Physics 831 Quiz #1 - Friday, Sep. 7

1. Consider two single-particle levels of energy $-\epsilon$ and ϵ, which are populated by three electrons (each of which can be either spin-up or spin down). The system is then attached to a heat bath characterized by a temperature T. In terms of T and ϵ, find
   (a) the average energy
   (b) the $T \to 0$ limit of (a)
   (c) the $T \to \infty$ limit of (a)
   (d) the entropy
   (e) the $T \to 0$ limit of (d)
   (f) the $T \to \infty$ limit of (d)

2. Consider two single-particle levels of energy $-\epsilon$ and ϵ, which can be populated by indistinguishable spin-zero bosons. The system is attached to a bath that can exchange particles and energy and is characterized by a temperature T and a chemical potential μ. In terms of μ,T and ϵ, find (assume $\mu < -\epsilon$)
   (a) an expression for the average number of particles.
   (b) the $T \to 0$ limit of (a)
   (c) the $T \to \infty$ limit of (a)

3. Beginning with the grand canonical partition function $Z_{GC}(\alpha = -\beta\mu, \beta, V)$, derive an expression for the specific heat at constant volume, $C_V \equiv dE/dT|_{N,V}$ in terms of derivatives of Z_{GC}.

4. Beginning with:

$$TdS = dE + PdV - \mu dQ,$$

prove:

$$\left.\frac{\partial E}{\partial \mu}\right|_{V,T} = T \left.\frac{\partial Q}{\partial T}\right|_{V,\beta\mu}$$