Question: A positively charged particle (charge q), is accelerated by passing through a voltage V_K. The acceleration leaves the charged particle with kinetic energy $KE = qV_K$. The particle approaches a small conducting sphere of radius R, which is at a positive voltage V_{sphere}. The potential energy of the particle at the surface is thus qV_{sphere}.

1. $KE = qV_K$

a) What is the minimum accelerating voltage, $V_{K\,min}$, necessary for the particle to reach the sphere?

$E = KE + PE$

$E = qV_K + qV_{sphere}$ → Conservation of Energy: The total mechanical energy of the system is conserved in the absence of non-conservation forces

$\frac{qV_K}{q} = \frac{qV_{sphere}}{q}$ ← So $KE = PE$

$V_K = V_{sphere}$ → $\boxed{V_{Kmin} = V_{sphere}}$

↓ The charged particles KE after being accelerated through a voltage V_K is equal to the potential energy it gains when approaching the positively charged sphere

b) What is the cross section for colliding with the sphere? Assume $V_K > V_{Kmin}$.

Express your answer in terms of R, V_K and V_{sphere}.

Conservation of Energy

$qV_K = qV_{sphere}$

Angular momentum is conserved: $L = mvr$ → distance from the center of the sphere

$mv_0 = mv_f$ (incoming velocity; speed when it grazes the sphere)

$mv_0(A) = mv_f R_{sphere}$ → impact parameter $A = R\frac{v_f}{v_0}$ = max collision

$A = \frac{v_f}{v_0} R_{sphere}$

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + qV_{sphere}$ (↗ electric potential) → the initial KE equals the sum of the final KE and the potential energies

$\frac{1}{2}mv_0^2 = qV_K$ (initial; initial)

$v_0^2 = \frac{2qV_K}{m}$

$\frac{1}{2}mv_f^2 = q(V_K - V_{sphere})$ (final; final)

$v_f^2 = \frac{2q}{m}(V_K - V_{sphere})$

$A = R_{sphere}\sqrt{\frac{(2q/m)V_K - (2q/m)V_{sphere}}{(2q/m)V_K}}$

$= R_{sphere}\sqrt{\frac{V_K - V_{sphere}}{V_K}}$

Cross section → $\sigma = \pi A^2$

$= \pi\left(R^2\left(1 - \frac{V_{sphere}}{V_K}\right)\right)$