Physics 831 Quiz #4 - Friday, Sep. 26

1. Suppose someone has calculated the coefficients, B_i, in the following expression for the grand canonical partition function,

$$\frac{P}{T} = \frac{1}{V} \ln Z = \rho_0 \left(B_1 e^{\beta\mu} + B_2 e^{2\beta\mu} + B_3 e^{3\beta\mu} \cdots \right),$$

where $B_1 = 1$, and B_i are functions of T, i.e., they don't depend on μ.

   (a) Find the coefficients, C_i, for $i = 1, 2, 3$, in an expansion of the density,

$$\rho = \rho_0 \left(C_1 e^{\beta\mu} + C_2 e^{2\beta\mu} + C_3 e^{3\beta\mu} \cdots \right).$$

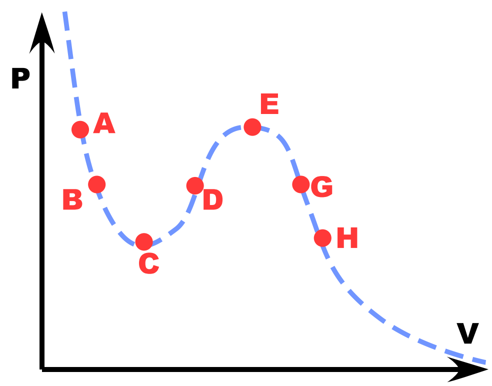
   Express C_i in terms of B_i.

   (b) Consider the virial expansion,

$$P = \rho T \left(1 + A_2 \frac{\rho}{\rho_0} + A_3 \frac{\rho^2}{\rho_0^2} + \cdots \right)$$

   Express A_2 in terms of $B_1, B_2 \cdots$.

2. Consider the isotherm (fixed temperature) on the $P - V$ diagram below. List all pairs of points that coexist at equilibrium.



3. A brilliant colleague of yours derives a coexistence curve by plotting an isobar (constant pressure) in a T vs. x graph. She states that the coexistence condition is that the shaded areas above and below the lines are equal. If she is correct, what was the intrinsic quantity x? For example, x might be ρ, ρ/T, S/V, the free energy density $\cdots$
Hint: $TdS = dE + PdV - \mu dN$, $TS = E + PV - \mu N$.