your name(s)____

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Physics 851 Exercise #1

Work in groups of 3 (assigned in class) to complete this assignment. You can use the following link to get some templates (with some of these steps already completed)
https://people.nscl.msu.edu/ pratt/phy851
Templates can be found at the bottom of the web page. C++ users will have to install EIGEN3 package.

Using either C++ or python, write a program to create and manipulate the following $\boldsymbol{3 \times 3}$ matrix,

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{1} & \boldsymbol{2i} & \mathbf{3} \\ \boldsymbol{-2i} & \mathbf{2} & \boldsymbol{-2i} \\ \mathbf{3} & \boldsymbol{2i} & \mathbf{3} \end{pmatrix}.$$

1. On your laptop, create the matrix $\boldsymbol{H}$, then find its inverse. Multiply them together and print the product, showing that its unity.
2. Find the eigenvalues and eigenvectors. Print out the eigenvectors as a matrix, and print out the eigenvector with the lowest eigenvalue.
3. Demonstrate that for each eigenvector, $\boldsymbol{v_\ell}$, that $\boldsymbol{H v_\ell = \lambda_\ell v_\ell}$.
4. Show that if the matrix of eigenvalues is called $\boldsymbol{U^\dagger}$, with each column representing an eigenvector, that the matrix $\boldsymbol{U H U^\dagger}$ is diagonalized with the eigenvalues found above.
5. Choose a constant $\boldsymbol{B}$ so that the lowest eigenvalue of $\boldsymbol{H - B}$ has an absolute value larger than the absolute value of any other eigenvalue of $\boldsymbol{H - B}$. Then take a vector $\boldsymbol{v}$ with all its elements set to unity. Then write a loop where you contract $\boldsymbol{H - B}$ and $\boldsymbol{v}$ to get a new vector $\boldsymbol{v}$,

   $$\boldsymbol{v = (H - B) v},$$

   then normalize $\boldsymbol{v}$ and repeat $\boldsymbol{n}$ times. Demonstrate that for large $\boldsymbol{n}$ you reproduce the eigenvector of $\boldsymbol{H}$ with the lowest eigenvalue, i.e. the ground state wave function if $\boldsymbol{H}$ is a Hamiltonian.

Solution:
See python template posted on course web page.