your name(s)____

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Physics 851 Exercise #2

Neutral Kaon Oscillations: There are two kinds of neutral kaons one can make using down and strange quarks,

$$|K^0\rangle = |d\bar{s}\rangle,\ \ |\bar{K}^0\rangle = |s\bar{d}\rangle.$$

If it weren't for the weak interaction, the two species would have equal masses, and the Hamiltonian (for a kaon with zero momentum) would be

$$H_0 = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}.$$

However, there is an additional term from the weak interaction that mixes the states,

$$H_m = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}.$$

The masses of a neutral kaon are 497.6 MeV, without mixing, but after adding the mixing term the masses differ by 3.56μeV. The two eigenstates are known as K_S (K-short) and K_L (K-long), because they decay with quite different lifetimes.

1. What is ϵ?

Solution:
A unitary transformation will change σ_x into σ_z, which will transform H_m into:

$$H_m = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

which diagonalizes H. The mass difference it thus 2ϵ. So, $\epsilon = 3.56$ μeV/2=1.78 μeV.

2. If one creates a kaon in the K_0 state at time $t = 0$, find the probability it would be measured as a $\bar{K}^0$ as a function of time.

Solution:

$$\begin{aligned} |\psi(t=0)\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \\ \psi(t)\rangle &= e^{-iHt/\hbar}|\psi(t=0)\rangle \\ &= e^{-iMt/\hbar}\left[\cos(\epsilon t/\hbar) - i\sigma_x \sin(\epsilon t/\hbar)\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \cos(\epsilon t/\hbar) \\ -i\sin(\epsilon t\hbar) \end{pmatrix}. \end{aligned}$$

Probability of being in $|\bar{K}_0\rangle$ state is $\sin^2(\epsilon t/\hbar)$

3. A beam kaons is created in the K_0 channel and has a kinetic energy of 600 MeV per kaon. Plot the probability that the kaon is in the K_0 state as a function of the distance traveled, x. Ignore the fact that the kaons decay.

Solution:
The energy is $E = m + KE$ and $\gamma = E/m$, or $\gamma v = \sqrt{(E/m)^2 - 1} = 1.966$. The proper time is

$$\tau = \frac{x}{\gamma v c} = \frac{x}{c\sqrt{(E/m)^2 - 1c}} = x/(1.966c).$$

Probability of being in original K_0 state is

$$P_{K0}(x) = \cos^2(\epsilon\tau/\hbar) = \cos^2(x\epsilon/(\gamma v\hbar c))$$

Using $\hbar c = 1.97326^{-7}$ eV m,

$$P_{K0}(x) = \cos^2(\epsilon x/(\gamma v/c\hbar c) = \cos^2(4.59x)$$

where x is in meters.

4. Repeat (c), but take into account the decays. The states

$$|K_S\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle),$$
$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle),$$

known at K-short and K-long, represent the eigenstates of the Hamiltonian. The lifetime of a K_L is 51.2 ns, and the lifetime of the K_S is 0.0896 ns. Note that the wave function should be modified by the factor $e^{-t/(2\tau)}$ to take decays into account decays of lifetime τ.

Solution:
Write the original $|K_0\rangle$ as

$$|\psi(t=0)\rangle = \frac{1}{2}\left[\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix}\right]$$

so

$$|\psi(\tau)\rangle = \frac{1}{2}e^{-i\bar{M}t/\hbar}\left[\begin{pmatrix}1\\1\end{pmatrix}e^{-i\epsilon t/\hbar - t/2\tau_L} + \begin{pmatrix}1\\-1\end{pmatrix}e^{i\epsilon t/\hbar - t/2\tau_S}.\right]$$

The probability of being in the original state is thus

$$P_{K0}(t) = \frac{1}{4}\left|e^{-i\epsilon t/\hbar - t/2\tau_L} + e^{i\epsilon t/\hbar - t/2\tau_S}\right|^2.$$

In terms of x make substitution $t \to x/(\gamma v c)$,

$$P_{K0}(t) = \frac{1}{4}\left|e^{-i\epsilon x/(\gamma(v/c)\hbar c) - x/(2\gamma v\tau_L)} + e^{i\epsilon x/(\gamma(v/c)\hbar c) - x/(2\gamma v\tau_S)}\right|^2$$

$$= \frac{1}{4}\left\{e^{-x/(\gamma v\tau_L)} + e^{-x/(\gamma v\tau_S)} + 2\cos(2\epsilon t/\hbar)e^{-x/(2\gamma v\tau_L)}e^{-x/(2\gamma v\tau_S)}\right\}.$$

The short component and the interference term die quickly and leaves the long component. The probability one is in the original K_0 state then is roughly 1/4 multiplied the slow decay factor for K_L, i.e.,

$$P_{K0} \sim (1/4)e^{-x/(\gamma v \tau_L)}.$$

Given that γv $1.97c$ and that the speed of light is 30 cm/ns, this gives a decay length, $\lambda = \gamma v \tau_L$ of around 30 meters.

FYI: If the above were exactly true, the K_S state would be even under CP while the K_L would be odd under CP. Here, CP is an operator that changes particles to anti-particles. If the particle-antiparticle symmetry were exact, the CP operator would commute with the Hamiltonian and the eigenstates of the Hamiltonian, K_S and K_L, would have to be eigenstates of CP. The K_S would then decay to two pions and the K_L could decay to three pions. However, there is an additional small CP violating term in the Hamiltonian which allows K_L to have a small probability of decaying to two pions. This was the first experimental laboratory observation of CP violation. CP violation is required to explain the preponderance of matter vs. anti-matter in the universe.