Physics 831 Quiz #1 - Friday, Sep. 8

1. Beginning with the fact that the number of ways to arrange N_s systems into the levels $i = 1, 2...m$ is:

$$I = \frac{N_s!}{n_1! n_2! \cdots n_m!},$$

where n_i is the number of systems in state i, show that the entropy defined as:

$$S \equiv \frac{1}{N_s} \ln I = -\sum_i p_i \ln p_i, \quad \text{where } p_i \equiv n_i / N_s.$$

In the proof, assume $N_s \to \infty$ and use Stirlings formula, $\ln N! \approx N \ln N - N \cdots$.

2. Consider two single-particle levels of energy $-\epsilon$ and ϵ, which can be populated by indistinguishable Fermions. The system is attached to a bath that can exchange particles and energy and is characterized by a temperature T and a chemical potential μ. In terms of μ,T and ϵ, find

   (a) an expression for the average number of particles.

   (b) the $T = 0$ limit of (a)

   (c) the $T = \infty$ limit of (a)

3. Beginning with:

$$TdS = dE + PdV - \mu dQ,$$

prove:

$$\left.\frac{\partial T}{\partial \mu}\right|_{V,S} = -\left.\frac{\partial Q}{\partial S}\right|_{V,\mu}$$