Physics 831 Quiz #2 - Friday, Sep. 12

1. Consider a massless **ONE**-dimensional gas of bosons with spin degeneracy $(2S+1)$. Assuming zero chemical potential, find the coefficients A and B in the expressions below for the pressure,

$$P = A(2S+1)T^2.$$

Note that in 1-d pressure is defined by $dE = -PdL + TdS$, and feel free to set $c = 1$ and express your answer in terms of Riemann-Zeta functions.

2. Show that if the previous problem is repeated for Fermions that:

$$A_{Fermions} = \gamma A_{Bosons},$$

and find the constant γ.

3. Consider a **MASSLESS TWO**-dimensional gas of spinless bosons. **SHOW** whether the system can undergo Bose condensation.

4. Imagine you live in a strange world where, instead of $\epsilon = \sqrt{p^2+m^2}$, the energy of a particle is represented through the momentum by some arbitrary relation $\epsilon(p)$. Beginning with the expression for the pressure,

$$\frac{PV}{T} = \ln Z_{GC} = \frac{V}{(2\pi\hbar)^3}\int d^3p \;\ln\left[1 + e^{-(\epsilon-\mu)/T} + e^{-2(\epsilon-\mu)/T} + e^{-3(\epsilon-\mu)/T} + \cdots\right],$$

derive an expression for the pressure of a gas of such bosons in terms of the phase space filling factor,

$$f(\epsilon) = \frac{e^{-(\epsilon-\mu)/T}}{1 - e^{-(\epsilon-\mu)/T}}.$$

The expression should have the form:

$$P = \int d^3p \; f(\epsilon_p) \cdot [\text{Some function of } p].$$

5. For the problem above, show that in the low density limit that one still obeys the ideal gas law:

$$P = \rho T.$$