your name____________________________

Physics 831 Quiz #1 - Friday, Sep. 6

1. (5 pts) Consider $N_s \to \infty$ systems. Each system has states i populated with probability p_i. The number of systems in state i are $n_i = p_i N_s$. If the ignorance I is defined as:

$$I = \frac{N_s!}{\prod_i n_i!},$$

and if the entropy is defined as

$$S \equiv \frac{\ln I}{N_s},$$

show that

$$S = -\sum_i p_i \ln p_i.$$

You may wish to know that $\ln N! \to N \ln N - N + \cdots$.

your name____________________________

2. (4 pts) Consider a spin-1 particle (could have $m = 1, 0, -1$) that is in one of two energy levels, 0 and ϵ, i.e. the energy is independent of m and there are 6 total states possible.

   (a) What is the entropy when $T = 0$?

   (b) What is the entropy when $T \to \infty$?

3. (4 pts) Fill out the following table. If a system adjusts itself to maximize the universe's entropy, which of these quantities will be either a maxima or minima for having the quantities in the left column fixed (or in the case of μ or T being connected to baths with those quantities fixed).

Fixed	Min. or Max.	Maximized or minimized quantity
V, Q, E	max	S
V, Q, T		
V, μ, T		
$V, \alpha \equiv -\mu/T, E$		
P, Q, T		

Some potentially useful information: $F = E - TS$, $P = (TS - E + \mu Q)/V$, $G = PV + E - TS$, $H = E + PV$.

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4. (12 pts) Consider 2 identical bosons (A given level can have an arbitrary number of particles) in a 2-level system, where the energies are 0 and ϵ. In terms of ϵ and the temperature T, calculate:

   (a) The partition function Z_C

   (b) The average energy $\langle E \rangle$. Also, give $\langle E \rangle$ in the $T = 0, \infty$ limits.

   (c) The entropy S. Also give S in the $T = 0, \infty$ limits.

   (d) Now, connect the system to a particle bath with chemical potential $\mu < 0$. Calculate $Z_{GC}(\mu, T)$. Find the average number of particles, $\langle N \rangle$ as a function of μ and T. Also, give the $T = 0, \infty$ limits.
   Hint: For a grand-canonical partition function of non-interacting particles, one can state that $Z_{GC} = Z_1 Z_2 \cdots Z_n$, where Z_i is the partition function for one single-particle level, $Z_i = 1 + e^{-\beta(\epsilon_i - \mu)} + e^{-2\beta(\epsilon_i - \mu)} + e^{-3\beta(\epsilon_i - \mu)} \cdots = 1/(1 - e^{-\beta(\epsilon - \mu)})$, where each term refers to a specific number of bosons in that level.