Physics 831 Quiz #3 - Wednesday, Oct. 14

1. A molecule of mass m has internal excitations consistent with that of a **TWO-DIMENSIONAL** harmonic oscillator with tightly packed levels, $\hbar\omega << T$. Initially, a gas of such molecules is at temperature T_i before expanding and cooling adiabatically to a temperature T_f . Neglect quantum degeneracy of the momentum states for the following questions. (HINT: A two-dimensional oscillator behaves like two independent one-dimensional oscillators.)

   (a) Find the average energy per particle in terms of the temperature T, the mass m and $\hbar\omega$.

   (b) Derive an expression for the initial entropy per particle in terms of m, T_i, $\hbar\omega$ and the initial density ρ_i.

   (c) After adiabatically cooling to temperature T_f , find the density ρ_f. Give answer in terms of T_i, T_f and ρ_i.

2. (Extra Credit) Consider a fluid with an ideal gas equation of state, $P = \rho T$, and a mass density $\rho_m = m\rho$. The energy density is that of a non-interacting gas, $\epsilon = (3/2)\rho T$. At time $t = 0$, the temperature is uniform, $T = T_0$, and the collective velocity is zero everywhere, but the density varies exponentially (as far as the eye can see),

$$\rho(x, t = 0) = \rho_0 e^{-x/\lambda}.$$

Solve for the evolution of the density $\rho(x,t)$, the collective velocity $v(x,t)$, and the temperature $T(x,t)$, by solving the equations:

$$\begin{aligned}
(\partial_t + v\partial_x)v(x,t) &= -\frac{\partial_x P(x,t)}{m\rho(x,t)}, \\
(\partial_t + v\partial_x)\rho(x,t) &= -\rho(x,t)\partial_x v(x,t), \\
(\partial_t + v\partial_x)\epsilon(x,t) &= -[P(x,t) + \epsilon(x,t)]\partial_x v.
\end{aligned}$$

Hint: Use your intuition and assume **SIMPLE** forms for the time and spatial dependence of $v, \rho, T, \cdots$.