Physics 831 Practice Quiz #8 - Friday, Dec. 5

YOUR NAME:_____

1. Brilliant Betty calculates the pressure in the grand canonical ensemble and finds,

$$P(T,\mu) = A\alpha T \frac{e^{\beta\mu}}{(1-\alpha e^{\beta\mu})}, \qquad \alpha \equiv \frac{(mT)^{3/2}}{\hbar^3}.$$

   (a) Calculate the integrated correlation function,

$$F \equiv \int d^3r \langle (\rho(r=0) - \langle\rho\rangle)(\rho(r) - \langle\rho\rangle)\rangle,$$

   in terms of T, μ, α, the constant A and the volume V. Use the fact that

$$Z = \mathrm{Tr}\ \exp\left\{-\beta H + \beta\mu \int d^3r\ \rho(\mathbf{r})\right\}.$$

2. Consider a set of harmonic oscillators arranged in the $x-y$ plane in a square lattice with N oscillators in area A. Further, assume the oscillators move only in the z direction. Assume the fundamental frequency of the oscillators is ω_0 and that after coupling the oscillators, the speed of sound is c_s.

   (a) For small T, derive the specific heat per volume, $C = (1/V)d\langle E\rangle/dT$ in terms of N, A, T, c_s, $\hbar$, ω_0 and Newton's universal constant of gravitation G.

   (b) Do the same for the limit of $T \to \infty$.

3. Consider the following three-dimensional field theories where the order parameter is a function of x, y and z:

   (A) A theory where $\phi(r)$ is the order parameter and is a real field with a free-energy density,

   $$f = A\phi^2 + B\phi^4 + \frac{1}{2}\kappa|\nabla\phi|^2.$$

   (B) A theory where $\phi(r)$ is the order parameter and is a complex field with a free-energy density,

   $$f = C\phi^*\phi + D(\phi^*\phi)^2 + \frac{1}{2}\kappa|\nabla\phi|^2.$$

   (C) A theory where the magnetization density $\vec{m}(x)$ is the order parameter, and $\vec{m}$ can point in any of three directions, and the free-energy density is

   $$f = E\sum_i m_i m_i + F(\sum_i m_i^2)^2 + \frac{1}{2}\kappa\sum_i |\nabla m_i|^2.$$

   (D) The same as above, except where the magnetization can only point parallel to the z axis, i.e., $m_x = m_y = 0$.

   (a) Which pairs of the the above theories, e.g. (A & C), will be of the same universality class.

   (b) Which theories might lead to spontaneous breaking of a continuous symmetry?

   (c) Which theories might lead to Goldstone bosons?