(1.4A+1.4A TeV)

III. Models

1. Pre-equilibrium

   messy — often parametric

2. Hydrodynamics (QGP, T≳160 MeV)

   relativistic, viscous, Israel-Stewart eq.s

$$\partial_t \pi_{ij} = -\frac{1}{\tau_{IS}}\left(\pi_{ij} - \pi_{ij}^{(\mathrm{NS})}\right) + \cdots$$

viscous part of SE tensor

3. Hadron simulation (T≲160 MeV)

   Boltzmann sampling

Also add interfaces, correlation “after-burners”…

IV. Phenomenology

As a philosophical movement:

From Wikipedia:

There are several assumptions behind phenomenology that help explain its foundations:

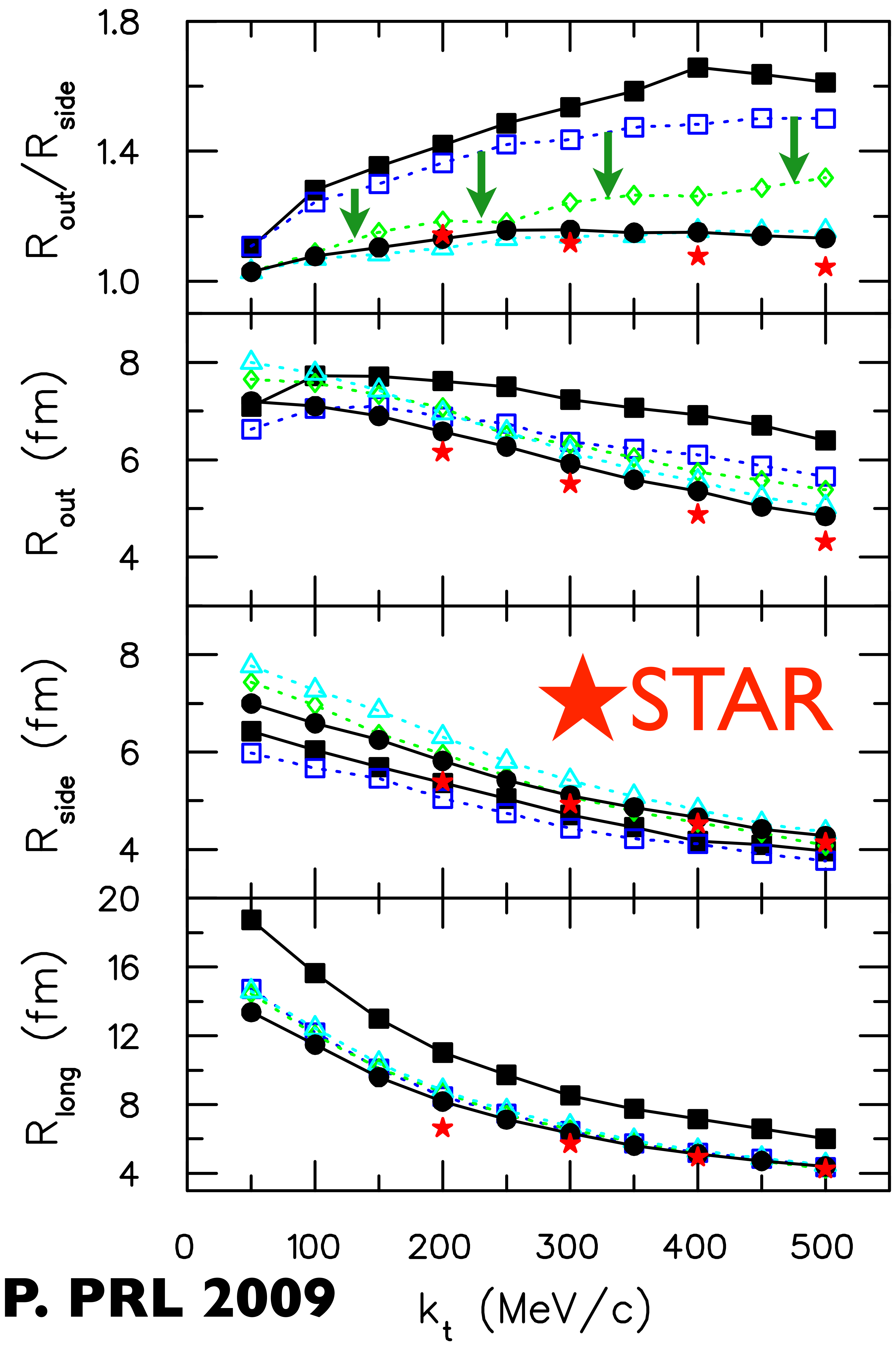
1. **Phenomenologists reject the concept of objective research.** They prefer grouping assumptions through a process called phenomenological epoché.
2. They believe that analyzing daily human behavior can provide one with a greater understanding of nature.
3. They assert that persons should be explored. This is because persons can be understood through the unique ways they reflect the society they live in.
4. **Phenomenologists prefer to gather "capta", or conscious experience, rather than traditional data.**
5. **They consider phenomenology to be oriented toward discovery, and therefore they research using methods that are far less restrictive than in other sciences.**

To a physicist:

- **Experiment (momenta and IDs of tracks)**

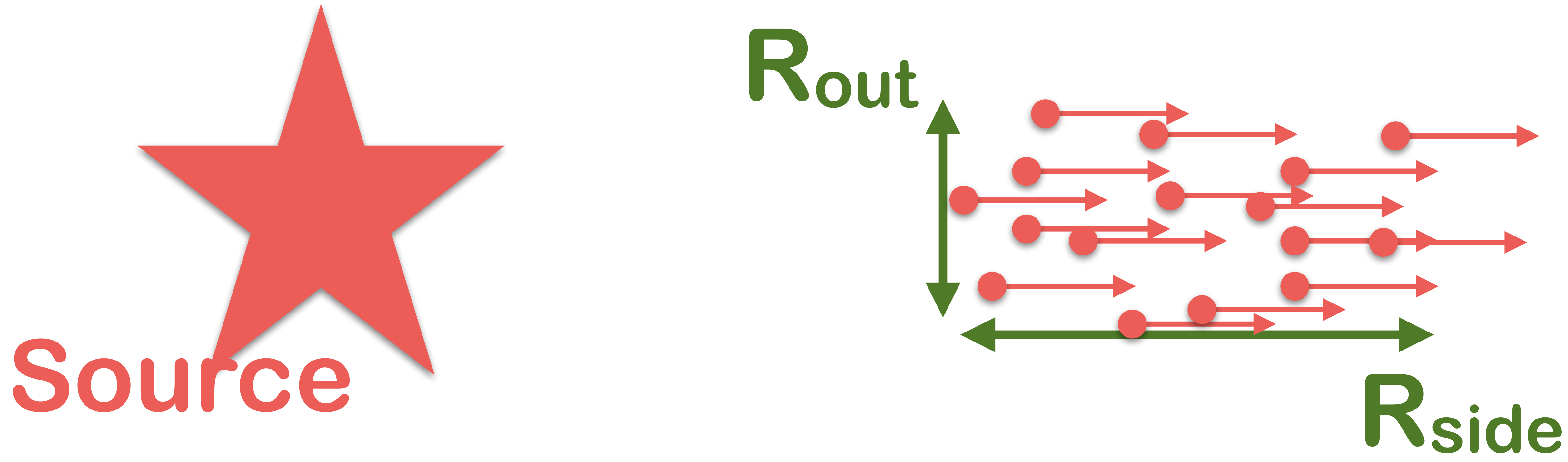
  ➜ Evolution of ε, P, v, ρ…
- **Can be heuristic or semi-quantitative**

IV. Phenomenology
Eq. of State



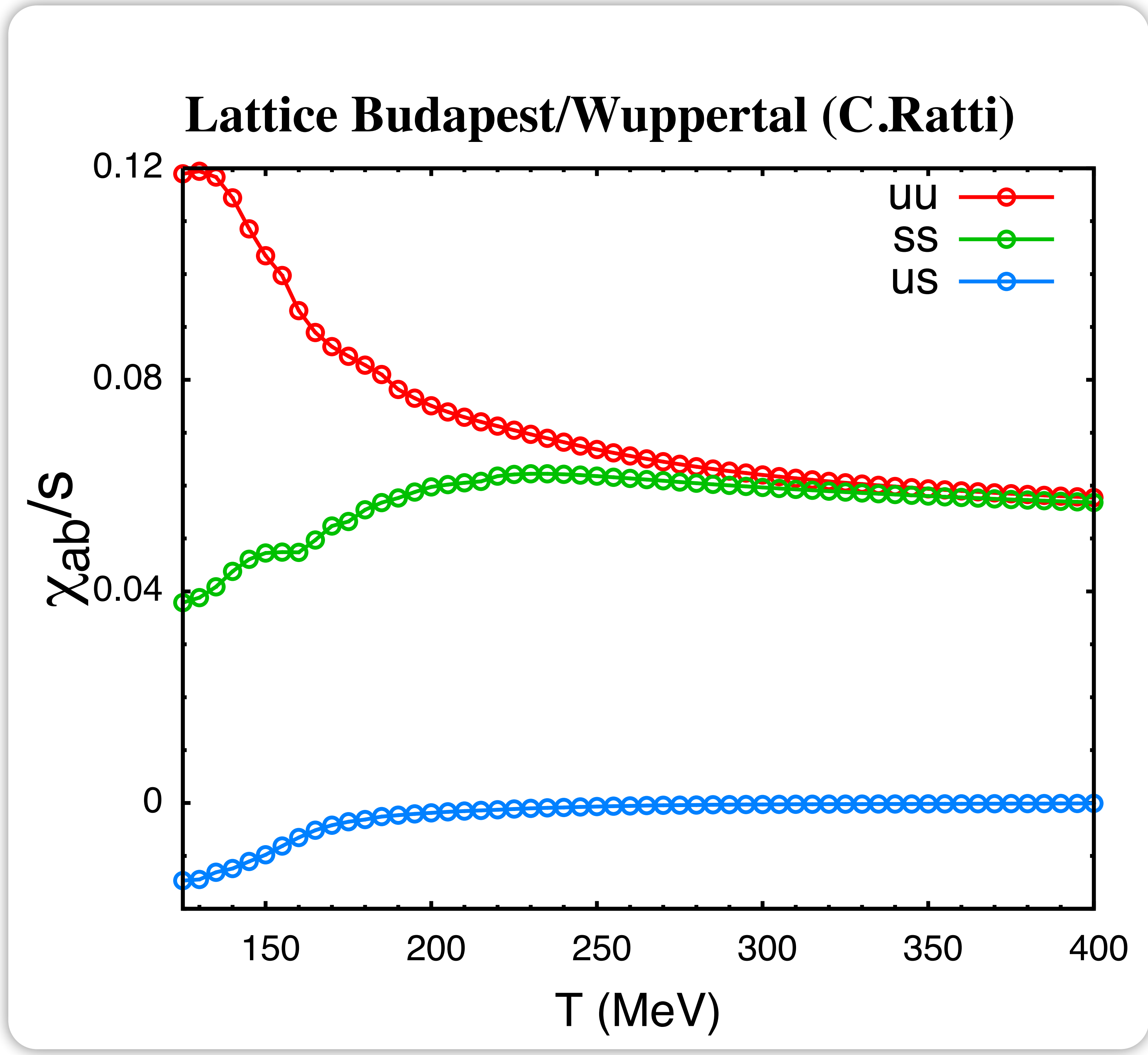
S.P. PRL 2009

- **Femtoscopic radii**
- **Interferometric correlations give shape of phase space cloud for given momentum**



- **For stiffer Eq. of state R_{out}/R_{side} decreases (blue to green)**
- **Eq. of state also affects spectra multiplicities, elliptic flow…**

