your name____________________________

*Physics 831 Quiz #6 - Monday, Oct. 31 2016*

Work in groups of one. This is a **CLOSED BOOK, CLOSED NOTE** quiz.

1. (10 pts) At time $t_0$, the density of ink molecules in a fluid is given by the expression,

$$\rho(x, t = 0) = \rho_0 + a \sin(kx).$$

The ink molecules diffuse according to a diffusion constant $D$. Find the density of ink molecules as a function of time?

2. An ideal gas of particles of mass $m$ is initially: at a uniform temperature $T_0$, has zero collective velocity, and as far as the eye can see, the number density profile initially has an exponential profile in the $x$ direction:

$$\rho(x, t = 0) = \rho_0 e^{-x/\lambda}.$$

The gas then expands expands hyrodynamically. For a solution, assume the velocity is independent of $x$ and rises linearly in time, $v = \alpha t$.

(a) (5 pts) Find $\alpha$.

(b) (5 pts) What is the temperature, $T(x, t)$?

(c) (5 pts) What is the density profile, $\rho(x, t)$?

The equations of hydrodynamics are:

$$\begin{aligned}
\frac{D}{Dt} v &= -\frac{\partial_x P}{m\rho}, \\
\frac{D}{Dt} \rho &= -\rho \partial_x v, \\
\frac{D}{Dt} &= \partial_t + v \partial_x.
\end{aligned}$$

---

Solutions:

1.

$$\begin{aligned}
\partial t \rho &= D \partial_x^2 \rho, \\
\rho &= \rho_0 + A(t) \sin(kx), \\
\dot{A} &= -k^2 D A, \\
A(t) &= a e^{-Dk^2 t}.
\end{aligned}$$

2.

$$\begin{aligned}
(a)\ \partial t v &= \alpha = -\frac{T}{m} \frac{\partial_x \rho}{\rho}, \\
\alpha &= \frac{T}{m\lambda}. \\
(b)\ v &= \alpha t, \quad \text{independent of } x, \\
\nabla \cdot \mathbf{v} &= 0, \quad \text{no expansion or cooling} \\
T &= T_0, \ \text{stays constant.} \\
(c)\ \frac{D}{Dt} \rho &= \rho \nabla \cdot \mathbf{v} = 0, \quad \rho \text{ stays fixed if you move with fluid} \\
\rho(x, t) &= \rho(x - \alpha t^2/2, t = 0) \\
&= \rho_0 e^{-(x - \alpha t^2/2)/\lambda}.
\end{aligned}$$

The profile of the fluid moves in the positive $x$ direction with constant acceleration.