Statistical Mechanics
Review Problem

Daniel Douglas and Daniel Rhodes

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1.

Show that the Liouville theorem,

$$d\left[\Delta\Omega\right] = d\left[\frac{1}{(2\pi\hbar)^3}\Pi_i \Delta x_i \Delta p_i\right] = 0,$$

in a fixed time step, follows from Hamilton's equations and the equality of mixed partials:

$$\frac{\partial H}{\partial p_i} = \frac{dx_i}{dt}$$
$$\frac{\partial H}{\partial x_i} = -\frac{dp_i}{dt}$$
$$\frac{\partial^2 H}{\partial x_i \partial p_i} = \frac{\partial^2 H}{\partial p_i \partial x_i}$$

2.

A beam of particles has a transverse width of 1 mm and its transverse momentum has a spread of 1 keV/c at its focal plane. Farther down the beamline, the width of the beam is measured to be 5 mm. Use Liouville's theorem to find the spread in momentum at this point on the beamline.