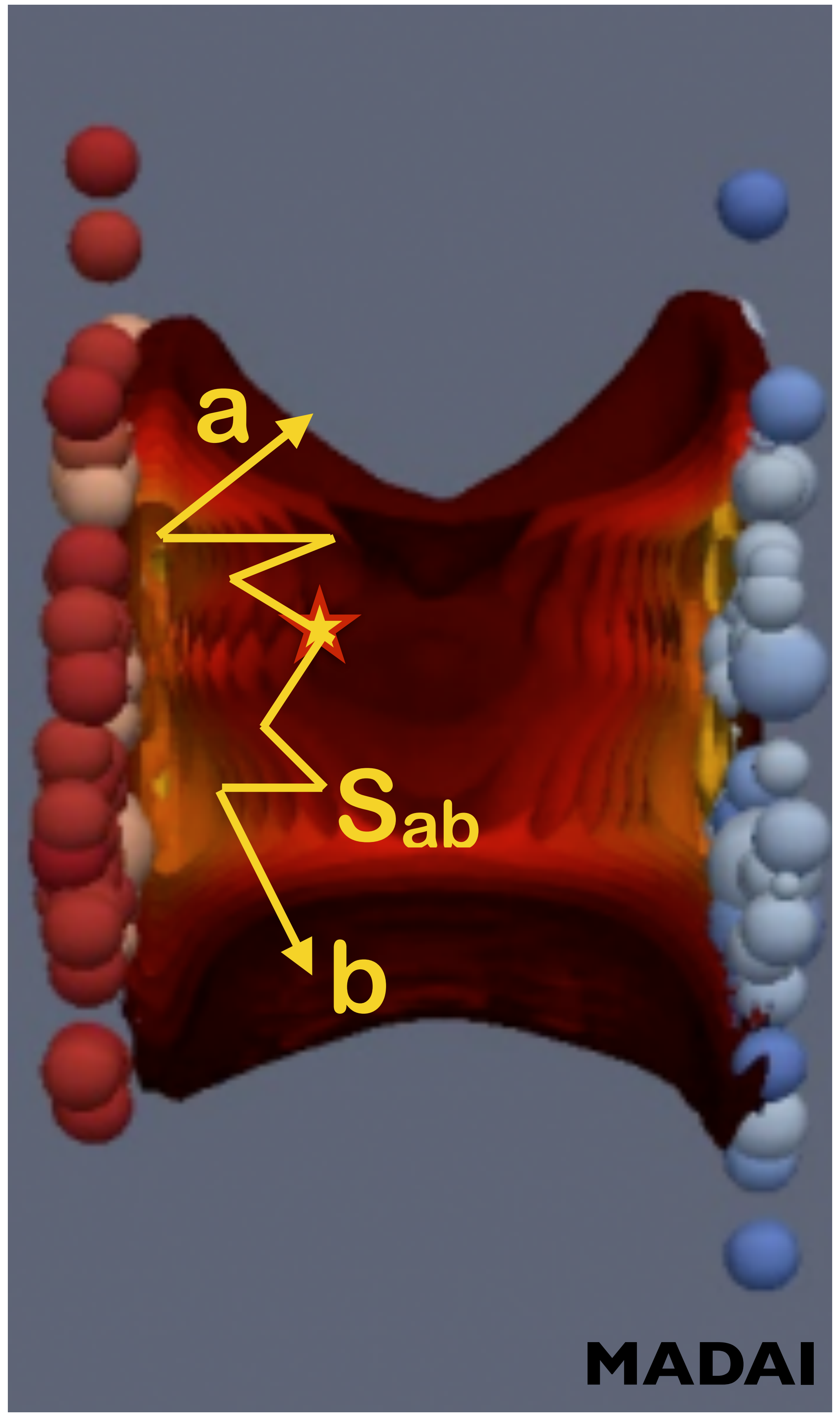
IV. Phenomenology
Susceptibility

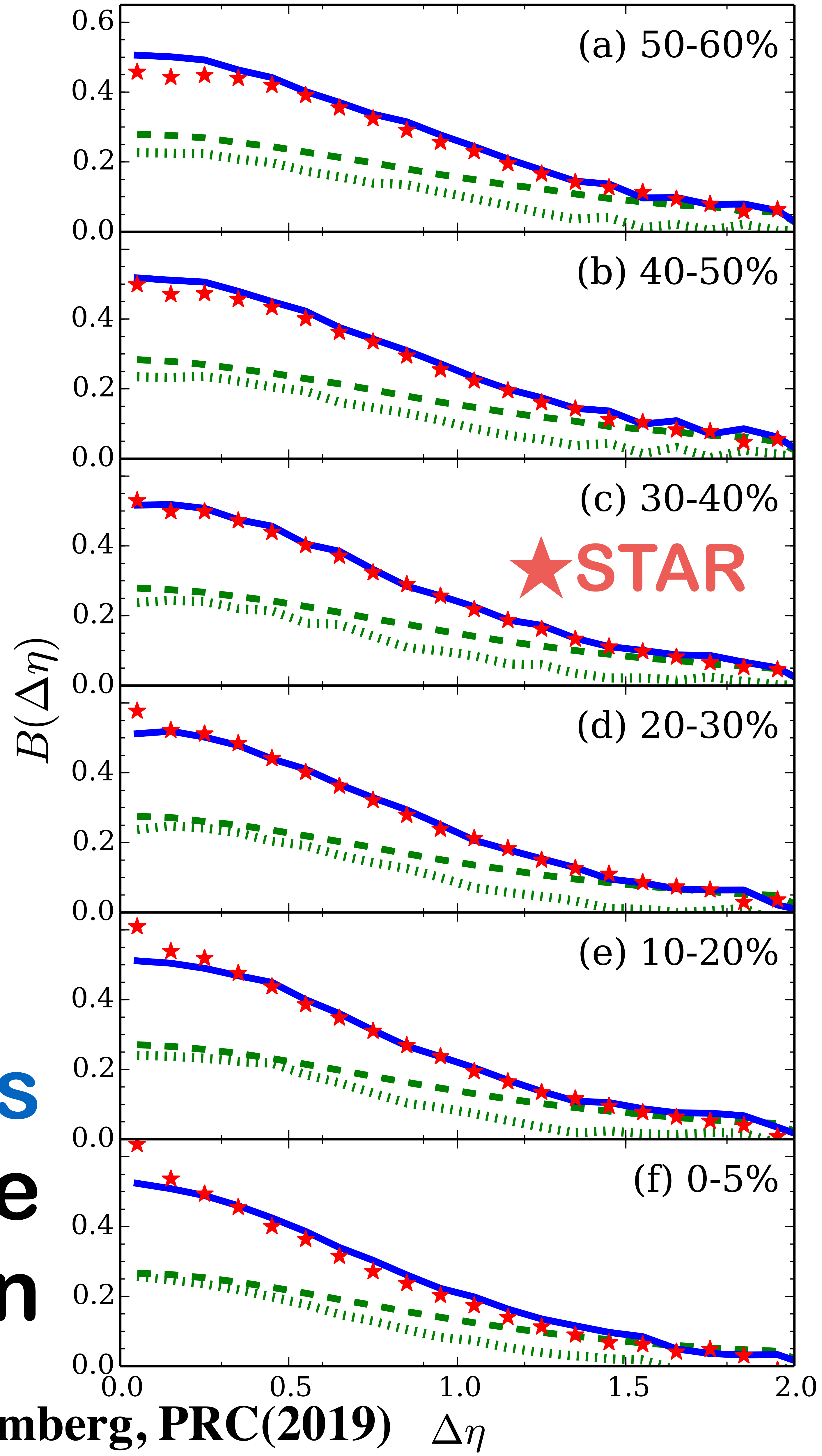


$$C_{ab}(t,\mathbf{r}_1,\mathbf{r}_2) = \langle \delta\rho_a(t,\mathbf{r}_1)\delta\rho_b(t,\mathbf{r}_2)\rangle$$

$$\partial_t C_{ab} + \nabla_1\cdot(\mathbf{v}_1 C_{ab}) + \nabla_2\cdot(\mathbf{v}_2 C_{ab}) - D\nabla_1^2 C_{ab} - D\nabla_2^2 C_{ab} = S_{ab}(t,\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2)$$

$$S_{ab}(t,\mathbf{r}) = -s\frac{D}{Dt}\frac{\chi_{ab}(t,\mathbf{r})}{s}$$





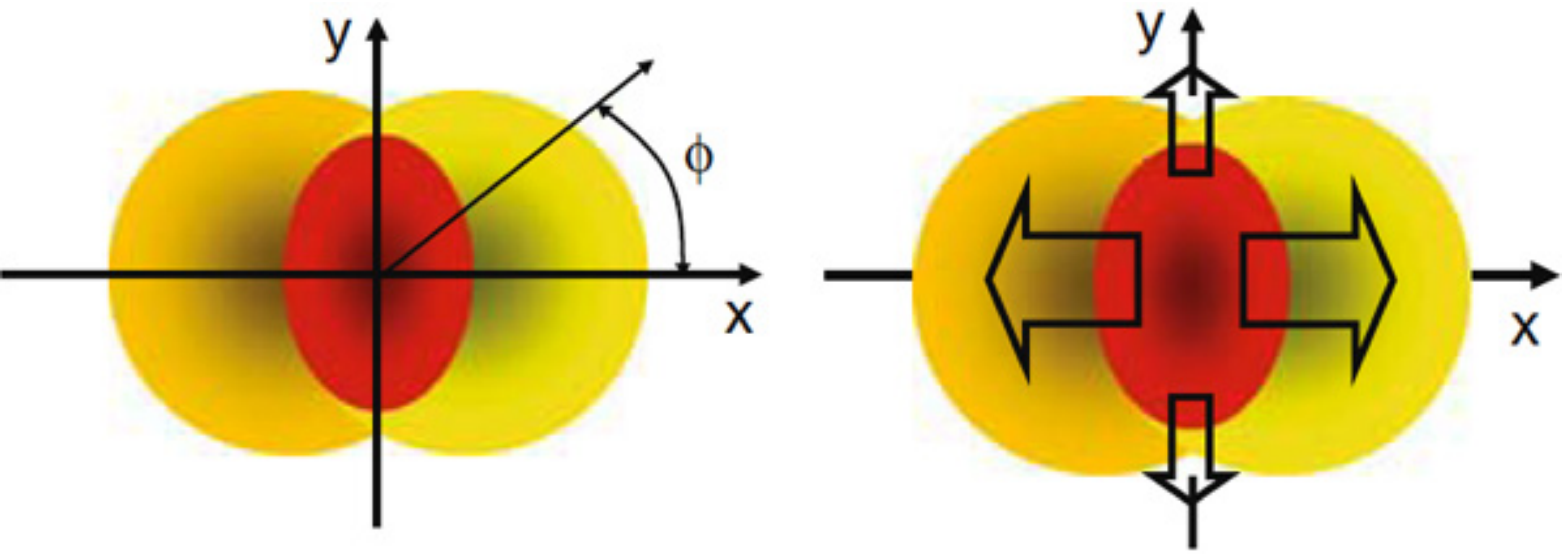
S.P. and C.Plumberg, PRC(2019)

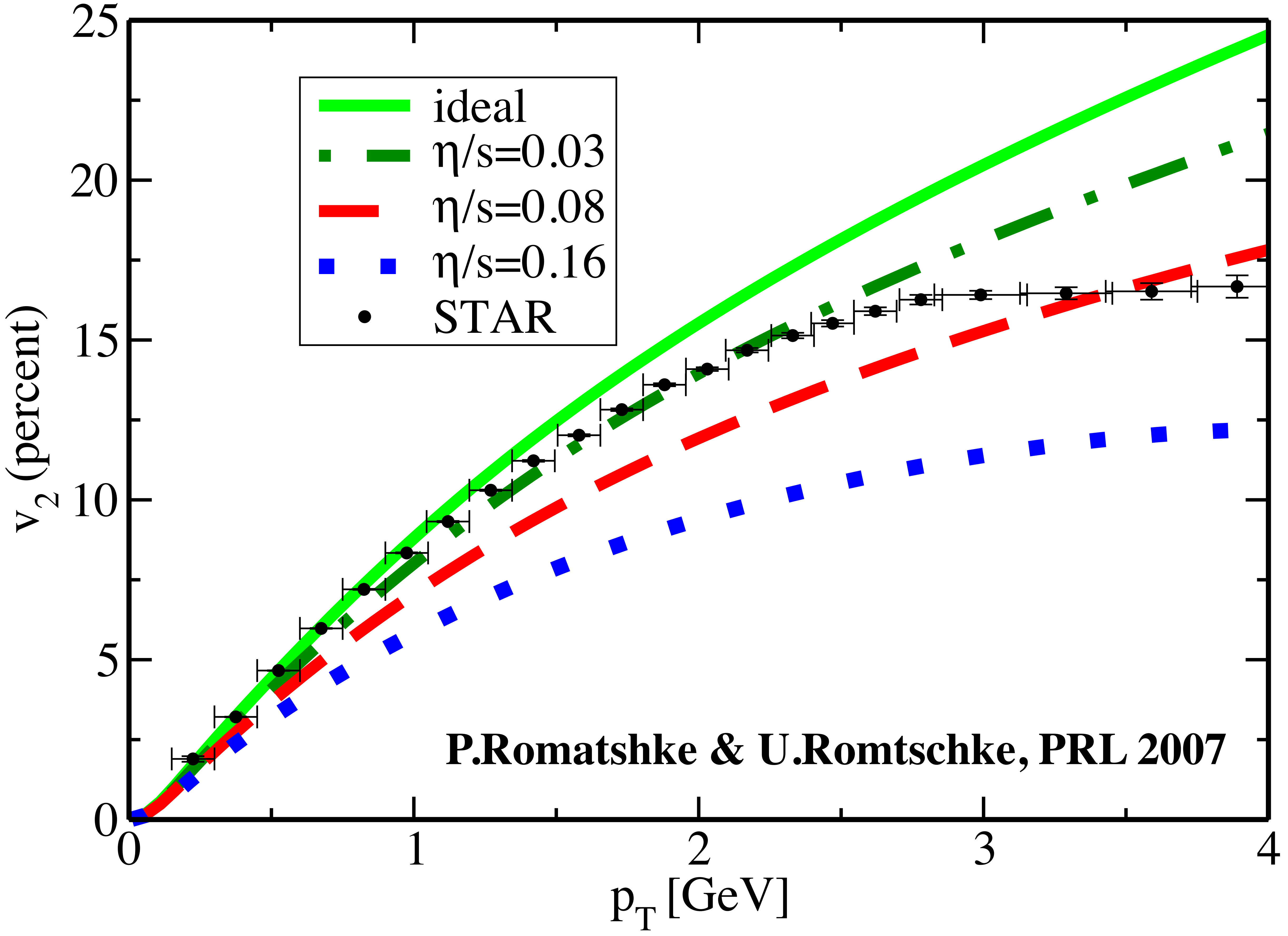
Charge-balance correlations

Early production of charge → broader correlation

IV. Phenomenology
Viscosity

$$v_2 \equiv \langle \cos 2\phi \rangle$$

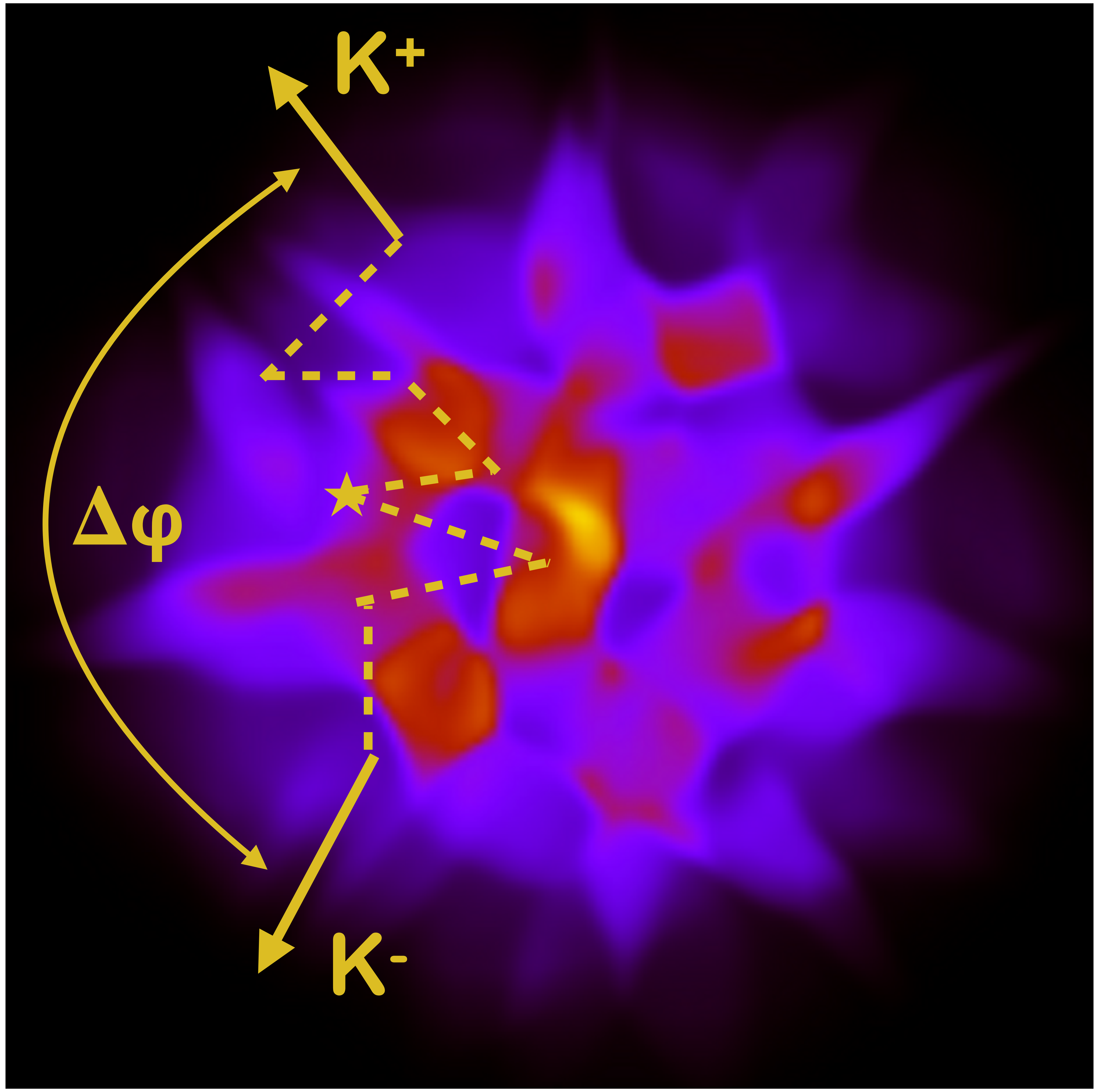




Suggests low viscosity (close to uncertainty limit)

