your name(s)____

____

Physics 852 Exercise #1 - Friday, Jan. 16nd

Consider two kinds of spinless particles, whose masses are m_A and m_B. The particles exist in a one-dimensional world. We define field operators,

$$\Phi_A(x) = \sum_k \frac{1}{\sqrt{LE_A(k)}} \left(a_k e^{ikx} + a_k^\dagger e^{-ikx} \right),$$
$$\Phi_B(x) = \sum_k \frac{1}{\sqrt{LE_B(k)}} \left(b_k e^{ikx} + b_k^\dagger e^{-ikx} \right).$$

Here, L is some large length. The interaction Hamiltonian is

$$H_{\text{int}} = g \int dx\ \Phi_A(x)\Phi_B(x)^2. \tag{0.1}$$

Now, let $m_A > 2m_B$, so that the heavier A particle can decay into two lighter B particles. Also assume the decay energy is sufficiently high that the lighter particles move relativistically, $E_B(k)^2 = (\hbar c)^2 k^2 + m_B^2$.

1. Calculate the matrix element $\mathcal{M} = \langle k_{B1}, k_{B2} | H_{\text{int}} | k_A = 0 \rangle$. Use the orthogonality of the momentum states:
$$\int dx\ e^{ik_1 x} e^{ik_2 x} = L\delta_{k_1,-k_2}.$$
Your answer should contain a Kronecker delta.

2. Calculate the decay rate, Γ, for the reaction $A \to 2B$ in lowest order perturbation theory. Express your answer in terms of m_A, m_B and g.

3. We have been working in units where m_A and m_B have units of energy. What are the dimensions of g? Check the dimensional consistency of your answer for Γ.

Solutions:
a)

$$\begin{aligned}
\langle f | H_{\text{int}} | i \rangle &= \langle 0 | b_k b_{k'} H_{\text{int}} a_{K=0}^\dagger | 0 \rangle \\
&= \int dx e^{i(k+k')x} \frac{2g}{L^{3/2}(E_B(k)E_B(k')m_A)^{1/2}} \\
&= L\delta_{k,-k'} \frac{2g}{L^{3/2} E_B m_A^{1/2}}.
\end{aligned}$$

b)

$$
\begin{aligned}
\Gamma &= \frac{2\pi}{\hbar}\sum_k |\langle f|H_{\text{int}}|i\rangle|^2\delta(\epsilon_f-\epsilon_i) \\
&= \frac{2\pi}{\hbar}\int_0^\infty \frac{dk}{2\pi} L|\langle\cdots\rangle|^2 \\
&= \frac{4g^2}{\hbar m_A}\int \frac{dk}{E_B^2}\delta(2E_B-m_A) \\
&= \frac{4g^2}{\hbar m_A E_B^2}\frac{1}{2dE_B/dk} \\
&= \frac{4g^2}{\hbar^2 c m_A^3/4}\frac{(m_A/2)/2}{\sqrt{(m_A/2)^2-m_B^2}} \\
&= \frac{4g^2}{\hbar^2 c m_A^2\sqrt{(m_A/2)^2-m_B^2}}.
\end{aligned}
$$

c)

$$
\begin{aligned}
[\Phi] &= [L]^{-1/2}[E]^{-1/2}, \\
[E] &= [g][L][\Phi]^3, \\
[g] &= [E]/([L][\Phi]^3]) \\
&= [E]/([]^{-1/2}[E]^{-3/2}) \\
&= [E]^{5/2}[L]^{1/2}.
\end{aligned}
$$