Consider the effective potential for an attractive force: $F = -\alpha/r^{3/2}$

Consider a particle of mass m with angular momentum L.

a) What is the radius of a circular orbit?

$$U_{eff} = \underbrace{L^2/2mr^2}_{\text{conservation of angular mom./ centrifugal potential}} - \underbrace{2\alpha/r^{1/2}}_{\text{potential from force}}$$

$$d/dr\, U_{eff} = -L^2/mr^3 + \alpha/r^{3/2} = 0$$

$$L^2/mr_{mm}^3 = \alpha/r_{mm}^{3/2}$$

$$r_{mm} = \left(L^2/m\alpha\right)^{2/3}$$

b) What is the angular frequency, $\dot{\theta}$, of the orbit?

$$\frac{(mr_{mm}^2\dot{\theta})^2}{mr_{mm}^3} = \frac{\alpha}{r_{mm}^{3/2}}$$

$$mr_{mm}\dot{\theta}^2 = \alpha/r_{mm}^{3/2}$$

$$\dot{\theta} = \sqrt{\frac{\alpha}{mr_{mm}^{5/2}}} = \frac{\alpha^{4/3}m^{1/3}}{L^{5/3}}$$

c) What is k_{eff} at the minimum?

- Curvature of U_{eff}

$$k_{eff} = \frac{d^2}{dr^2} U_{eff} \Big|_{r=r_{min}} = \frac{3L^2}{m r_{min}^4} - \frac{3\alpha}{2 r_{min}^{5/2}}$$

d) What is the angular frequency for small vibrations about the minimum?

$$\omega_r = \sqrt{k_{eff}/m} = \sqrt{\frac{3L^2/m r_{min}^4 - 3\alpha/2 r_{min}^{5/2}}{m}}$$

$$= \frac{\sqrt{3}\, \alpha^{4/3} m^{1/3}}{\sqrt{2}\, L^{5/3}} = \dot{\theta}\sqrt{\frac{3}{2}}$$