your names______________________________________

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Physics 321 Exercise, Nov. 27, 2023

Turn in your own assignment, but discuss with all yourn neighbors.

1. (10 pts) Consider a mass m that moves according to the following potential,

$$V(x, y, z) = \frac{kz^2}{(z^4 + a^4)(x^2 + y^2)}$$

Which of the following quantities are conserved? (Momentum is $\vec{p}$, angular momentum is $\vec{L}$ and the energy is E)
Circle the conserved quantities.

a) p_x
b) p_y
c) p_z
d) $p_x + p_y$
e) $p_x - p_y$
f) $p_x + p_z$
g) $p_x - p_z$
h) $p_y + p_z$
i) $p_y - p_z$
j) L_x
k) L_y
l) L_z
m) $L_x + L_y$
n) $L_x - L_y$
o) $L_x + L_z$
p) $L_x - L_z$
q) $L_y + L_z$
r) $L_y - L_z$
s) E
t) $|\vec{p}|$
u) $|\vec{L}|$

Solution: There is rotational symmetry about the z axis. So only L_z and E are conserved (for any trajectory).

2. (10 pts) Consider a mass m that moves according to the following potential,

$$V(x, y, z) = \frac{k}{(x+y)^2}.$$

Which of the following quantities are conserved? (Momentum is $\vec{p}$, angular momentum is $\vec{L}$ and the energy is E)
Circle the conserved quantities (for any trajectory).

a) p_x
b) p_y
c) p_z
d) $p_x + p_y$
e) $p_x - p_y$
f) $p_x + p_z$
g) $p_x - p_z$
h) $p_y + p_z$
i) $p_y - p_z$
j) L_x
k) L_y
l) L_z
m) $L_x + L_y$
n) $L_x - L_y$
o) $L_x + L_z$
p) $L_x - L_z$
q) $L_y + L_z$
r) $L_y - L_z$
s) E
t) $|\vec{p}|$
u) $|\vec{L}|$

Solution: Define the variables

$$\tilde{x} = (x+y)/\sqrt{2}, \quad \tilde{y} = (x-y)/\sqrt{2}, \quad \tilde{z} = z$$
$$\tilde{p}_x = m(\dot{x}+\dot{y})/\sqrt{2}, \quad \tilde{p}_y = m(\dot{x}-\dot{y})/\sqrt{2}, \quad \tilde{p}_z = m\dot{z}.$$

The potential and kinetic energies are

$$T = \frac{m}{2}(\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \dot{\tilde{z}}^2),$$
$$V = \frac{k}{2\tilde{x}^2}.$$

One can then see that the conserved quantities are:
$E, \tilde{p}_y, \tilde{p}_z, \tilde{L}_x$, and linear combinations of $\tilde{p}_y$ and $\tilde{p}_z$.
In the original coordinate system these are: $E, (p_x - p_y)/\sqrt{2}, p_z, (L_x + L_y)/\sqrt{2}$ and linear combinations of $(p_x - p_y)$ and p_z. Thus you should circle: E, $(p_x - p_y)$, p_z and $L_x + L_y$.