Physics 831 Quiz #7 - Friday, Oct. 17

1. Assume the free energy for a complex field in ONE dimensions is given by:

$$F = \int dx \frac{1}{2}\left(A\,|\phi|^2 + \kappa\,|\partial_x\phi|^2\right).$$

   Define the correlation Γ as

$$\Gamma(x) \equiv \langle \phi^*(0)\phi(x)\rangle.$$

   Fourier transforms in one dimensions are defined by:

$$\tilde{\phi}_k \equiv \frac{1}{\sqrt{L}}\int dx\ e^{ikx}\phi(x), \qquad \phi(x) = \frac{1}{\sqrt{L}}\sum_k e^{-ikx}\phi_k.$$

   (a) Calculate $\Gamma(x)$.

   (b) What is the critical exponent ν?
   The correlation length ξ behaves as $\xi \sim t^{-\nu}$ as $t = (T - T_c)/T_c \to 0$.

2. The partition function for a two-dimensional spin system in a magnetic field is:

$$Z = \text{Tr}\ \exp\left\{-\beta\int dxdy\ \left[h_0(x,y) - \mu B m(x,y)\right]\right\},$$

   where $m(x,y)$ is the magnetization density. After some work, the correlation function is determined to be,

$$\Gamma(x,y) = \langle (m(0) - \bar{m})(m(x,y) - \bar{m})\rangle = \Gamma_0 e^{-(x^2+y^2)/2R^2},$$

   where $\bar{m}$ is the average magnetization density. In terms of μ, T, Γ_0 and R, find the susceptibility,

$$\chi \equiv \frac{d\bar{m}}{dB}.$$

3. Two phase transitions have different critical exponents. You can safely conclude, (circle all correct answers)

   - They come from different universality classes
   - They have different dimensionalities
   - They have different order parameters

4. In a two-dimensional magnetic substance, spins become aligned when the temperature is lowered below the critical temperature, even though the material is in a field-free region. This is an example of: (circle all correct answers)

   - spontaneous symmetry breaking
   - explicit symmetry breaking
   - breaking a continuous symmetry
   - breaking a discrete symmetry