your name(s)____

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Physics 851 Exercise #6 - Monday, October 18th

Continuing from Chapter 4, number 3 in the homework: You considered the matrices

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

1. Write the matrices

$$S_\pm = S_x \pm i S_y.$$

Show what happens when $S_\pm$ acts on each of the three basis vectors. (The basis vectors are the eigenvectors of S_z).

2. Write down the matrix that performs a rotation by an angle ϕ about the z axis, $R_z(\phi) = e^{-iS_z\phi/\hbar}$.

3. In this basis, we will define $\hat{x}, \hat{y}$ and $\hat{z}$ in terms of the basis vectors,

$$|m=1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}),$$

$$|m=0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \hat{z}$$

$$|m=-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y}).$$

Show that

$$\hat{x} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

$$\hat{y} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i \\ 0 \\ i \end{pmatrix},$$

$$\hat{z} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

4. Apply the rotation matrix, $R_z(\phi)$, on $\hat{x}$, then express the result in terms of $\hat{x}$, $\hat{y}$ and $\hat{z}$.

5. Find the matrix U that transforms the coordinate system into one where

$$\hat{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \hat{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

6. Extra Credit: Transform $R_z(\phi)$ to this new basis, i.e. find $R'_z = U R_z U^\dagger$.