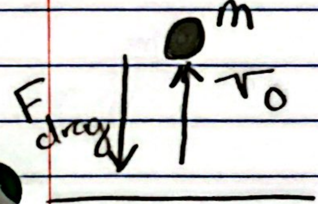
Tingliang Sun
Kyndrel Allen

PHY 321

Projectile motion Problem

A projectile with mass m is launched vertically upward from ground level at time $t=0$ with initial velocity v_0 and is acted upon by gravity and air resistance. assume drag force is proportional to velocity, with a drag coefficient k. Derive an expression for the time, when the projectile achieves its max height



No motion in the x-direction so net force can be expressed by

$$F_{net} = ma$$

acceleration a is the sum of gravitational acceleration and acceleration due to air resistance

$$a = g - \frac{F_{drag}}{m}$$

F_{drag} is proportional to the velocity and goes in the opposite direction thus

$$F = -F_{drag} - F_g$$
$$F = -kv - mg$$

We can then set up the differential equation to ultimately solve dt:

$$m \cdot dv/dt = -kv - mg$$
$$-dt = \frac{dv}{\frac{kv}{m} + g}$$

after setting up the differential equation we can integrate both sides

$$\int_{v_0}^{0} \frac{1}{\frac{kv}{m}+g}\, dv = -\int_{0}^{t} 1\, dt$$

next can use u substitution

$u = g + k/m\, v$

$du = \frac{k}{m} \cdot dv$

when $v = u$, $u = g$

$$\int_{v_0}^{0} g+\frac{k}{m} v\, dv = \frac{-m}{k} \int_{g+\frac{k}{m}v_0}^{g} \frac{1}{u}\, du$$

Ultimately the integration can be solved for time.

$$t = \frac{m}{k} \cdot \ln\left(\frac{\frac{k}{m}v_0 + g}{g}\right)$$