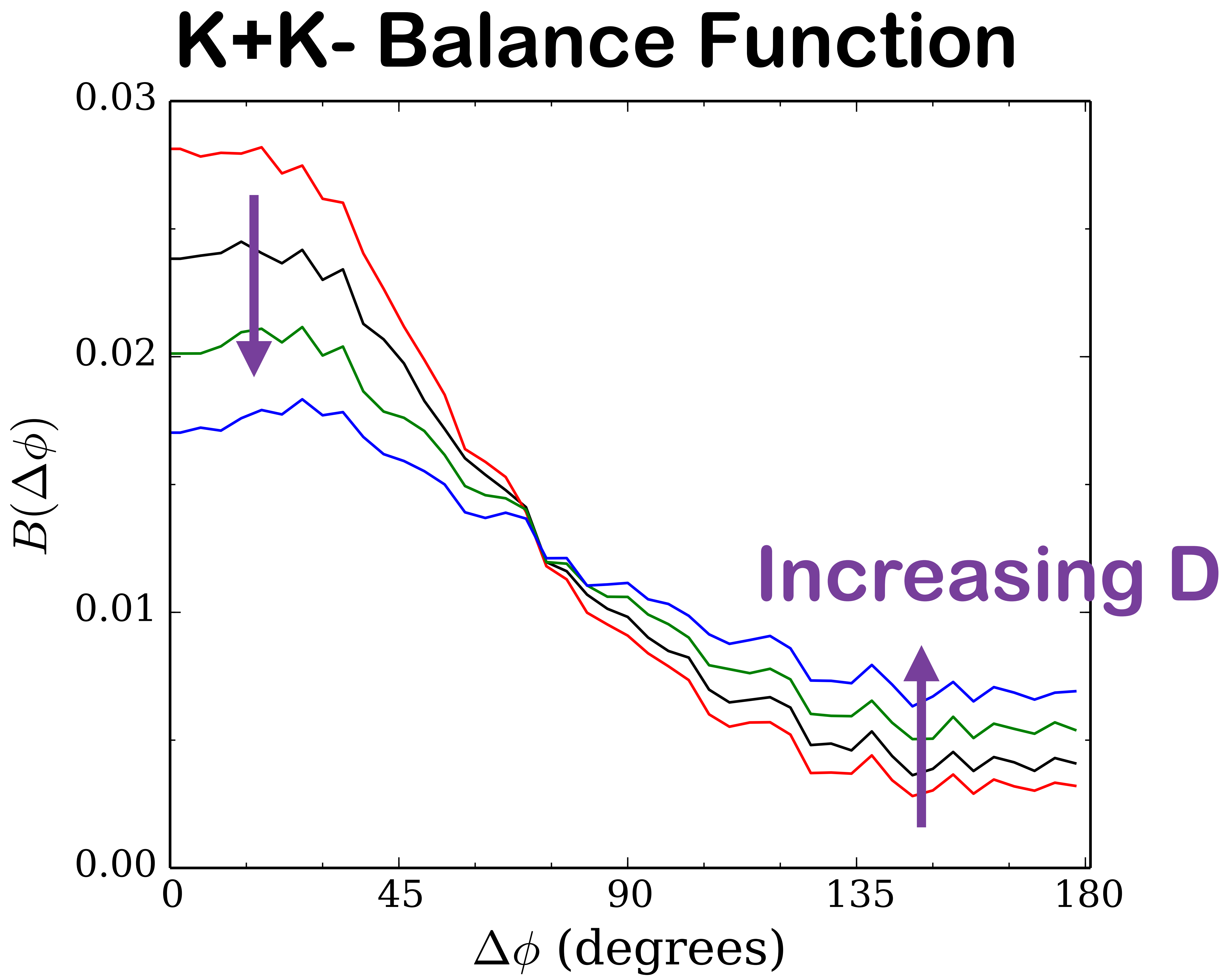
P.Danielewicz and M.Gyulassy, PRD(1985)

IV. Phenomenology — Diffusivity



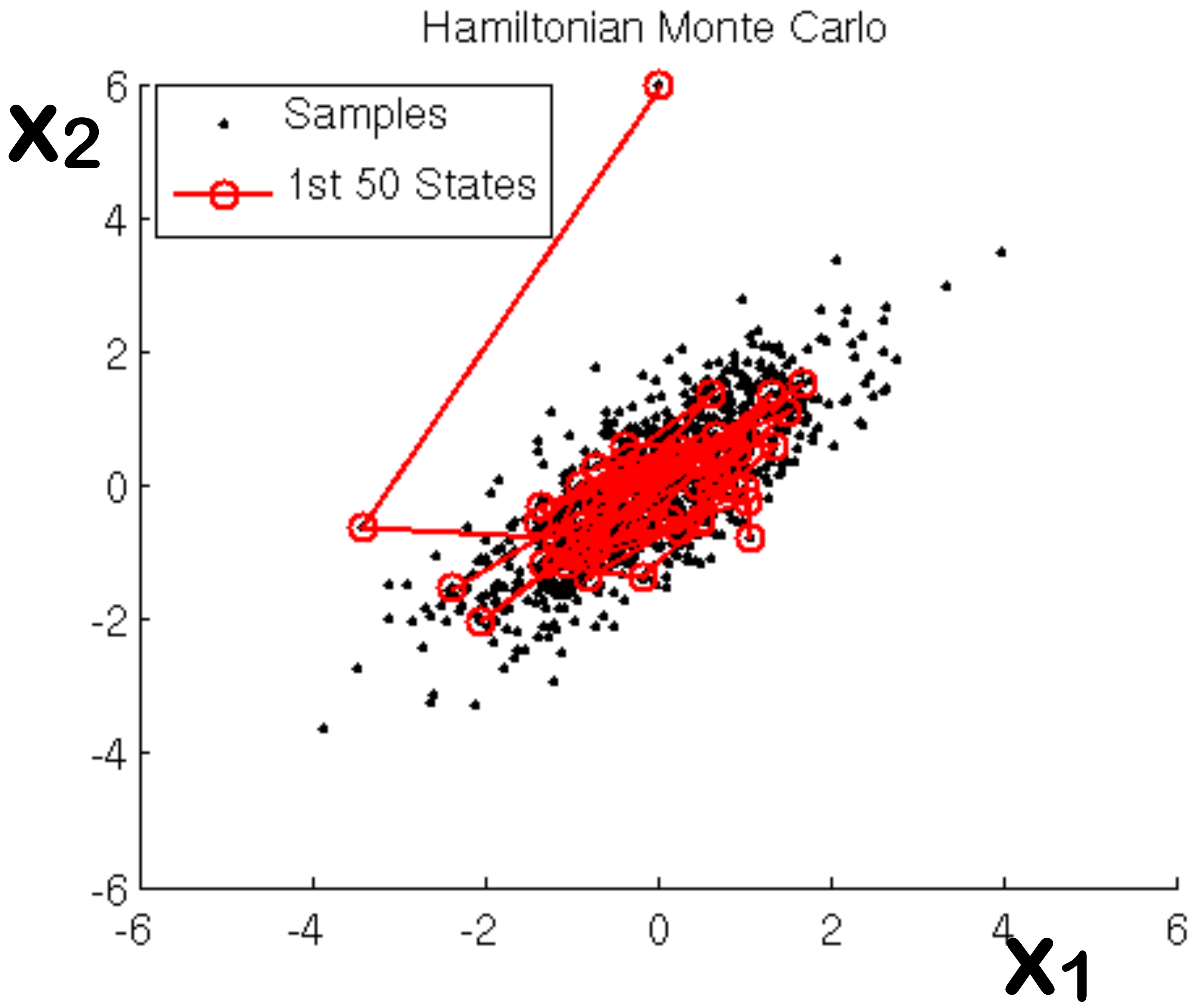
Strangeness made early

∴ kaon separation determined by diffusivity



V. Bayesian Analysis

Many parameters (dozens)
— all affect many observables (dozens of plots)
to proceed...



Markov-Chain Monte Carlo

- Simultaneously vary N model parameters x_i
- Perform random walk weight by likelihood

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{ -\sum_a \frac{\left(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})}\right)^2}{2\sigma_a^2} \right\}$$

- Use all observables y_a
- Obtain representative sample of posterior

V. Bayesian Analysis

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\sum_a \frac{(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})})^2}{2\sigma_a^2}\right\}$$

Difficulties:

1. **Calculating $y_a^{(\mathrm{model})}$ is expensive**
2. **Too much data**
   - **heterogenous, many plots**
   - **correlated uncertainties**

V. Bayesian Analysis

To address these issues:

MADAI Collaboration
Models and Data Analysis Initiative
(active 2010-2017)

Cyber-enabled Discovery and Innovation

MICHIGAN STATE UNIVERSITY · Duke UNIVERSITY · THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL · renci

1st MADAI Collaboration Meeting, SANDIA 2010

V. Bayesian Analysis

Data Distillation

1. Experiments reduce PBs to 100s of plots

2. Choose which data to analyze
   Does physics factorize?

3. Reduce each plot to a few values, y_a
   (use principle components)

4. Calculate global principal components, z_a

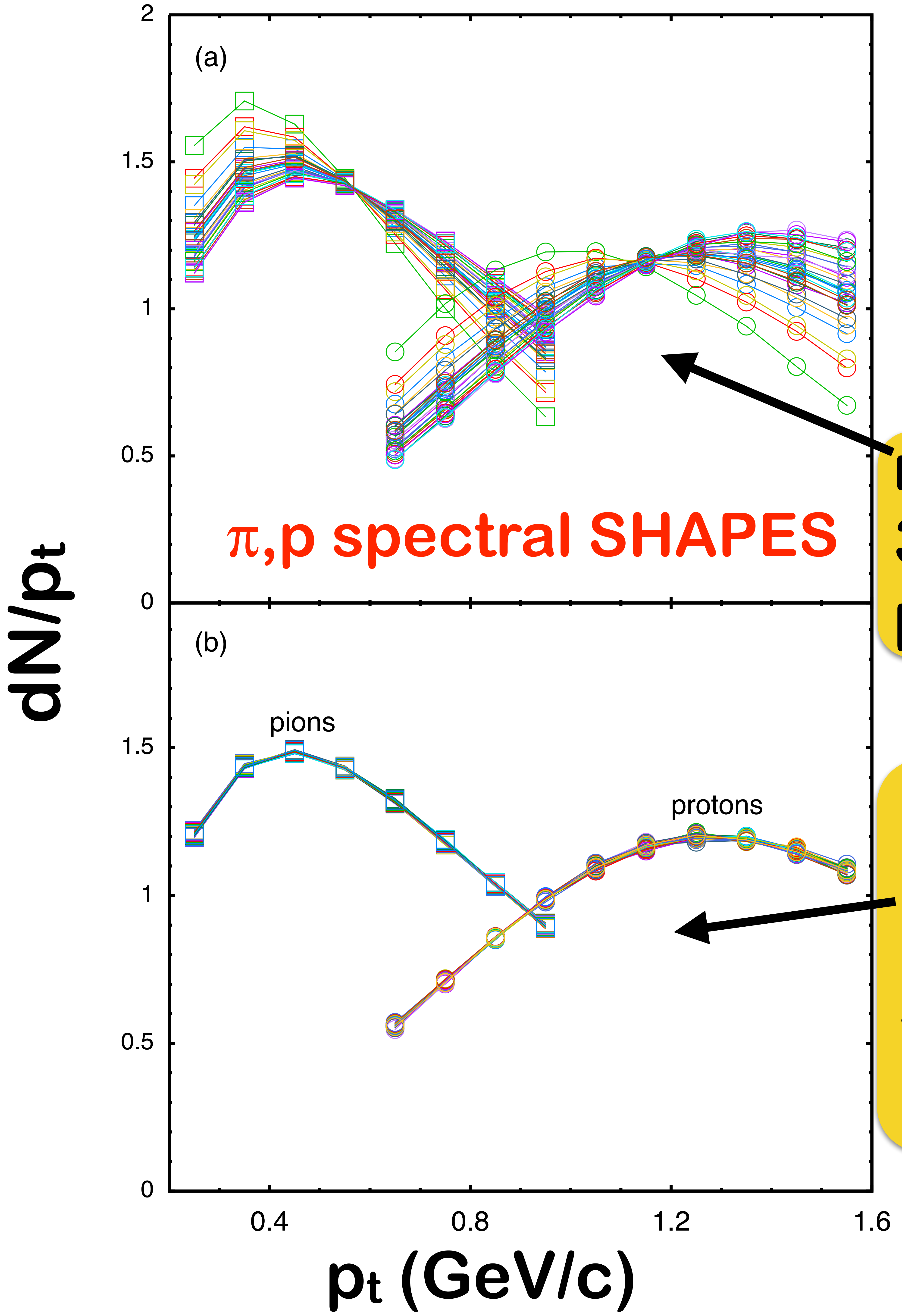
$$\mathcal{L} \sim \exp\left\{ \frac{-1}{2} \sum_a \left(z_a - z_a^{(\mathrm{exp})}\right)^2 \right\}$$

