Physics 831 Practice Quiz #2 - Monday, Dec. 4

YOUR NAME:____________________________

1. Consider a two-dimensional Fermi gas of spin-1/2 particles of mass m at low temperature T and fixed density (number per area) ρ. Here the chemical potential will be measured relative to $p = 0$, i.e.,

$$f = \frac{e^{-\beta(p^2/2m-\mu)}}{1 + e^{-\beta(p^2/2m-\mu)}}$$

   (a) What is the single-particle density of states per volume $D(\epsilon)/V$ at $\epsilon = \mu$?

   (b) What is the net kinetic energy per particle at $T = 0$? Give answer in terms of $\epsilon_f = \mu$ and other given constants.

   (c) What is the increase in the excitation energy per particle to order T^2? Give answer in terms of ρ and D/V.

2. Consider a Hamiltonian for a particle moving in one dimension,

$$H = \sqrt{m^2 + p^2} - A\ln(x/x_0) + Bx, \quad A > 0, B > 0.$$

   which confines the particle to $0 < x < \infty$. Using the generalized equipartition theorem, or the virial theorem, find the average value of $\langle x \rangle$ as a function of T.