your name(s)____

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Physics 852 Exercise #10 - Friday, April. 2nd

Chirality

Consider the chirality operator,

$$\gamma_5 = i\gamma_0\gamma_x\gamma_y\gamma_z = i\beta\beta\alpha_x\beta\alpha_y\beta\alpha_z = -i\alpha_x\alpha_y\alpha_z.$$

1. Show that γ_5 is Hermitian.
2. Show that $\gamma_5^2 = \mathbb{I}$. (this shows that γ_5 behaves as a scalar under rotations and boosts)
3. What are the eigenvalues of γ_5
4. Show that $(1+\gamma_5)/2$ and $(1-\gamma_5)/2$ are projection operators.
5. Show that γ_5 commutes with the Hamiltonian for massless particles,

   $$H = \vec{\alpha}\cdot\vec{p},$$

   but does not commute with H if a mass term

   $$H_M = \beta m$$

   is added.
6. Write γ_5 in the chiral representation.
7. "Prove" that

   $$\frac{1}{3!}\sum_{ijk}\epsilon_{ijk}\alpha_i\alpha_j\alpha_k\alpha_\ell = i\gamma_5\alpha_\ell = \frac{1}{2}\sum_{ij}\epsilon_{ij\ell}\alpha_i\alpha_j = i\Sigma_\ell.$$

   You can use the fact that γ_5 is rotationally invariant.
8. For massless particles, the Dirac equation is

   $$(\vec{\alpha}\cdot\hat{p})u_{\vec{p},s} = u_{\vec{p},s}$$
   $$(\vec{\alpha}\cdot\hat{p})v_{-\vec{p},s} = -v_{-\vec{p},s}.$$

   Exploiting the information above, show that for massless particles,

   $$\gamma_5 u_{\vec{p},s} = (\vec{\Sigma}\cdot\hat{p})u_{\vec{p},s},$$
   $$\gamma_5 v_{-\vec{p},s} = -(\vec{\Sigma}\cdot\hat{p})v_{-\vec{p},s}.$$

Comment: In the standard model the weak interaction couples only to neutrinos of a given chirality, e.g. the terms coupling to neutrinos appears as $(1-\gamma_5)\gamma_\mu\Psi(x)$. The operator γ_5 has odd parity, so the operator $(1-\gamma_5)$ mixes even and odd parity maximally. Thus, in the famous experiment of Chien-Shiung Wu https://en.wikipedia.org/wiki/Chien-Shiung_Wu, the direction of neutrinos (a vector) lined up with the direction of the magnetic field (a pseudo vector) thus demonstrating that in the weak interaction the choice of right-handed vs. left-handed coordinate systems is no longer arbitrary, and represents a striking violation of parity conservation.