5. Resolving power of RHIC/LHC
   data reduced to $\lesssim$10 numbers!

V. Bayesian Analysis

Checking the Distillation

Spectral information encapsulated by two numbers, dN/dy & $\langle p_t \rangle$



model spectra from 30 random points in parameter prior

74 pion spectra: with $573 < \langle p_t \rangle_\pi < 575$ MeV

44 proton spectra: with $1150 < \langle p_t \rangle_p < 1152$ MeV

V. Bayesian Analysis

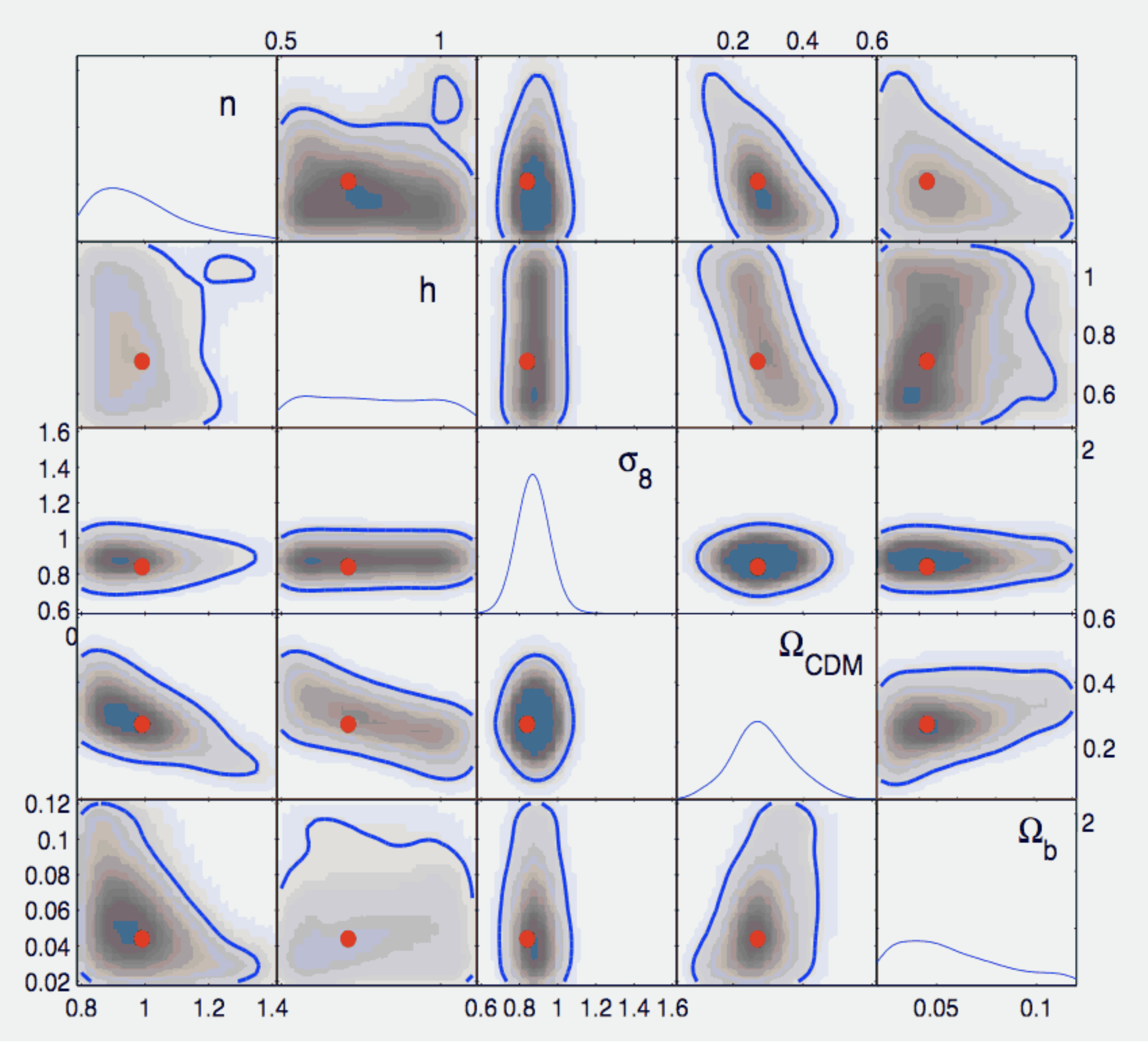
Model Emulators

1. Run the model ~1000 times
   Semi-random points (LHS sampling)

2. Determine Principal Components
   $(y_a - \langle y_a \rangle)/\sigma_a \rightarrow z_a$

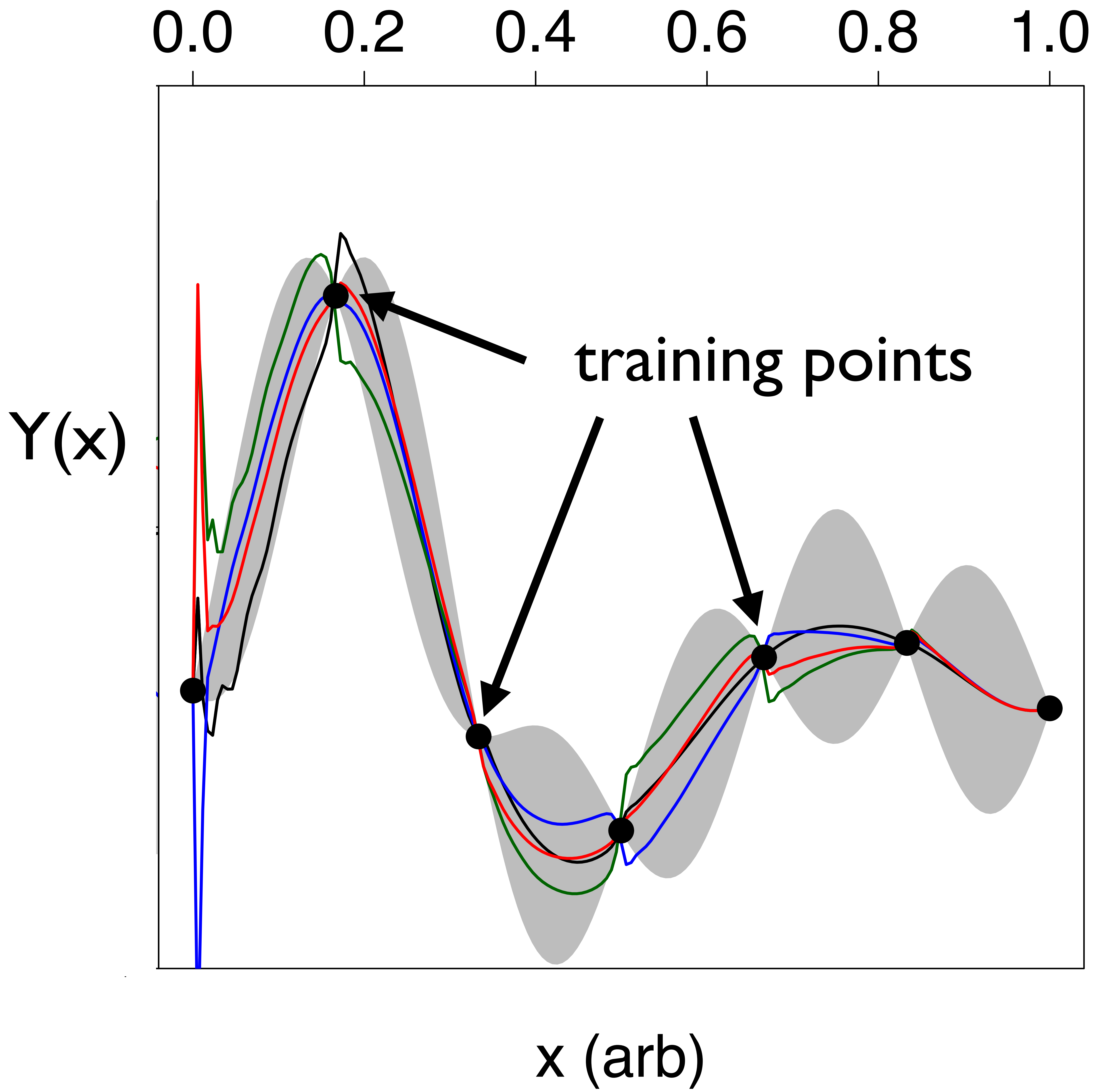
3. Emulate z_a (Interpolate) for MCMC
   Gaussian Process...

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\frac{1}{2}\sum_a (z_a^{(\text{emulator})}(\mathbf{x}) - z_a^{(\text{exp})})^2\right\}$$



S. Habib,K.Heitman,D.Higdon,C.Nakhleh&B.Williams, PRD(2007)

V. Bayesian Analysis



Gaussian Process Emulator

- Reproduces training points
- Assumes localized Gaussian covariance
- Must be trained, i.e. find “hyper parameters”
- Other methods also work

V. Bayesian Analysis

14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

30 Observables

- π,K,p Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Sizes
- v_2 Weighted by p_t

Initial State Parameters

$$\epsilon(\tau = 0.8\text{fm}/c) = f_{\text{wn}}\epsilon_{\text{wn}} + (1 - f_{\text{wn}})\epsilon_{\text{cgc}},$$

$$\epsilon_{\text{wn}} = \epsilon_0 T_A \frac{\sigma_{\text{nn}}}{2\sigma_{\text{sat}}} \{1 - \exp\left(-\sigma_{\text{sat}} T_B\right)\} + (A \leftrightarrow B)$$

$$\epsilon_{\text{cgc}} = \epsilon_0 T_{\text{min}} \frac{\sigma_{\text{nn}}}{\sigma_{\text{sat}}} \{1 - \exp\left(-\sigma_{\text{sat}} T_{\text{max}}\right)\}$$

$$T_{\text{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\text{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

5 parameters for RHIC, 5 for LHC

V. Bayesian Analysis

Equation of State and Viscosity

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

2 parameters for EoS, 2 for η/s

Review the Grand Plan

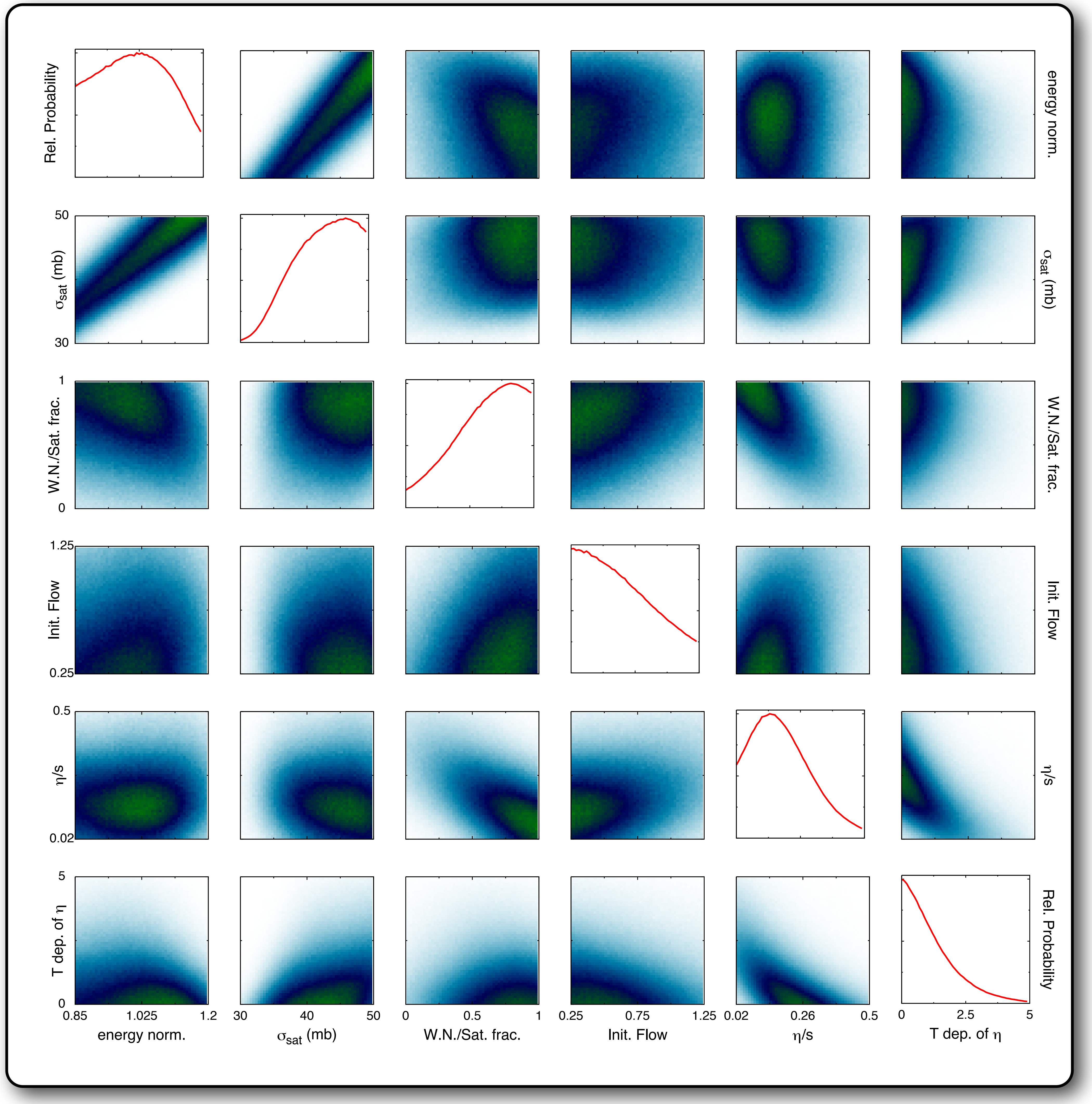
1. **Choose observables**
2. **Distill Data**
3. **Parameterize model**
4. **Run full model hundreds of times (Latin hyper-cube sampling)**
5. **Build & Tune emulator**
6. **Perform MCMC with emulator**
7. **Analyze sensitivities**

