Physics 831 Quiz #2 - Friday, Sep. 22

1. Consider one-dimensional classical non-relativistic particles acting through a potential,

$$V(x) = V_0 \exp\{x^2/(2L^2)\}.$$

Using the equipartition and virial theorems, show that

$$\left\langle x^2 V(x) \right\rangle = TL^2.$$

2. Consider a TWO-dimensional non-relativistic gas of spin-1/2 Fermions of mass m at temperature T confined to a area A.

   (a) Find the density of single-particle states $D(\epsilon)$.

   (b) Find the the change of the chemical potential $\delta\mu(T, \rho)$ necessary to maintain a constant density per unity length, ρ, while the temperature is raised from zero to T. Give answer to order T^2 as a function of μ, T, m and $\hbar$.

3. Consider a massless TWO-dimensional gas of spin-1/2 fermions at zero chemical potential. The energy per unit area has the form,

$$\frac{E}{A} = BT^3.$$

Derive the coefficient B. You can leave answer in terms of the Riemann-Zeta function, $\xi(n) = \sum_{i=1}^{\infty} i^{-n}$.