your name(s)____

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Physics 851 Exercise #5 - Monday, Oct. 11th

Consider two nucleons, mass = 939 MeV/c^2. They bind into a deuteron with a binding energy of B=2.2 MeV. Consider a potential,

$$V(r) = \begin{cases} \infty, & r < 0 \\ -V_0/[1 + e^{(r-a)/a}], & r > 0 \end{cases},$$

where $a = 0.707$ fm. Perform the following calculations numerically.

1. Find V_0 so that the binding energy is indeed $B = 2.2$ MeV. You can treat this as a one-dimensional problem, where $r < 0$ is suppressed by an infinite repulsive potential. Also, don't forget to use the reduced mass $\mu = M/2$. Schrödinger's equation for an s-wave is the same as for a one-dimensional problem,

$$-\frac{\hbar^2}{2\mu}\partial_r^2\phi_0(r) + V(r)\phi_0(r) = E\phi_0(r).$$

   The boundary condition is that $\phi_0(r = 0) = 0$. You can assume that for large r the wave function behaves as e^{-qr}, with q chosen according to the binding energy. Integrate numerically to $r = 0$, then repeat with different values of V_0 until you get $\phi(0) = 0$.

2. What is the r.m.s. radius? (in fm)

$$R^2 = \frac{\int dr\, r^2|\phi_0(r)|^2}{\int dr\, |\phi_0(r)|^2}.$$

FYI: $\hbar c = 197.327$ MeV fm. If masses are in units of MeV/c^2, energies are in MeV, momenta are in units of MeV/c, and distances are in units of fm, Schrödinger's equation,

$$-\frac{(\hbar c)^2}{2\mu c^2}\partial_r^2\phi_0(r) + V(r)\phi_0(r) = E\phi_0(r),$$

becomes

$$-\partial_r^2\phi_0(r) = \left\{-q^2 - \frac{2\mu c^2}{(\hbar c)^2}V(r)\right\}\phi_0(r).$$

and $\mu c^2 = 469.5$ MeV, while $q^2 = 2\mu c^2 B/(\hbar c)^2$. In these units $\hbar$ always appear as the combination $\hbar c$, and mc^2 is in units of MeV. One can then ignore the factors of c and treat energies, masses and momenta as if they are all in the same units.