VI. RESULTS

Two Calculations

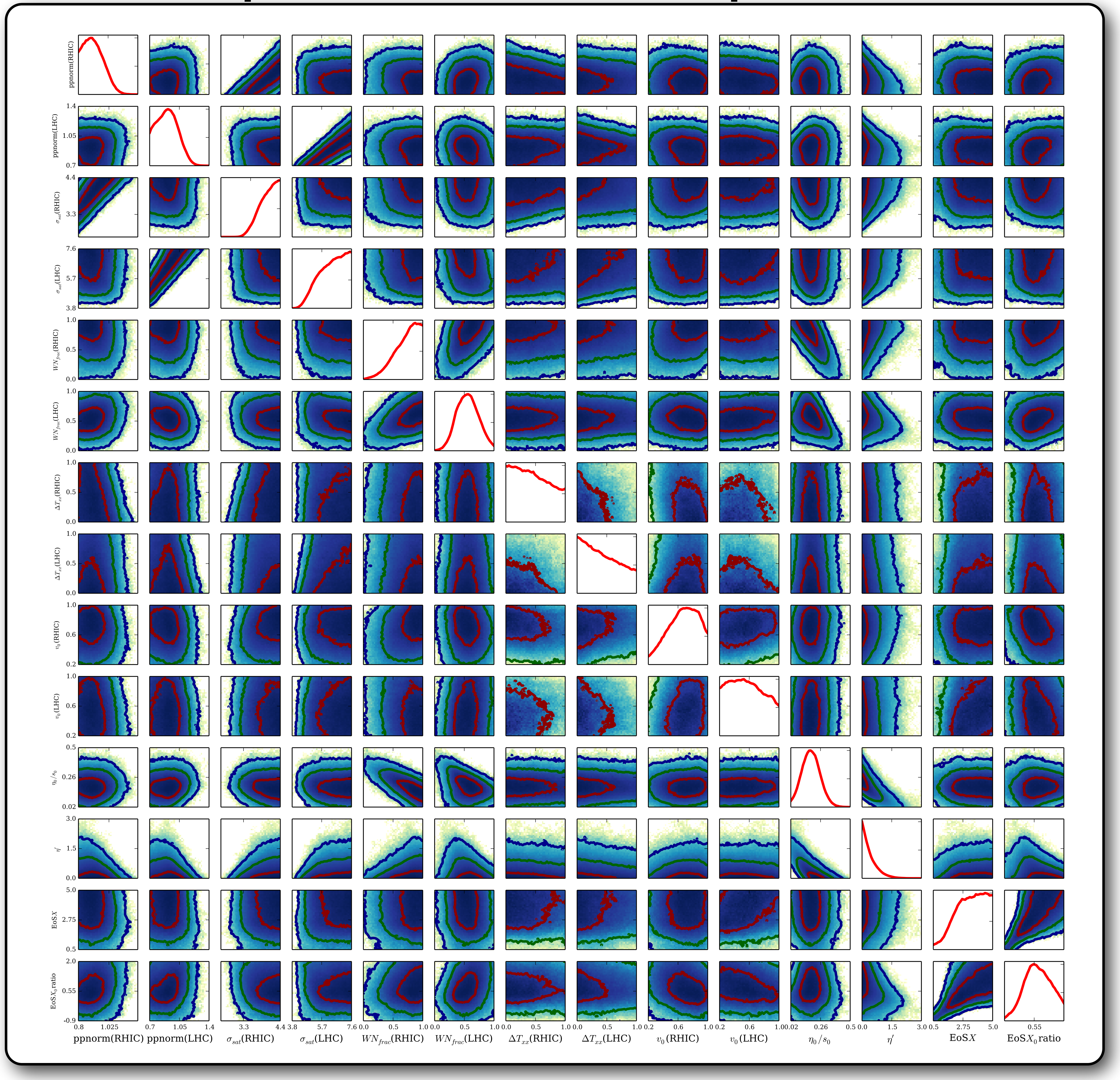
J.Novak, K. Novak, S.P., C.Coleman-Smith & R.Wolpert, PRC 2014

RHIC Au+Au Data
6 parameters

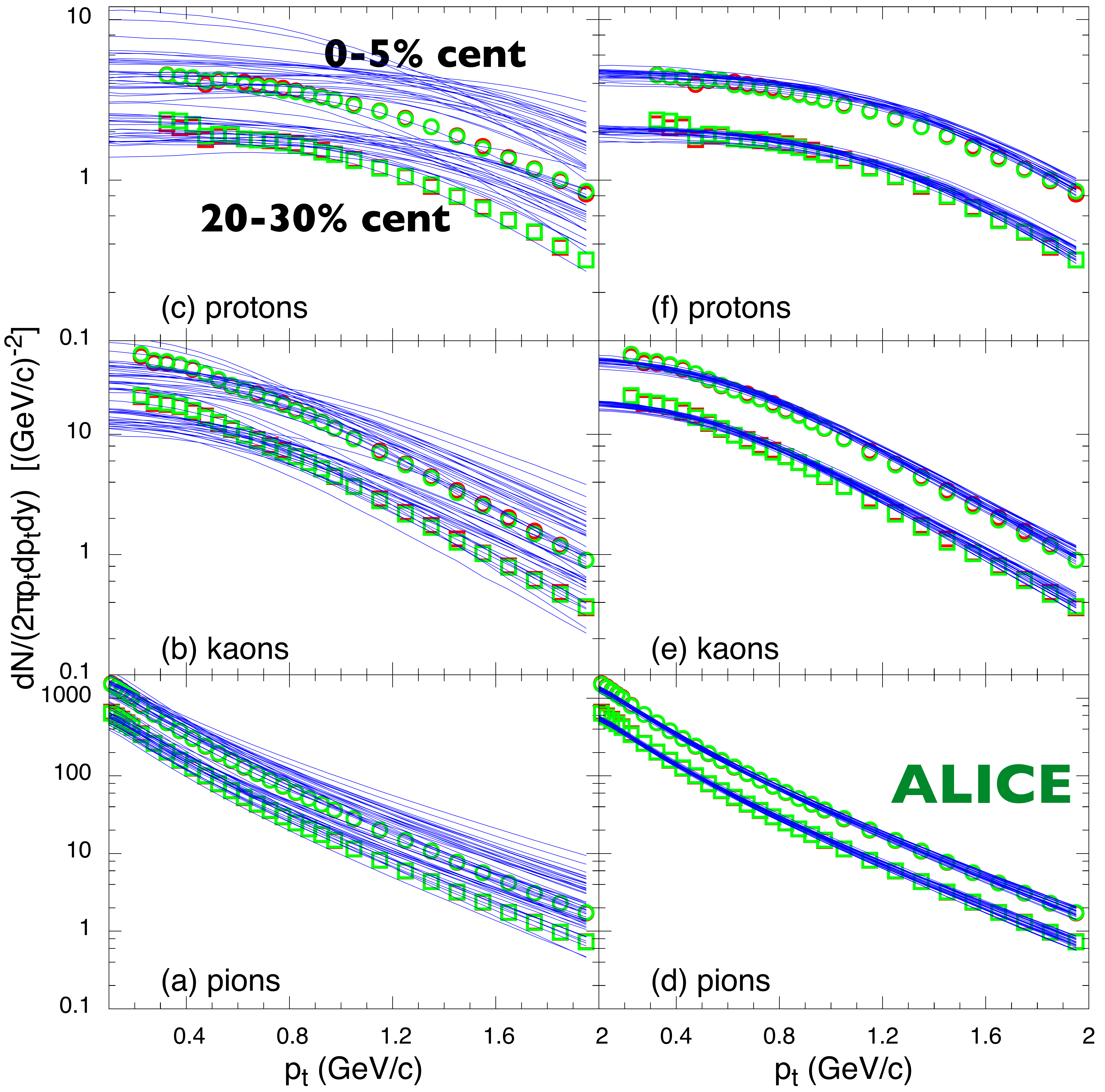


S.P., E.Sangaline, P.Sorensen & H.Wang, PRL 2015

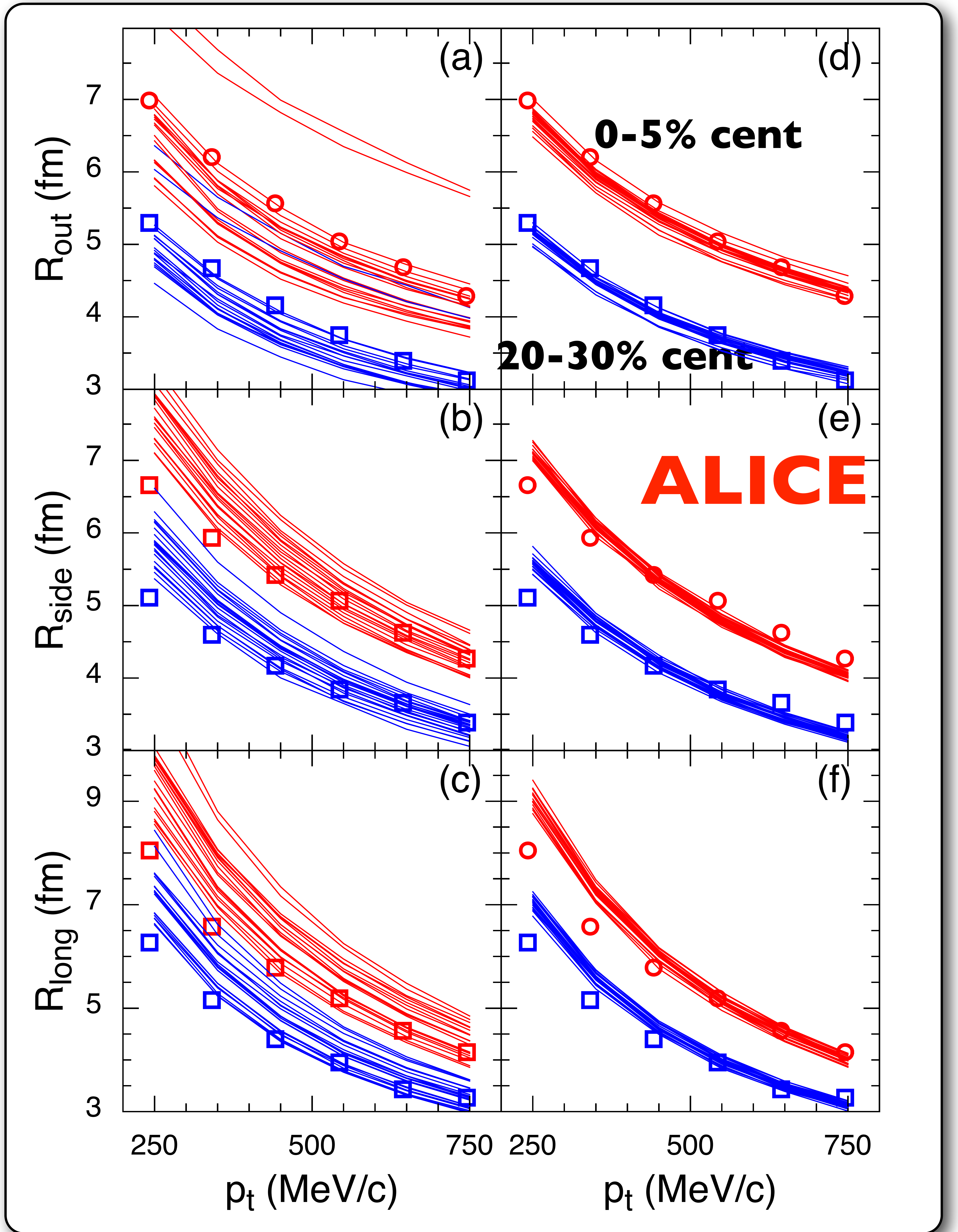
RHIC Au+Au and LHC Pb+Pb Data
14 parameters, include Eq. of State



