your name(s)____

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Physics 852 Exercise #11 - Friday, April. 15th

Consider the scalar field operator (in one dimension),

$$\Phi(x,t) = \frac{1}{\sqrt{L}} \sum_k \frac{1}{\sqrt{2\omega_k}} \left[a_k e^{-i\omega_k t + ikx} + a_k^\dagger e^{i\omega_k t - ikx} \right],$$

$\omega_k = E_k/\hbar$. One creates a state $|\eta\rangle$, which for positive times is defined as

$$|\eta(t)\rangle_I = \exp\left\{ i \int dt' dx\, j(x,t')\Phi(x,t') \right\} |0\rangle,$$

where $j(x,t')$ is a real function and is zero for $t' > t$.

1. Find the commutation relation for $[\Phi(x,t), \dot{\Phi}(x',t)]$.
2. Show that the $\langle \eta | \eta \rangle = 1$. (hint: should be very short)
3. Rewrite $|\eta(t)\rangle$ using the operators a_k and $a_k^\dagger$ instead of $\Phi(x,t)$, and assume that $j(t' > t) = 0$. Your expression should use the Fourier transforms of $j(x,t)$,
$$\tilde{j}(k,\omega) \equiv \int dx\, dt\, e^{-i\omega t + ikx} j(x,t).$$
4. Find an expression for $\langle \eta | a_k^\dagger a_k | \eta \rangle$. Note that $\langle \eta | a_k^\dagger a_k | \eta \rangle$ in the Schrödinger representation is the same as $\langle \eta(t) | a_k^\dagger(t) a_k(t) | \eta(t) \rangle$ in the Heisenberg representation.
5. Find an expression for the net number of particles.
6. Find an expression for $\langle \eta | a_k^\dagger a_q^\dagger a_q a_k | \eta \rangle$.