You may work in groups of three. Turn in one solution for the group.

1. Consider two neutrons (could have spin projections $m = -1/2, 1/2$) that are in one of three single-particle energy levels, $-\epsilon$, 0 and ϵ.

   (a) (2 pts) List the system energy levels and their degeneracies

   (b) (2 pts) Calculate the average system energy as a function of the temperature

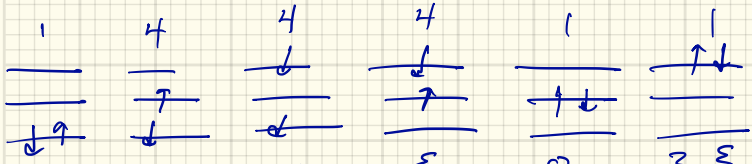
   (c) (2 pts) Calculate the entropy as a function of the temperature

   (d) (2 pts) What is the average energy at $T = 0$ and at $T = \infty$?

   (e) (2 pts) What is the entropy at $T = 0$ and at $T = \infty$?

   (f) (10 pts) Assuming a chemical potential μ and temperature T, i.e. the number of particles is not fixed, find the average number of particles in the system.

a)



b) $Z = e^{2\varepsilon/T} + 4e^{\varepsilon/T} + 5 + 4e^{-\varepsilon/T} + e^{-2\varepsilon/T}$

$\langle E \rangle = \dfrac{-2\varepsilon e^{2\varepsilon/T} - 4\varepsilon e^{\varepsilon/T} + 4\varepsilon e^{-\varepsilon/T} + 2\varepsilon e^{-2\varepsilon/T}}{Z}$

c) $S = \ln Z + \beta \langle E \rangle$

d) $\langle E \rangle_{T=0} = -2\varepsilon, \quad \langle E \rangle_{T=\infty} = 0$

e) $S_{T=0} = 0, \quad S_{T=\infty} = \ln(15)$

f) $\langle N \rangle = 2\dfrac{e^{(\varepsilon+\mu)/T}}{1+e^{(\varepsilon+\mu)/T}} + 2\dfrac{e^{\mu/T}}{1+e^{\mu/T}} + 2\dfrac{e^{(-\varepsilon+\mu)/T}}{1+e^{(-\varepsilon+\mu)/T}}$