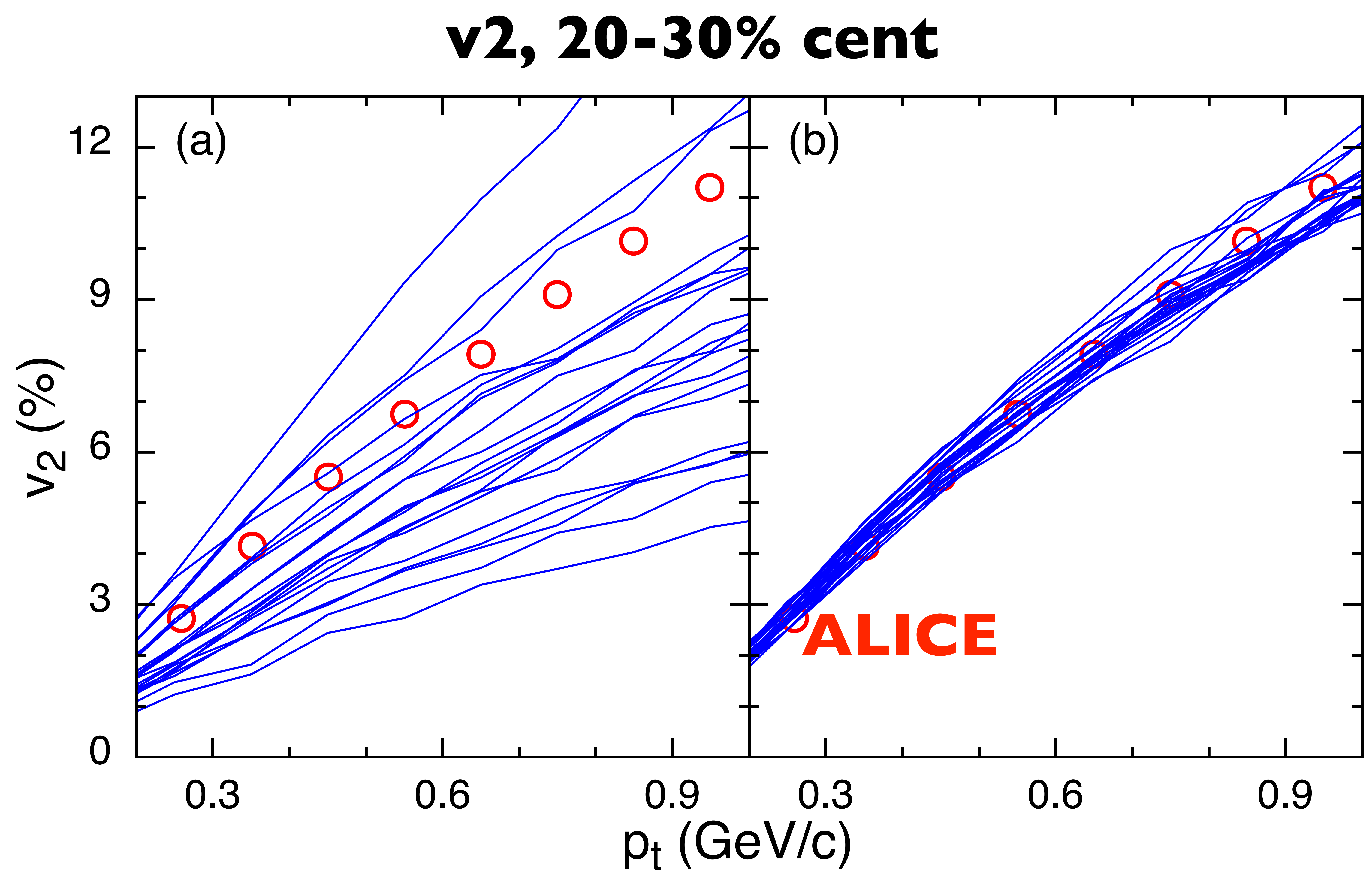
Sample Spectra from Prior and Posterior



Sample HBT from Prior and Posterior



Sample v_2 from Prior and Posterior

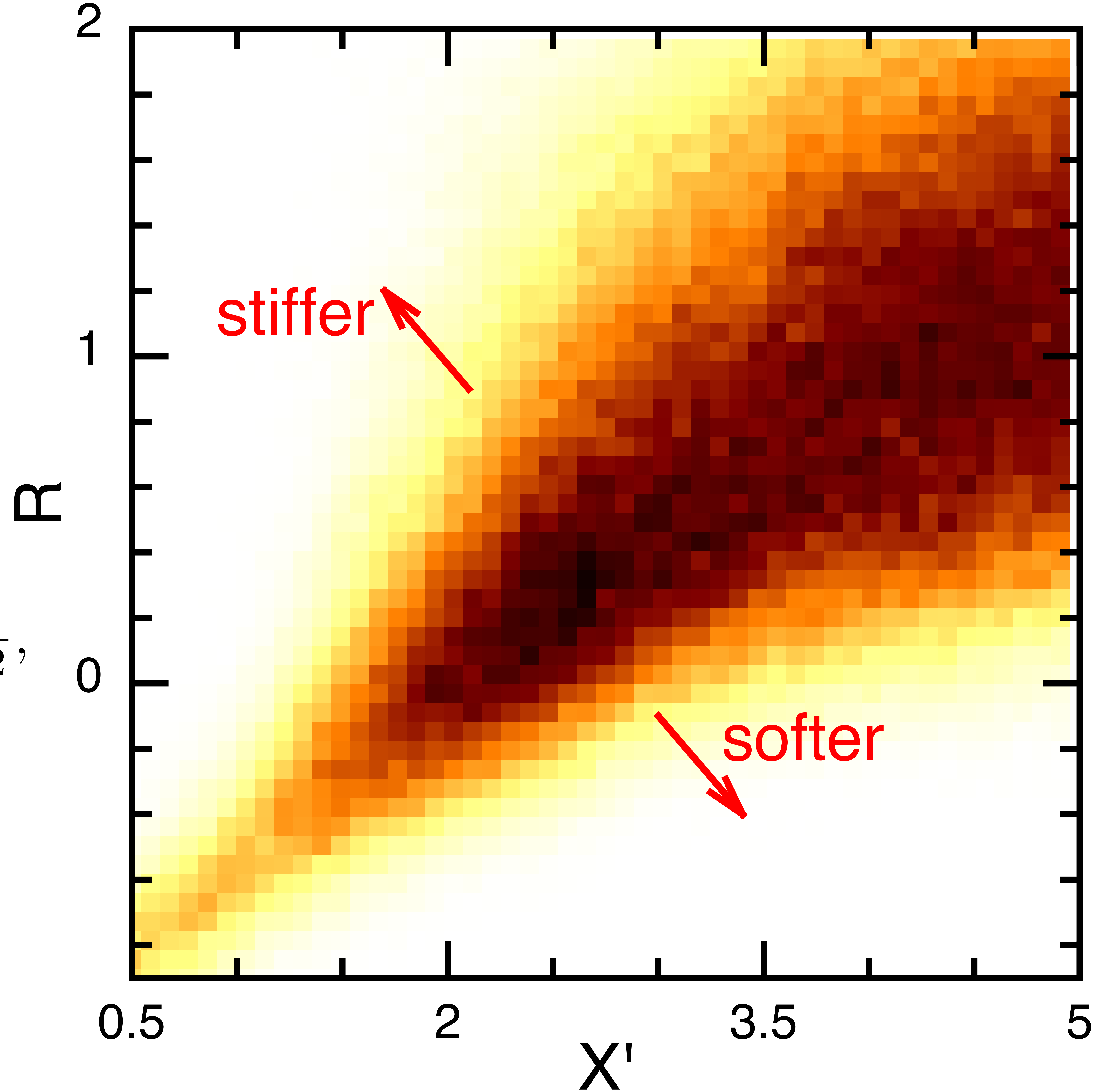


Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

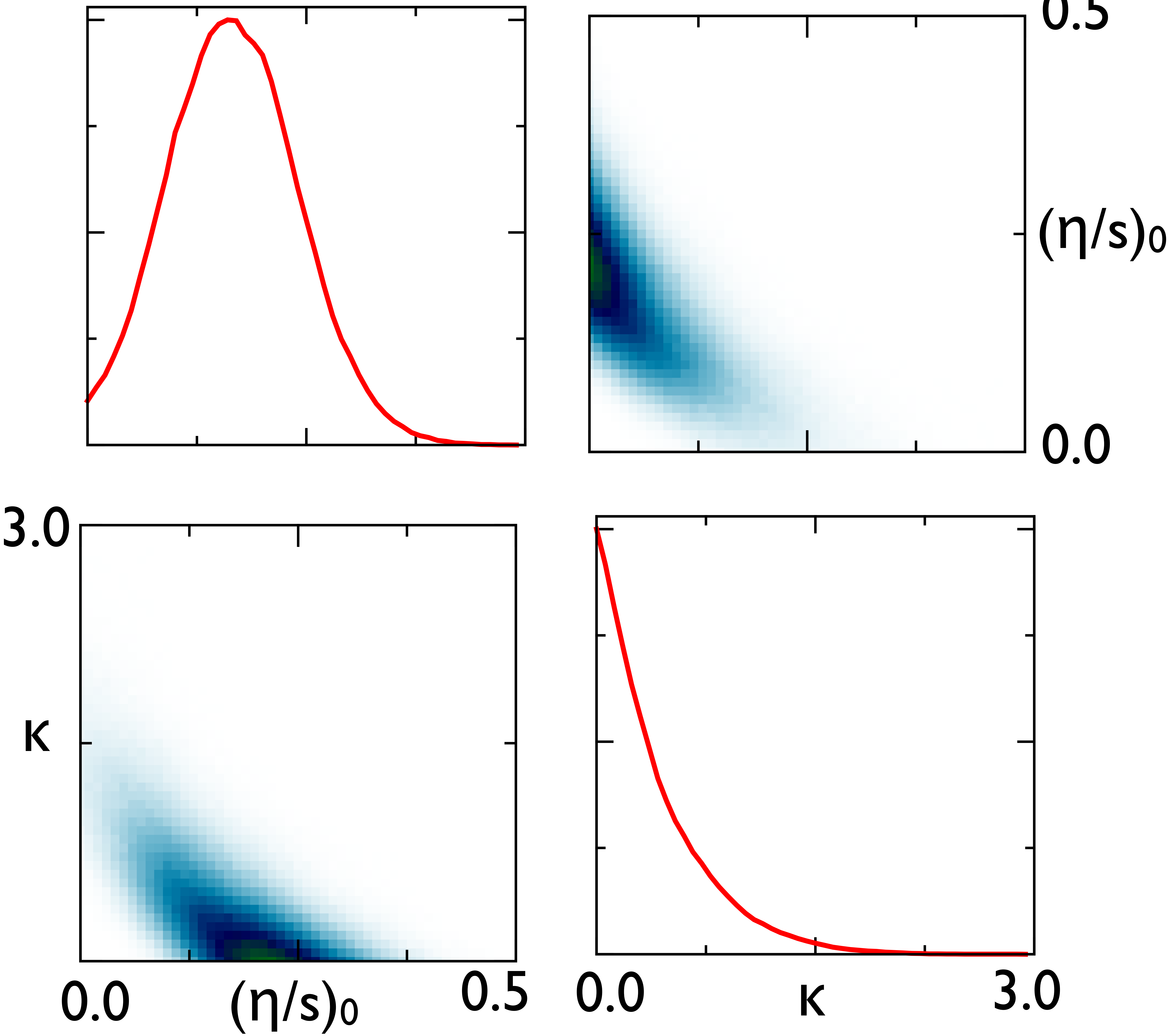
$$x \equiv \ln \epsilon/\epsilon_h$$



Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
UNCONSTRAINED
CONSTRAINED
BY DATA
HADRON GAS
c_s^2 (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

$\eta/s(T)$

$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$

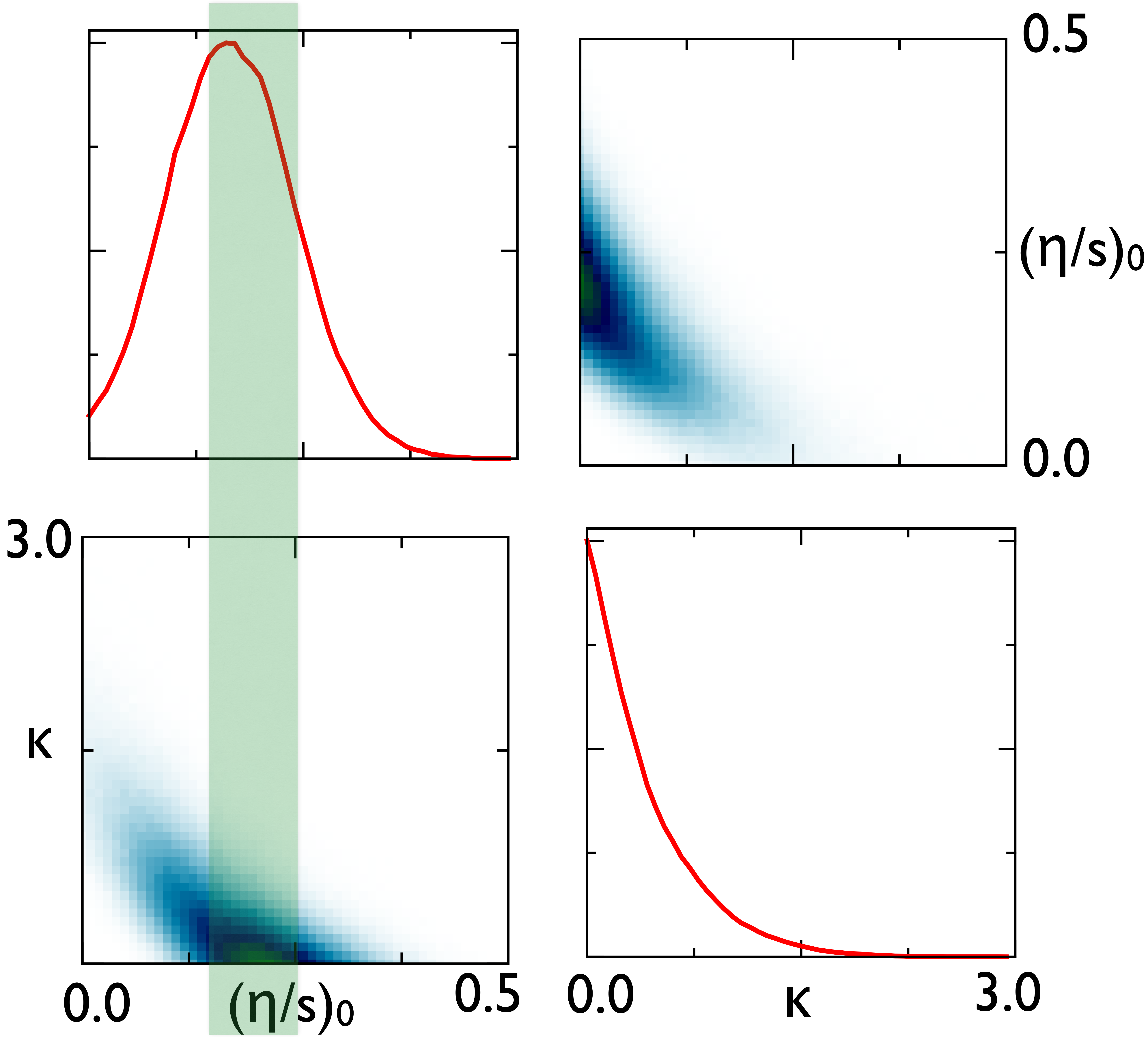


What should you expect for η/s at T=165 MeV?

- ADS/CFT: 0.08
- Perturbative QCD: > 0.5 ($\sigma \approx$ 3 mb)
- Hadron Gas: $\approx$ 0.2 ($\sigma \approx$ 30 mb)

Extracted η/s at T=165 MeV consistent with expectations for hadron gas!

