your name(s)________________________________

____

Physics 851 Exercise #11 - Monday, Nov. 22nd

Consider a one-dimensional world where a particle of mass m experiences the attractive potential,

$$V_0(x) = -\frac{\hbar^2}{mb}\delta(x).$$

A particle in the bound state of the well then experiences a small external potential,

$$V_p(t) = v_0 \cos\omega t,$$
$$\hbar\omega > \frac{\hbar^2}{2mb^2}.$$

1. What is the bound-state energy B of the original well (ignore the external potential)? If you know, or can look up the answer, just write it down.
2. What is the energy, E, and wavenumber k of the liberated particles?
3. Again ignoring the small external potential, find the wave function where at large times (long after V_p is turned off) there is an outgoing plane wave $e^{ikx}/\sqrt{L}$ with $k > 0$, i.e. it moves in the positive x direction. For this boundary condition have an outgoing wave for $x > 0$ and incoming waves for both $x < 0$ and for $x > 0$. This wave function describes that of a created particle with asymptotic momentum k. At some large time ($vt >> L$), the incoming waves disappear and there is only an outgoing wave.
4. Calculate the overlap of the outgoing wave function

$$\alpha(k) \equiv \langle k|\psi_0\rangle,$$

where $|k\rangle$ is the state described above and $|\psi_0\rangle$ is the bound state. Give your answer in terms of k and b.

5. What is the rate at which one liberates the particle?

Solutions ——————————————————————

1. Using the BC for a delta function,

$$\psi_0 = e^{-q|x|},$$
$$q\frac{\hbar^2}{m}\psi(0) = \frac{\hbar^2}{mb}\psi(0),$$
$$q = \frac{1}{b},$$
$$E = -\frac{\hbar^2}{2mb^2}.$$

The normalized wave function is

$$\psi_0(x) = \sqrt{q}e^{-q|x|},$$

2.

$$\begin{aligned} E_k &= -\frac{\hbar^2}{2mb^2} + \hbar\omega, \\ k &= \sqrt{\frac{2mE_k}{\hbar^2}}. \end{aligned}$$

3. Let ψ_+ refer to the wave function for a state that asymptotically goes as e^{ikx}.

$$\begin{aligned} \psi_+(x) &= \begin{cases} e^{ikx} + Ae^{-ikx}, & x > 0 \\ Be^{ikx}, & x < 0 \end{cases}, \\ 1 + A &= B, \\ \frac{\hbar^2}{2m}\left[ik(1-A) - ikB\right] &= \frac{\hbar^2}{m}\frac{1}{b}B, \\ 1 + A &= (1-A)\frac{ik/q}{(1/b) + ik/2}, \\ ikb(1-A) - ikB(1+A) &= 2(1+A) \\ A &= \frac{-1}{1+ikb} \\ B &= \frac{ikb}{1+ikB}. \end{aligned}$$

4. The overlap of two different energy eigenstates (of the same potential) is zero!

5. Zero!