your name(s)____

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Physics 851 Exercise #9 - Monday, Nov. 1st

Consider a particle of mass M confined to a two-dimensional circle of radius R. The particle moves in a periodic potential,

$$V(\phi + 2\pi/N) = V(\phi),$$

where N is an integer. Assuming the wave-function has the form,

$$\psi(\phi) = e^{im\phi} + Be^{-im\phi},\ \ 0 < \phi < 2\pi/N,$$

and that the eigenvalue of the rotation operator, $\mathcal{R}(2\pi/N)$, were $e^{i\gamma}$, i.e.,

$$\psi(\phi + 2\pi/N) = e^{i\gamma}\psi(\phi).$$

In your homework you showed that m could be found and that the allowed values of γ were $j\alpha$, where $\alpha = 2\pi/N$. You then considered a potential of the form,

$$V(\phi) = \beta \sum_{j=1,N} \delta(\phi - 2\pi j/N),$$

and found a transcendental equation for m,

$$\begin{aligned}
&0 = p\sin(m\alpha) + 2m\cos(m\alpha) - 2m\cos(j\alpha),\\
&\gamma = j\alpha,\ \alpha = 2\pi/N,\\
&p = 2M\beta R^2/\hbar^2.
\end{aligned}$$

Write a program (you can use any packages you wish, to solve for the lowest 4 values of m as a function of γ. Plot m (energy is $\hbar^2m^2/2MR^2$) as a function of γ for $-\pi \leq \gamma \leq \pi$. Assume $p\alpha = 5.0$ and make plots for $N = 4$ and for $N = 100$. Note that the function will be multi-valued because you will find the lowest four values of m for each γ.