Does not rise strongly in QGP



RESOLVING POWER OF OBSERVABLES

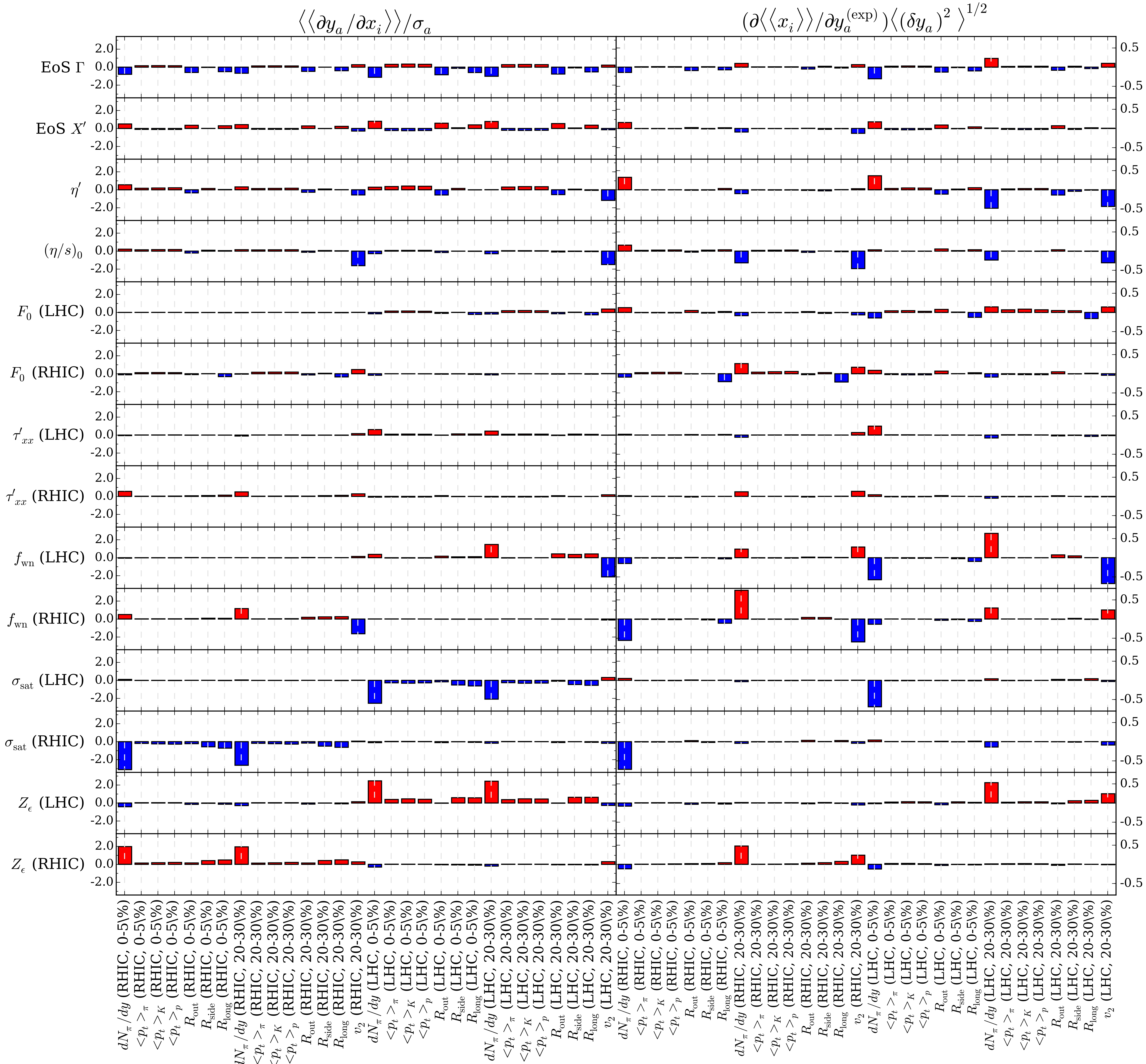
How does changing $y_{a,exp}$ or σ_a alter $\langle\langle x_i \rangle\rangle$ or $\langle\langle \delta x_i \delta x_j \rangle\rangle$?

We need $\dfrac{\partial}{\partial y_a^{(\mathrm{exp})}} \langle\langle x_i \rangle\rangle$ **NOT** $\dfrac{\partial}{\partial x_i} y_a^{(\mathrm{mod})}$

From covariances form MCMC trace + linear algebra….

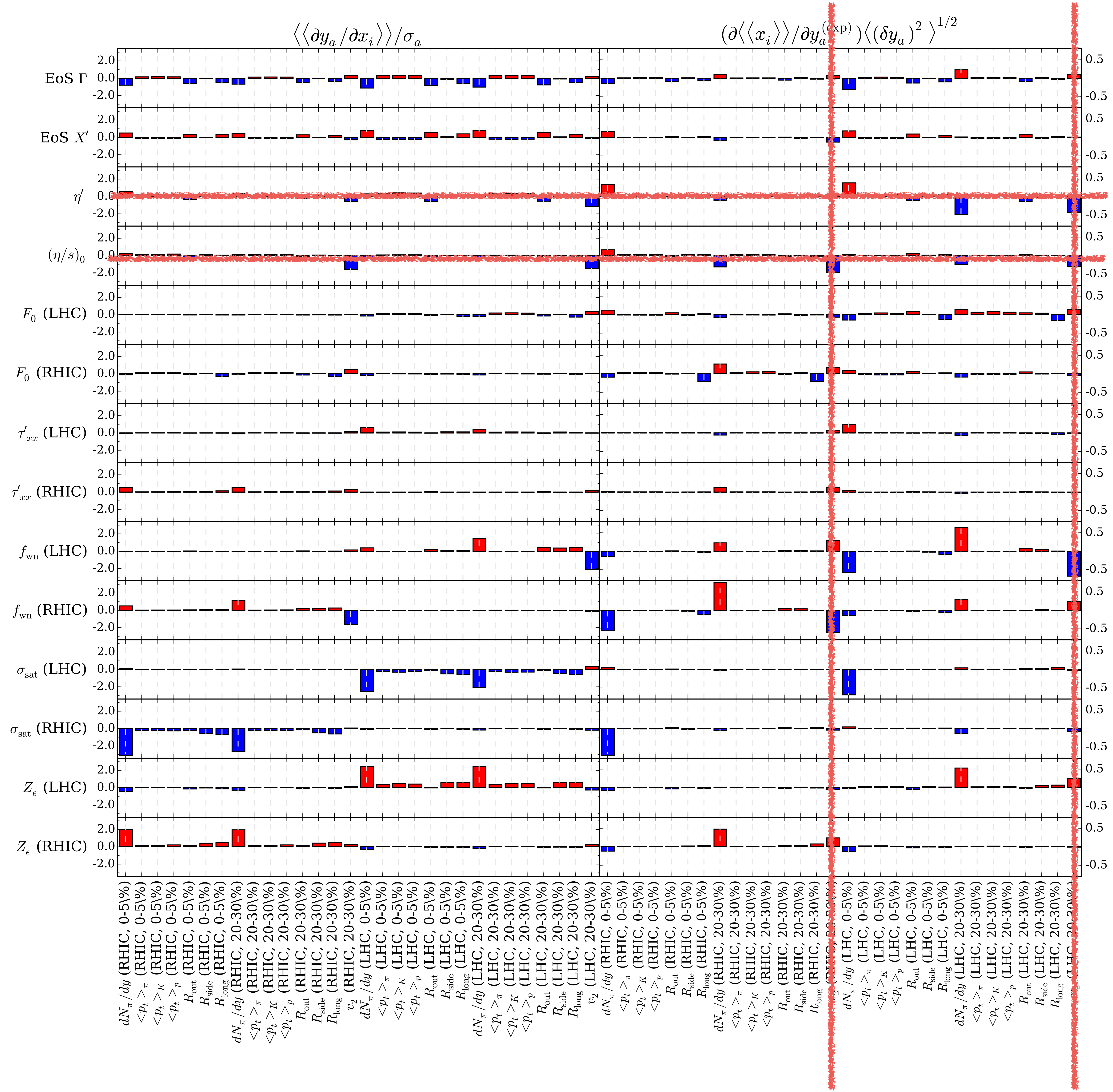
E.Sangaline and S.P., arXiv 2015

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\bigg|_{y_{b\neq a}}$$

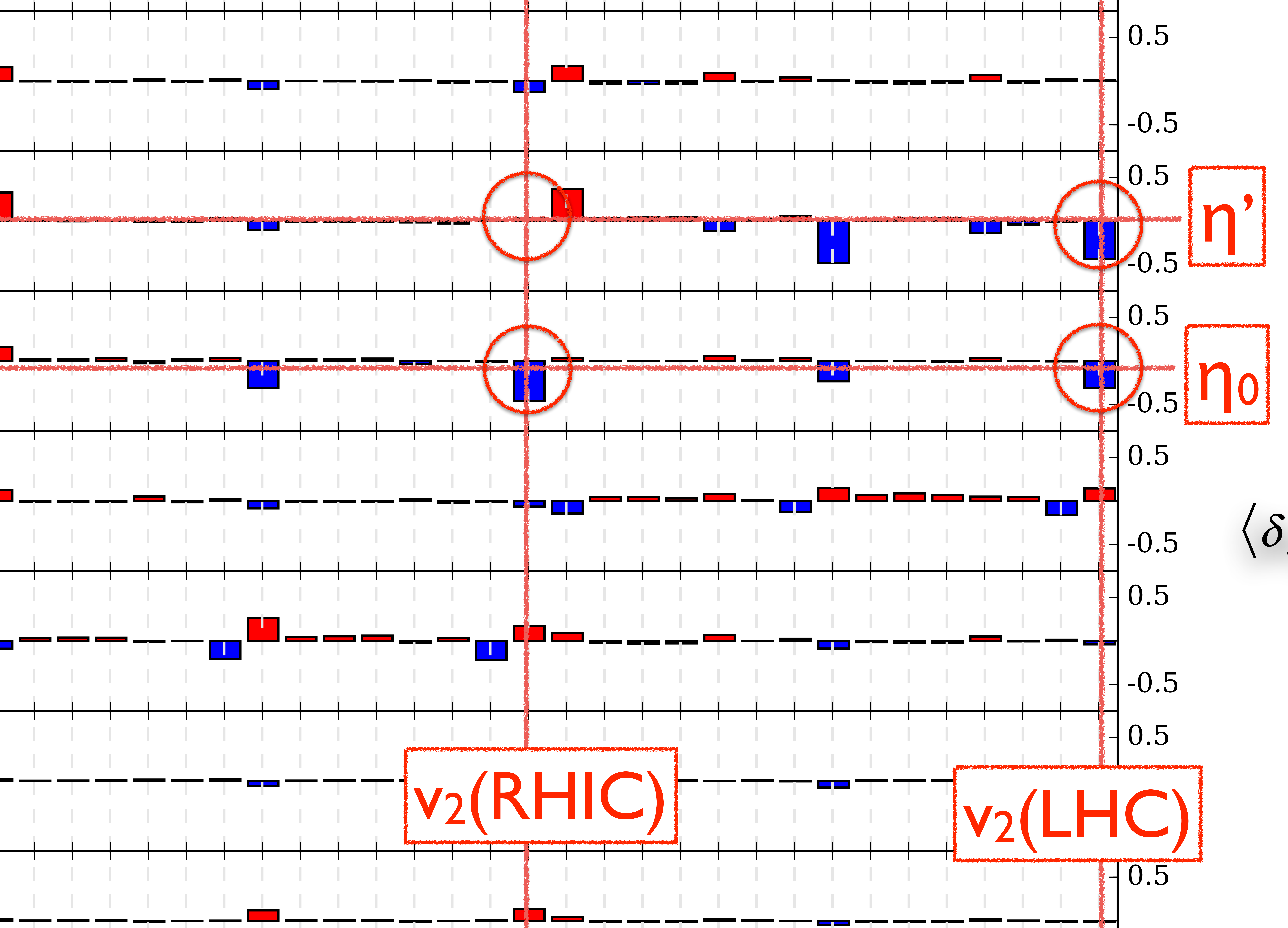


$$\langle \delta y_a \delta y_a \rangle^{1/2} \frac{\partial x_i}{\partial y_a}\bigg|_{y_{b\neq a}}$$

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\bigg|_{y_{b\neq a}}$$



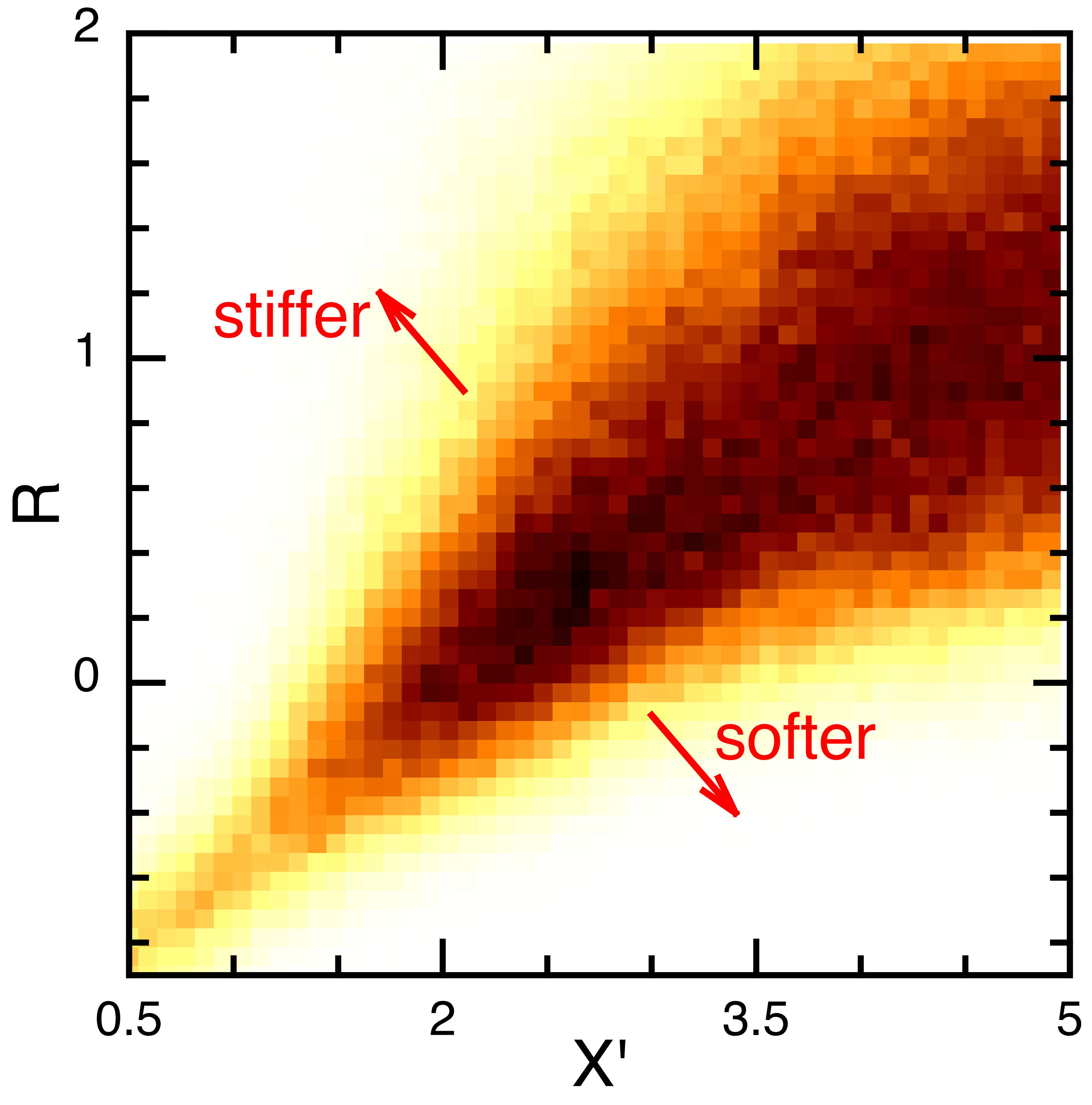
$$\langle \delta y_a \delta y_a \rangle^{1/2}\frac{\partial x_i}{\partial y_a}\bigg|_{y_{b\neq a}}$$

