your name(s)____

Physics 841 Quiz #10 - Friday, April 12-14

1. An antenna has a length of $2L$ and is set in space. The current in the antenna is

$$I = I_0 \cos\omega t \cos kx, \qquad -L < x < L.$$

In terms of I_0, ω and L,

(a) Find the lowest possible value of k. Assume there is no charge accumulation at the ends of the wire. (Use this value for the next several questions)

(b) Find the charge per unit length as a function of time.

(c) If the speed of a charge wave moving through the antenna is v, what is ω?

(d) Estimate the average radiated power, using the first term in the multipole expansion.

(e) In what limit is the multipole expansion accurate?

(f) If the antenna is to broadcast to Earth, how should the antenna be oriented?

(g) Given this optimum orientation, describe the polarization of the light when it hits Earth relative to the orientation of the antenna.

Solutions:

a) The current must disappear at the edge, so

$$I = I_0 \cos\omega t \cos kx, \quad kL = \pi/2,\ k = \frac{\pi}{2L}$$

b)

$$\begin{aligned}
\partial_t \rho &= -\partial_x j_x, \\
\partial_t \lambda &= \partial_x I, \\
\lambda &= \frac{k}{\omega} I_0 \sin\omega t \sin kx.
\end{aligned}$$

c) Charge wave behaves as $e^{i\omega t - kx}$, $\omega/k = v$ because v is constant, so

$$\omega = kv.$$

d)

$$\begin{aligned}
p_x &= \int_{-L}^{L} dx\ x \frac{k}{\omega} I_0 \sin kx \qquad (1) \\
&= \frac{2I_0}{k\omega}, \qquad (2) \\
P &= \frac{p_x^2 \omega^4}{3} \qquad (3) \\
&= \frac{4I_0^2\omega^2}{3k^2} \qquad (4)
\end{aligned}$$

Don't use k from previous problem, unless it is $k = \omega c$, not $k = \omega v$.

e) $v << c$

f) Antenna axis should be perpendicular to direction of *Earth*

g) Polarization of $\vec{E}$ field is parallel to antenna axis.

2. A small conducting sphere of radius a is placed in a region of originally uniform electric field $\vec{E} = E_0\hat{z}$.

   (a) Find the induced dipole moment of the sphere.

   (b) A long wavelength ($\lambda >> a$) electromagnetic wave with intensity (power/area) S traveling in the z direction and polarized in the x direction passes by the sphere. Find the induced dipole moment of the wave as a function of time.

   (c) Find the power radiated by the dipole.

   (d) Find the elastic scattering cross section of the sphere.

Solutions:

a)

$$\begin{aligned}
\Phi &= E_0 r\cos\theta - \frac{A}{r^2}\cos\theta, \quad A = E_0R^3 \text{ to satisfy BC,} \\
E_r(R,\cos\theta) &= -\left.\frac{\partial\Phi}{\partial r}\right|_{r=R} \\
&= E_0\left[-\cos\theta - 2\cos\theta\right] = -3E_0\cos\theta, \\
\sigma &= \frac{E_r}{4\pi} = \frac{3E_0\cos\theta}{4\pi}, \\
p_z &= R^2\int d\cos\theta\ (R\cos\theta)\sigma \\
&= E_0R^3.
\end{aligned}$$

b)

$$\begin{aligned}
S &= \frac{E_0^2}{4\pi}, \\
E_0 &= \sqrt{4\pi S}, \\
p_z(t) &= R^3\sqrt{4\pi S}\cos(\omega t),
\end{aligned}$$

c)

$$\begin{aligned}
P &= \frac{p_z^2\omega^4}{3} \\
&= R^6(4\pi S)\frac{\omega^4}{3} \\
&= \frac{4}{3}\pi R^6\omega^4 \\
&= \frac{64\pi^5R^6}{3\lambda^4}S.
\end{aligned}$$

d)

$$\begin{aligned}
\sigma &= P/S \\
&= \frac{64\pi^5R^6}{3\lambda^4}
\end{aligned}$$