your name(s)_____

Physics 831 Quiz #7 - Friday, Nov. 2, 2017
Work in groups of 2.

Consider a two-dimensional array of N coupled two-dimensional harmonic oscillators, i.e., the oscillators only move in the $x-y$ plane.

1. At low temperature, the specific heat per oscillator can be expressed as:

$$C_V = \frac{1}{N}\frac{dE}{dT} = \alpha T^n.$$

What is the power n? 2.

2. What is the specific heat at high temperature? 2

3. If one doubles the speed of sound in (1) the parameter α will:
   (a) quadruple
   (b) double
   (c) increase by $\sqrt{2}$
   (d) stay the same
   (e) fall by $1/\sqrt{2}$
   (f) fall by 1/2
   (g) fall by 1/4.

4. Assume the oscillators each have mass m, are arranged in a square lattice, and are coupled by springs of spring constant k. Derive the speed of sound in terms of m, k, and the number of oscillators per area, ρ.

$E \sim T^3$, $C_V \sim T^2$, $n = 2$

$E \sim \frac{1}{(2\pi\hbar)^3}\int \rho\, d\rho \cdot (\rho c_s) e^{-\rho c_s/T}$

$\sim c_s \left(T/c_s\right)^3 \sim \frac{1}{c_s^2}$

$\frac{1}{\rho}\delta\rho_{i+1/2} = -\frac{\delta l_{i+1/2}}{a}$, a = latt. sep.

$m\frac{dv_i}{dt} = +k\left(\delta l_{i+1/2} - \delta l_{i-1/2}\right)$

$= -\frac{ka}{\rho}\cdot a\frac{\partial \rho}{\partial x} = -\frac{ka^2}{\rho}\frac{\partial \rho}{\partial x}$

$\frac{\partial \rho}{\partial t} = -\rho\frac{\partial v}{\partial x}$

$\frac{\partial^2 \rho}{\partial t^2} = +\rho\left[\frac{ka^2}{m\rho}\right]\frac{\partial^2 \rho}{\partial x^2} = \frac{k}{m\rho}\frac{\partial^2 \rho}{\partial x^2}$

$c_s^2 = k/m\rho$