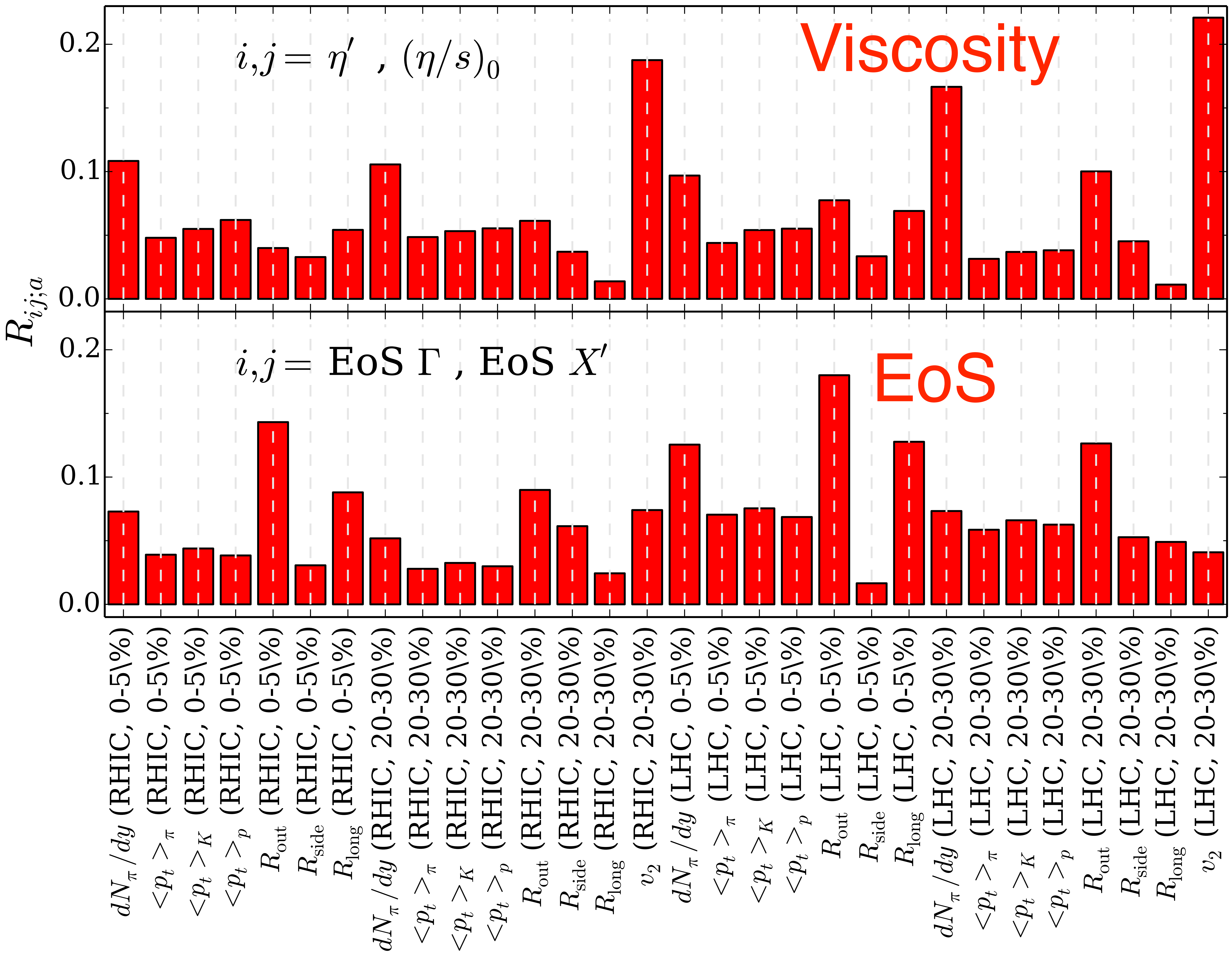


$$\left\langle \delta y_a \delta y_a \right\rangle^{1/2} \left.\frac{\partial x_i}{\partial y_a}\right|_{y_{b\neq a}}$$

$$\frac{d}{d\sigma_y}\sqrt{\begin{vmatrix} \langle\langle\delta x_1\delta x_1\rangle\rangle & \langle\langle\delta x_1\delta x_2\rangle\rangle \\ \langle\langle\delta x_1\delta x_2\rangle\rangle & \langle\langle\delta x_2\delta x_2\rangle\rangle \end{vmatrix}}\,\langle\delta y\delta y\rangle^{1/2}$$

2-Parameter Resolving Power





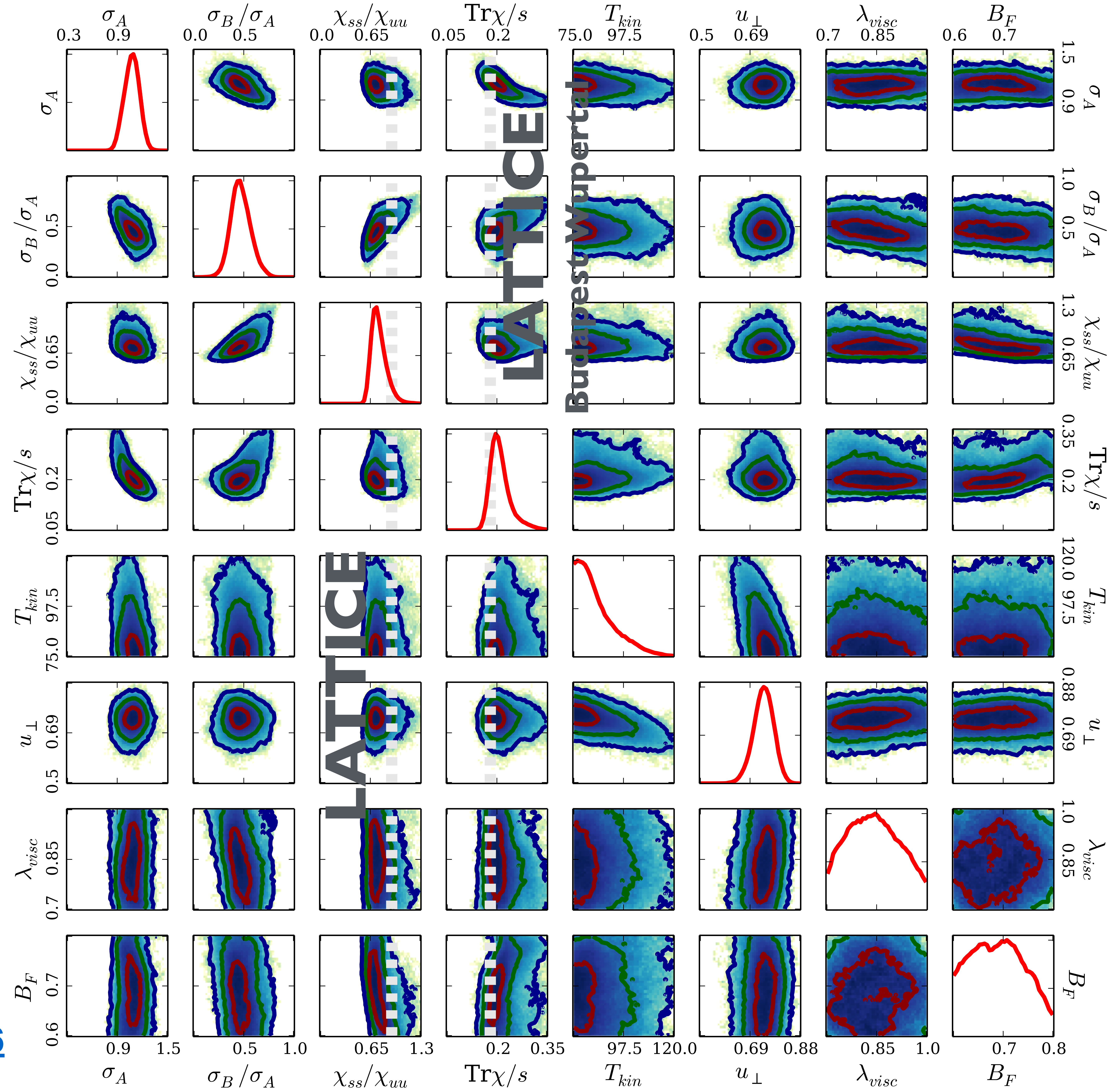
What determines EoS?

- Lots of observables
- Femtoscopic radii are important

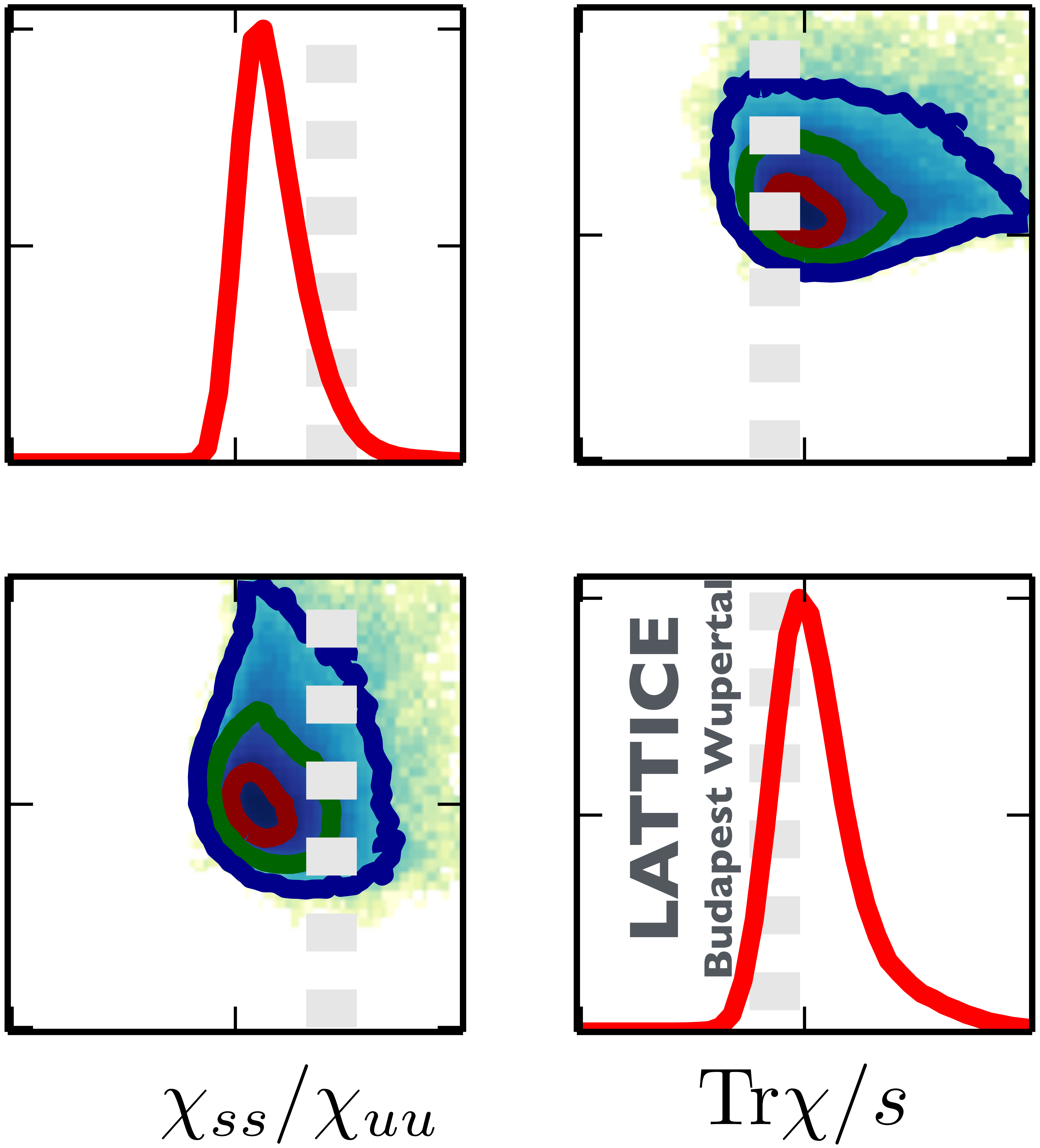
What determines viscosity?

- Both v_2 and multiplicities
- T-dependence comes from LHC v_2

Charge BFs and charge susceptibilities (analyzed with STAR data) parametric model



[S.P., C. Ratti and W.McCormack, PRC 2016]



Early production of u,d,s consistent with equilibrium at 25% level

S.P., C. Ratti and W.McCormack, PRC 2016

CONCLUSIONS

- Robust, emulation works splendidly
- Scales well to more parameters & more data
- Eq. of State and Viscosity can be extracted from data
- Eq. of State consistent with lattice gauge theory
- Early chemistry near (~25%) QGP equilibrium
- Heavy-Ion Physics can be a Quantitative Science!!!!