FINAL EXAM

PHYSICS 851, FALL 1998

1. (15 pt.s) Consider a spin 1/2 system. The projection operator P_z projects the component of the wave function that has positive spin along the z axis.

$$\langle\eta|P_z|\eta\rangle = |\langle z,\uparrow|\eta\rangle|^2$$

   (a) Express P_z as a matrix in the basis where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ denotes a state with positive spin along the z axis.

   (b) Write down the density matrix for a state that is an incoherent mixture of 50% positive spin along the y axis and 50% negative spin along the y axis.

   (c) If the Hamiltonian is defined as:

$$\mathcal{H} = \alpha + \beta\sigma_x$$

   Calculate the expectation of $\mathcal{H}$ for the state described in b.

2. (15 pt.s) Consider two flavors of neutrinos, the μ neutrino and the τ neutrino. Suppose that the Hamiltonian can be written as a free term plus a term that mixes the μ and τ neutrinos, which is proportional to α.

$$\mathcal{H} = \begin{pmatrix} m_\mu c^2 & 0 \\ 0 & m_\tau c^2 \end{pmatrix} + \alpha \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   If a neutrino starts as a μ neutrino, what is the probability, as a function of time, of being a τ neutrino?

3. (15 pt.s) A particle of mass m and charge e interacts with the vector potential

$$\begin{aligned} A_x &= 0 \\ A_y &= Bx \\ A_z &= 0 \end{aligned}$$

   (a) What is the magnetic field generated by the vector potential?

   (b) Find the ground state energy.

4. (15 pt.s) A particle of mass m is placed in a one-dimensional harmonic oscillator of characteristic frequency ω. The state is described by

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\{|n=0\rangle + |n=1\rangle\}$$

   at a time $t = 0$. Find the expectation of the operator X as a function of time.

5. (25 pt.s) A spherically symmetric potential has the form,

$$V(r) = \frac{\hbar^2}{2m}\beta\delta(r-a)$$

(a) As a function of the asymptotic momentum p, find the s-wave phase shift in terms of a, β and the particle's mass m. Assume $\beta > 0$.

(b) What is the cross section in the limit of zero relative momentum?

(c) Now, assume $\beta < 0$. Find the minimum magnitude $|\beta|$ necessary for the creation of a bound state. Express your answer in terms of m and a.
Hint: Solve the boundary conditions assuming the binding energy is zero.

6. (15 pt.s) This problem is set in one dimension. An electron is in the ground state of an attractive potential described by a delta function at $x = 0$. The ground state wave function is:

$$\psi(x) = \sqrt{Q}\exp -Q|x|$$

A uniform electric field is applied which varies in time as $\mathcal{E}\cos\omega t$. Assume that $\hbar\omega$ is greater than the binding energy of the potential.

(a) What is the density of states dN/dE of spin-up electrons in the continuum.

(b) Estimate the ionization rate in terms of Q, $\mathcal{E}$ and the mass of the particle m. You may leave the matrix element in the form of an integral.

7. (15 pt.s) Express the state $|j_1 = 1, j_2 = 1/2, m_1 = 0, m_2 = 1/2\rangle$ as a linear combination of eigenstates of total angular momentum, J and projection, M.