Physics 831 Quiz #9 - Wednesday, Nov. 7

YOUR NAME:_____

Consider the 1-d ising model on $N \to \infty$ spins, where each spin can have $\sigma_i = \pm 1$ and the Hamiltonian is:

$$H = -\sum_i J\sigma_i\sigma_{i+1} - \mu B\sigma_i,$$

and the partition function is given by:

$$\ln Z = N \ln(\lambda),\ \lambda = e^{\beta J}\cosh(\beta\mu B) + \sqrt{e^{2\beta J}\sinh^2(\beta\mu B) + e^{-2\beta J}}.$$

Consider the case where $B = 0$.

1. What is the average spin per particle $\langle \sum_i \sigma_i \rangle / N$ when $B = 0$?
2. What is the fluctuation of the net spin (assume N spins)? The fluctuation is defined as:

$$F = \frac{1}{N}\left\langle \left(\sum_i \sigma_i\right)^2 \right\rangle - \frac{1}{N}\left\langle \sum_i \sigma_i \right\rangle^2$$